Joint Channel Estimation and Co-Channel Interference Mitigation in Wireless Networks Using Belief Propagation

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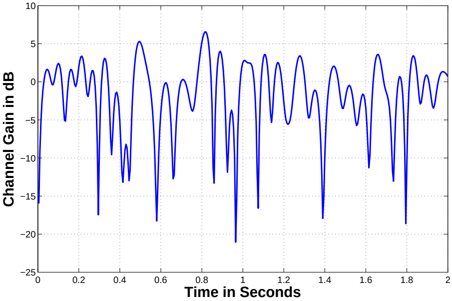
Northwestern University

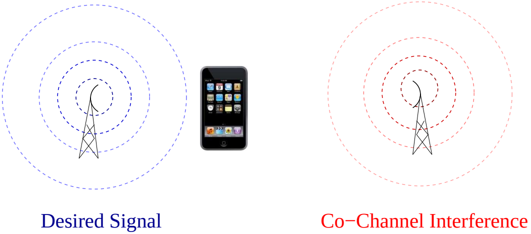
May. 21, 2008

Key Challenges in Wireless Networks

Channel Uncertainty (fading)



Interference



System Model

Interference:

$$Y_i = H_i X_i + H_i' X_i' + N_i \qquad i = 1 \dots n$$

- $N_i \sim \mathcal{CN}(0, \sigma_N^2)$,
- $H_i \sim \mathcal{CN}(0, \sigma_H^2)$, $H_i' \sim \mathcal{CN}(0, \sigma_{H'}^2)$,
- $X_i, X_i' \in \{\pm 1\}$.

Fading process

$$H_i = \alpha H_{i-1} + \sqrt{1-\alpha^2}\, W_i$$
$$H_i' = \alpha H_{i-1}' + \sqrt{1-\alpha^2}\, W_i' \qquad (\alpha \lesssim 1)$$

Problem

Given $Y_1 \dots Y_n$ and known pilots in $X_1 \dots X_n$, detect the remaining unknown symbols in $X_1 \dots X_n$.

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Observation

- $H_i' X_i'$ ($i = 1 \dots n$) is not Gaussian process; treating it as Gaussian leads to poor performance.

We propose

- Using statistical inference on factor graph to solve this problem.

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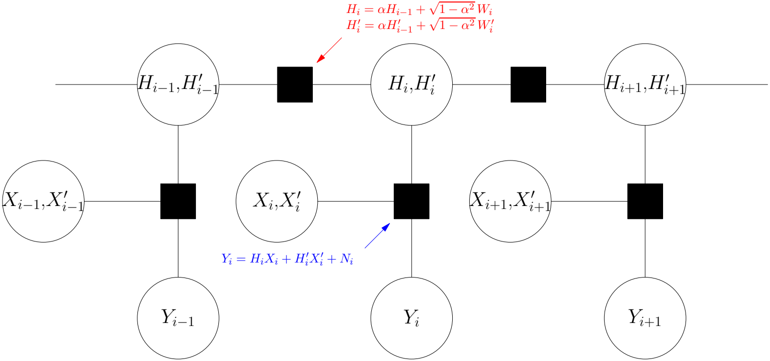
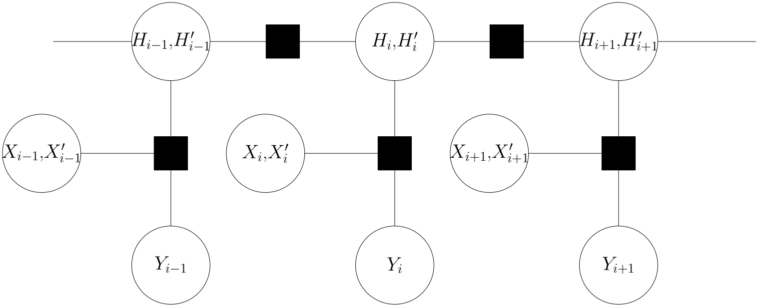


Message Passing

Optimal detection on the unknown $X_i \iff$ computation of $p(x_i|y_1^n)$.



The key is to compute $p(h_i, h'_i|y_1^{i-1})$ and $p(h_i, h'_i|y_{i+1}^n)$ recursively. It is similar to the BCJR algorithm.

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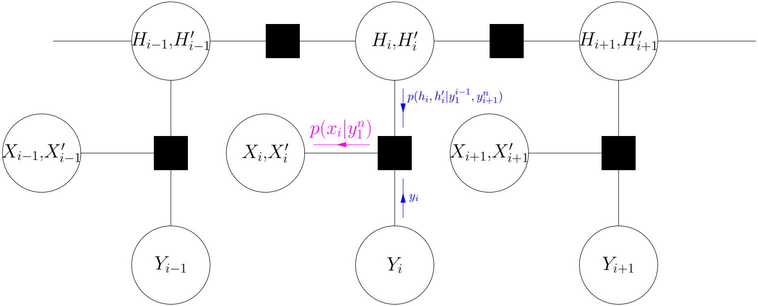

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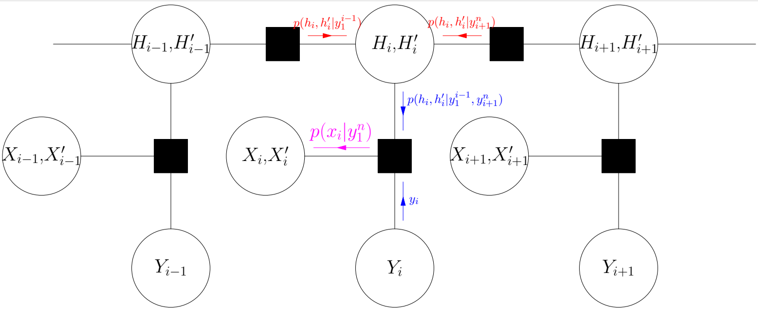

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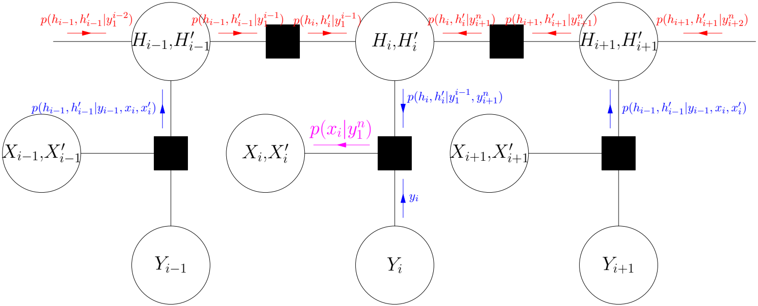

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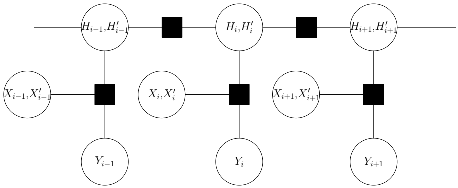
Practical Issues (1/2)



- All r.v.'s on graph are either Gaussian or discrete; the messages are of mixture Gaussian form.

$$\sum_j \rho_j \mathcal{CN}(h_i, h'_i; m_j, K_j).$$

- Enough to pass amplitudes, means and variances. No integrals.
- No cycle $\Rightarrow$ The BP algorithm is ultimately optimal.

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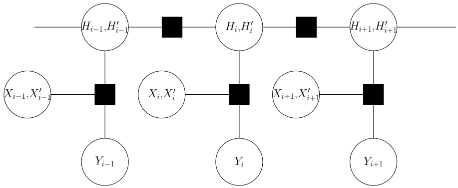

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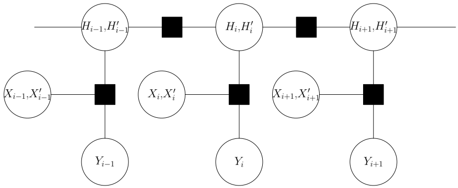

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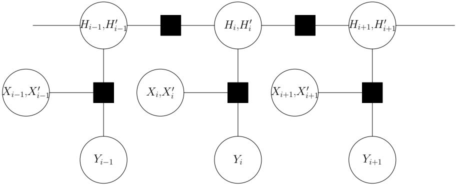

- The number of Gaussian components increases exponentially as belief propagates and can only be approximated from time to time. Here, we limit the maximum number of Gaussian components in each step.
- Complexity is constant per bit. (The constant is typically larger than linear estimation)

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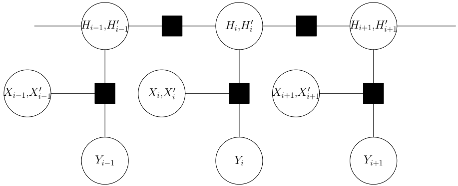

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Performance Evaluation

Simulation Conditions

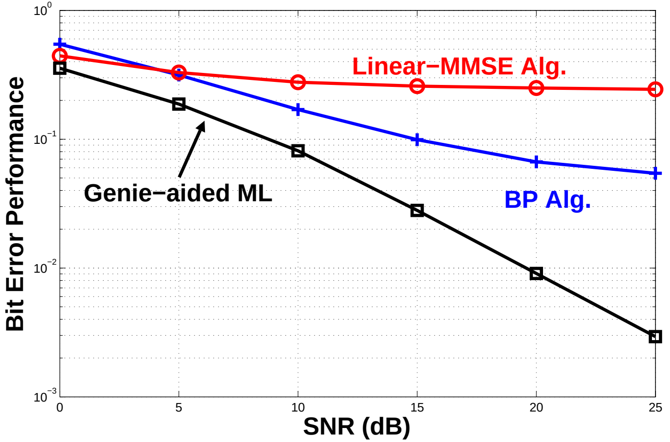
- uncoded system.
- snr $= \sigma_H^2/\sigma_N^2$ and $\alpha = .99$.
- block length 200; one pilot every 4 symbols.

We compare the following receivers:

1. BP.
2. Linear-MMSE channel estimation: Interference treated as white Gaussian noise.
3. Genie-aided ML: A genie reveals channel to receiver.

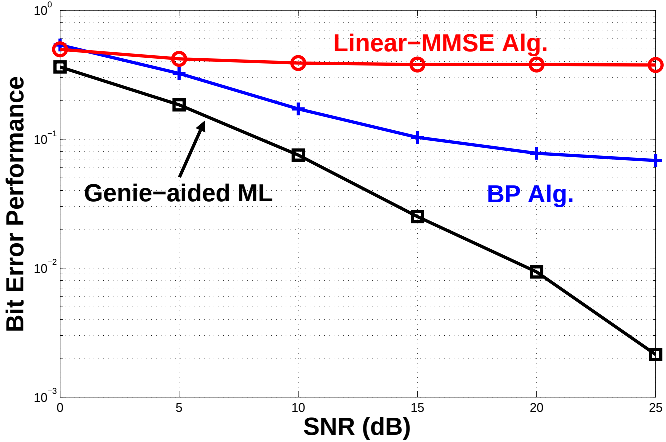
Medium Interference: Interference 3dB weaker

$$Y_i = H_i X_i + H_i' X_i' + N_i$$



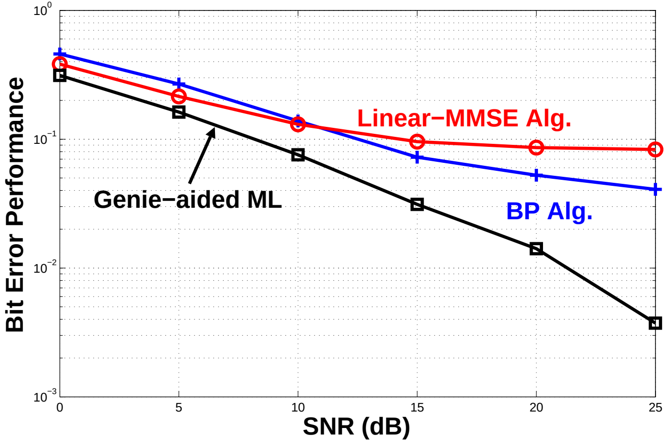
Strong Interference: Interference with same strength

$$Y_i = H_i X_i + H_i' X_i' + N_i$$



Weak Interference: Interference 10dB weaker

$$Y_i = H_i X_i + H_i' X_i' + N_i$$



Channel estimation under medium interference

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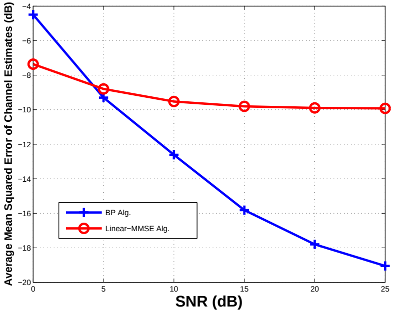

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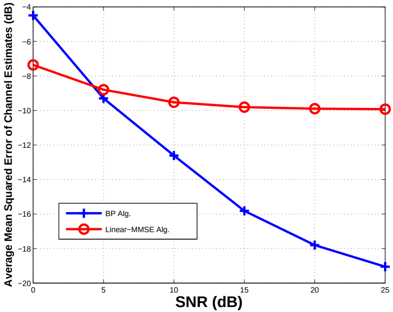
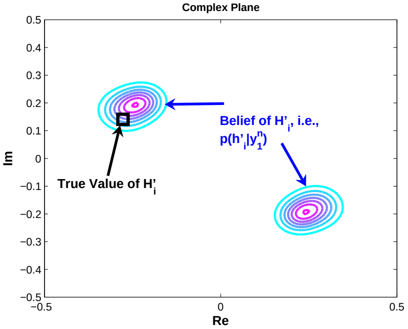
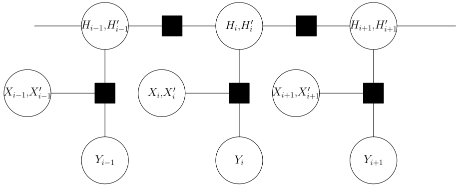
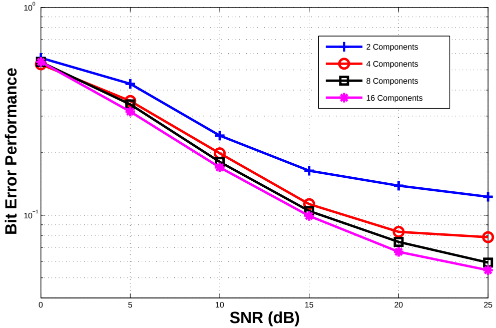




Effect of Mixture Approximation

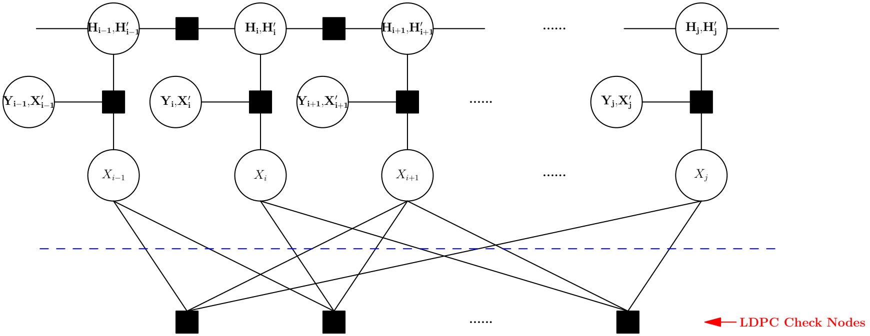




Conclusions and More Recent Results

- Joint channel estimation and interference mitigation based on BP.
- Unlike linear receivers, BP exploits the non-Gaussian statistics of the interference.
- Constant complexity per bit.
- Simulation shows significant gain over linear receiver.

More Recent Result: Factor graph with LDPC code



More Recent Result: BER performance with MIMO and LDPC