

# Blind Demodulation of Direct-Sequence CDMA Signals Using an Antenna Array

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## Abstract

Recent work on Direct-Sequence (DS) Code Division Multiple Access (CDMA) has shown that blind adaptive interference suppression techniques can be used for reliable demodulation, provided that the timing of the desired signal is known. The blind demodulator has since been used as a building block for a scheme for joint timing acquisition and demodulation. In this paper, we consider a blind adaptive demodulator that uses spatial diversity in the form of an antenna array in addition to the inherent time diversity of the DS signals exploited in previous work. Our results show that, as expected, the use of an antenna array leads to a substantial increase in system capacity, but that the convergence of an adaptive implementation using a stochastic gradient algorithm may be too slow for many applications. On the other hand, the large number of taps in the space-time filter imply that a faster least squares implementation is quite complex. Our results therefore point out the need for devising faster adaptive algorithms with reasonable complexity. It is shown that, while the blind demodulator requires knowledge of the DoA of the desired signal, a “sectorization” scheme can be used to get around this requirement.

## 1 Introduction

The conventional CDMA receiver (refer to Viterbi[1]) is a matched filter for the user of interest. This is in many ways, the simplest receiver. It assumes knowledge of the signature waveform and timing of only the desired user. The performance of this receiver, measured in terms of the bit-error rate, is severely limited by the near-far problem, that stringent power control is required. While the near-far problem can be addressed using a number of multiuser detection schemes (see Verdú[2] for

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a survey), these schemes are complex and require explicit knowledge of the interference parameters. An alternative approach is to use the Minimum Mean-Square-Error (MMSE) criterion for devising adaptive demodulators that only require knowledge of the desired signal (refer to Madhow and Hönlig[3], Rapajic and Vucetic[4], Abdulrahman, et al.[5] and Miller[6]).

The preceding adaptive MMSE detectors transcend the need for knowledge of the interference parameters by using training sequences for initial adaptation, followed by decision-directed adaptation. In a rapidly changing environment like in cellular mobile communications, however, this could result in poor utilization of the channel because any drastic change like a deep fade or a new user could entail the initiation of another training phase. Thus, it is imperative to devise a blind technique which would adapt to changes without any additional information. In Hönlig, et al.[7] a blind adaptive multiuser detection scheme was proposed. The detector derived is related to the philosophy of beam-forming (refer to Johnson and Dudgeon[8]) in array signal processing and the anchored minimum-energy adaptive equalizer of Verdú[9]. The basic idea is to decompose the receive filter into two components - one along the desired signal and another orthogonal to it (in a suitable Hilbert space). Thereafter, the output energy is minimized over all the possible receive filters which are in the form described above. The component along the desired user's signal is kept a constant and the orthogonal component is adapted. The receiver that is obtained from the minimization of the output energy called the Minimum Output Energy (MOE) detector, was shown to be the MMSE multiuser detector. The blind adaptation rule derived assumes the knowledge of the desired user's signature waveform and complete timing. Thus, it was shown that it is possible to have optimum near-far resistance with a demodulator that assumes only as much knowledge as the conventional detector.

The techniques proposed in [7] are applicable in a general Hilbert space setting, although the ideas have thus far been applied to using the time- diversity of CDMA signals for interference suppression. In this work, we evaluate the performance gain achievable by using, in addition, spatial diversity in the form of an antenna array. It is worth mentioning that spatial diversity has been shown to improve capacity even when used with the conventional matched-filter receiver Suard, et al.[10], and Naguib, et al.[11]. In Section 2 we describe the model, the blind demodulator with antenna diversity, and a stochastic gradient adaptation rule for our specific model. Section 3 is devoted to the simulation results that are obtained for various scenarios - number of users and directions. Comparisons with a system with a single element antenna are presented to show the amount of improvement obtained by using spatial diversity. We also address the case of imperfect

knowledge of the desired user's direction of arrival by means of a sectorization algorithm. Section 4 contains our conclusions.

## 2 Background

In this section we review the blind demodulator in [7] (to which we refer the reader for a detailed description and analysis) in the context of a system model including antenna diversity.

### 2.1 Model Description

The antipodal K-user, L-antenna element asynchronous CDMA white Gaussian channel is

$$\mathbf{y}(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^K A_k b_k[i] s_k(t - iT - \tau_k) \mathbf{a}_k + \sigma \mathbf{n}(t) \quad (1)$$

where  $\mathbf{n}(t)$  is a complex-valued White Gaussian L element noise vector with independent components and unit spectral density, the data  $b_k[i]$  are independent and equally likely to be -1 or +1,  $s_k(t)$  is the  $k^{\text{th}}$  user's signature waveform which is assumed to have unit energy, i.e.,  $\|s_k\| = 1$ ,  $A_k$  is the received amplitude of the  $k^{\text{th}}$  user,  $\tau_k$  are the relative offsets of the received asynchronous signals at the receiver and  $\mathbf{a}_k$  is the L element vector of antenna responses for the  $k^{\text{th}}$  user. For a linear array (depicted in Figure 1), the  $l^{\text{th}}$  component of this array response is given by

$$\mathbf{a}_k[l] = \exp\left(\frac{j2\pi d}{\lambda} \left(l - \frac{L+1}{2}\right) \sin(\theta_k)\right)$$

where  $d$  is the interelement spacing,  $\lambda$  the received wavelength and  $\theta_k$  the  $k^{\text{th}}$  user's direction of arrival. It is assumed that the spacing  $d$  is small enough that the time-offsets at different array elements are negligible.

For simplicity of exposition, we consider a synchronous system. Any asynchronous model can be viewed in terms of an equivalent synchronous model [3, 12] by considering each symbol falling in a given observation interval to correspond to a different user. We assume that perfect knowledge of the desired user's signature sequence, timing and DoA is available to the receiver. Knowledge of timing can be obtained using a joint timing acquisition and demodulation algorithm as in [12, 13], or using subspace based methods as in [14, 15]. Knowledge of DoA is less critical since the array response is insensitive to small changes in DoA. Thus, when the DoA is unknown,

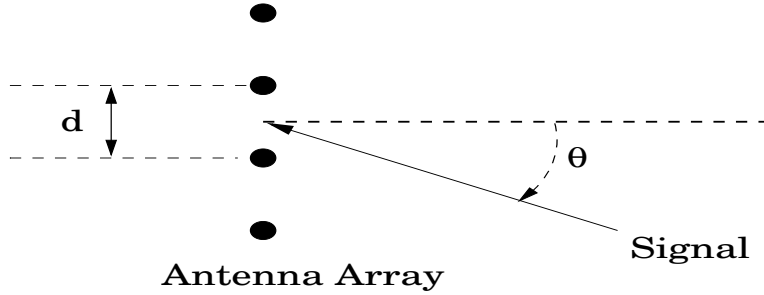


Figure 1: The antenna array with the defining parameters.

it suffices to use a “sectorization” technique in which a small number of blind demodulators are operated in parallel under different nominal DoAs. This is discussed further in Section 3.1.

For a synchronous system, it suffices to consider one-shot transmission, viz.,

$$\mathbf{y}(t) = \sum_{k=1}^K A_k b_k s_k(t) \mathbf{a}_k + \sigma \mathbf{n}(t), \quad t \in [0, T] \quad (2)$$

The signature sequence for the  $k^{\text{th}}$  user’s waveform is given by

$$s_k(t) = \sum_{j=0}^{N-1} \mathbf{s}_k[j] \psi(t - jT_c) \quad (3)$$

where  $T_c$  is the chip period,  $N$  is the processing gain (also equal to  $T/T_c$ ),  $\mathbf{s}_k$  is the spreading sequence of the  $k^{\text{th}}$  user, and  $\psi(t)$  is the spreading waveform.

At the receiver, chip-matched filtering followed by chip-rate sampling yields the following discrete-time model for the received matrix  $\mathbf{R}[n] \in \mathbb{C}^L \times \mathbb{C}^N$  used for the deciding on the  $n^{\text{th}}$  bit of the desired user  $b_1[n]$ :

$$\mathbf{R}[n] = \sum_{k=1}^K A_k b_k[n] \mathbf{S}_k + \mathbf{W}_n, \quad (4)$$

where  $\mathbf{S}_k[i, j] = \mathbf{a}_k[i] \mathbf{s}_k[j]$  and  $\mathbf{W}_n$  is a matrix of independent complex-valued Gaussian random variables with mean 0 and variance  $\sigma^2$ .

## 2.2 Blind Demodulation

The blind demodulator considered is that of [7]. The received signal (after chip-matched filtering and chip-rate sampling) is an  $L \times N$  matrix, as is the linear blind demodulator. Therefore, we

consider the following inner-product (denoted by  $\langle, \rangle$ )

$$\langle \mathbf{A}, \mathbf{B} \rangle = \text{trace}(\mathbf{B}^* \mathbf{A})$$

where  $\mathbf{B}^*$  is the adjoint (i.e., the complex conjugate transposed) of  $\mathbf{B}$ .<sup>1</sup>

We consider the following linear detector for user 1:  $\hat{b}_1[n] = \text{sgn}(\text{Re}[\langle \mathbf{C}, \mathbf{R}[n] \rangle])$ , where where the correlator  $\mathbf{C}$  is of the canonical form

$$\mathbf{C}_1 = \mathbf{S}_1 + \mathbf{X}_1 \tag{5}$$

$$\text{where } \langle \mathbf{S}_1, \mathbf{X}_1 \rangle = 0$$

and is chosen to minimize the output energy

$$E[|\langle \mathbf{R}[n], \mathbf{C} \rangle|^2]$$

when the sequence of received matrices  $\mathbf{R}[n]$  are given by (4). It is important to restrict  $\mathbf{C}$  to be of the form in (5), otherwise  $\mathbf{C} = \mathbf{0}$  trivially minimizes the output energy. It was shown in [7] that the canonical linear detector which minimizes the output energy is the MMSE detector as well. It was also shown that the MOE cost function is strictly convex, and thus has a unique global minimum.

We also give an expression for the near-far resistance of the desired user's signal, i.e.,  $\mathbf{S}_1$ , with respect to the interference space (refer to [3] or [7] for the derivations) denoted by  $\eta$ .

$$\eta = 1 - \rho_1^* \mathbf{R}_I^{-1} \rho_1 \tag{6}$$

where  $\rho_1 = (\rho_2, \dots, \rho_K)^T$  is the vector of normalized crosscorrelations of  $\mathbf{S}_1$  with the interfering signals, and  $\mathbf{R}_I$  is the  $(K-1) \times (K-1)$  matrix of normalized crosscorrelations for the interfering signals.

### 2.3 Stochastic Gradient Adaptation

A simple stochastic gradient-descent adaptation rule for the MOE cost criterion was derived in [7]. We present an adaptation of the same algorithm for our problem. Let the  $n^{\text{th}}$  output of the matched filter (here it means correlation of  $\mathbf{R}$  with  $\mathbf{S}_1$ ) be the random variable

$$Z_{MF}[n] = \langle \mathbf{R}[n], \mathbf{S}_1 \rangle \tag{7}$$

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<sup>1</sup>This is equivalent to the standard vector inner product if the entries of the  $L \times N$  matrix are rearranged as an  $LN \times 1$  vector.

Also, let the  $n^{\text{th}}$  output of the canonical linear detector be

$$Z[n] = \langle \mathbf{R}[n], \mathbf{S}_1 + \mathbf{X}_1 \rangle \quad (8)$$

The actual operation is shown in Figure 2.

The decision rule for demodulating the desired user's transmitted symbol is

$$\hat{b}_1[n] = \text{sgn}(\text{real}(Z[n])) \quad (9)$$

The stochastic gradient adaptation rule is

$$\mathbf{X}_1[n] = \mathbf{X}_1[n-1] - \mu \text{conj}(Z[n])(\mathbf{R}[n] - Z_{MF}[n]\mathbf{S}_1) \quad (10)$$

where  $\text{conj}(\cdot)$  refers to the complex conjugate. In an actual implementation we may need to make  $\mathbf{X}_1[n]$  exactly orthogonal to  $\mathbf{S}_1$  while updating  $\mathbf{X}_1[n]$ .

The Signal-to-Interference Ratio (SIR) for a linear detector in the canonical form is given by

$$\text{SIR} = \frac{A_1^2 |\langle \mathbf{C}_1, \mathbf{S}_1 \rangle|^2}{\sigma^2 \|\mathbf{C}_1\|^2 + \sum_{k=2}^K A_k^2 |\langle \mathbf{C}_1, \mathbf{S}_k \rangle|^2}$$

Since the linear MMSE detector achieves the maximum SIR (denoted by MSIR) over all linear detectors, a lower bound on its SIR is given by that of the decorrelating detector as follows:

$$\text{MSIR} \geq \text{SNR} + \eta_{dB} \quad (11)$$

where  $\eta_{dB} = 10 * \log_{10}(\eta)$ . The exact expression for MSIR is derived in [3] but for our discussions it is sufficient to consider the bound given above. One more performance parameter which is important is the *effort*  $\chi$ , given by

$$\chi = \|\mathbf{X}_1\|^2 / \|\mathbf{S}_1\|^2 \quad (12)$$

This is the energy of the adaptive component  $\mathbf{X}_1$  of the correlator relative to that of the nonadaptive component  $\mathbf{S}_1$ , and is a measure of the noise enhancement and the amount of work the adaptive algorithm has to do. The minimum effort required to suppress the interference is given by [7]

$$\chi_I = \frac{1}{\eta} - 1$$

which implies that the adaptive algorithm is expected to converge faster to an interference suppressing solution for a higher near-far resistance. This prediction is borne out by the simulation results of the next section.

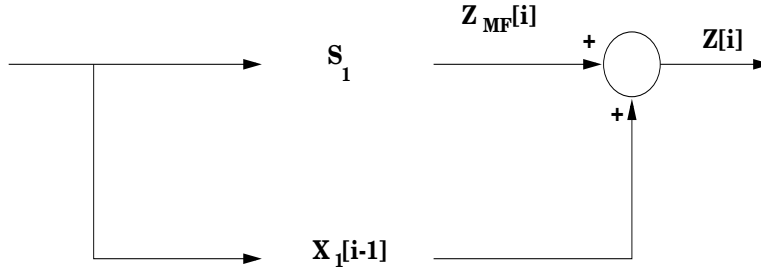


Figure 2: The blind multiuser detector with  $\mathbf{X}_1[i - 1]$  governed by stochastic gradient adaptation rule.

### 3 Simulation Results

In this section we present simulation results showing the comparison between the single element antenna and a four element antenna array. For all our simulation results we use a normalized version of the stochastic gradient-descent algorithm, in which the adaptive gain is a dimensionless quantity divided by an estimate of the power of the received matrix. The direction of arrival of the desired user is arbitrarily fixed at 45 degrees as measured in Figure 1. As mentioned later, the best performance is obtained if this angle is close to 0 degrees. The signature sequences and DoAs for all the other users are chosen randomly. In all simulations the desired signal to background-noise ratio is 20dB. The amplitudes of the interfering signals is chosen to be 20 times the amplitude of the desired signal, representing an extreme near-far situation. As mentioned earlier, we assume perfect knowledge of the desired user's signature waveform, direction of arrival and timing. We would also like to emphasize that we are considering a synchronous situation. All the graphs for the Signal-to-Interference-Ratio (SIR) are averaged over 50 runs. Error probability curves are not shown because in all cases, the probability of bit error matches almost exactly with  $Q(\sqrt{SIR})$  where  $Q(\cdot)$  is the complementary distribution function of an  $N(0, 1)$  Gaussian random variable. In all the plots shown we also specify the corresponding near-far resistance of the desired user's signal with respect to the interference space. This is an important parameter in all our discussions. In all our simulations, we initialize our algorithm with the matched filter, i.e., with  $\mathbf{X}_1 = \mathbf{0}$ . All simulations are either for a single element antenna, or for a four-element antenna array ( $L=4$ ) with an inter-element spacing of  $d = \lambda/8$ .

Figure 3 shows the performance for a system with a single element antenna and 6 interferers. The blind algorithm is able to separate the signals and is able to take the SIR to about 8dB at

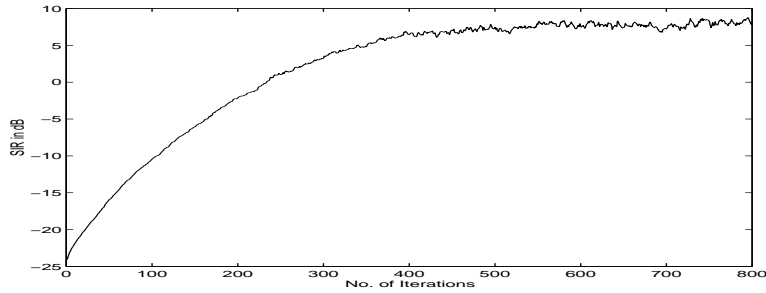


Figure 3: The single element case with 6 strong interferers,  $\eta = .3333$ .

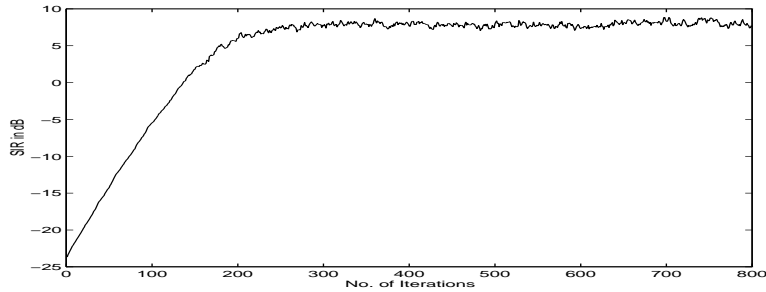


Figure 4: The four element case with 6 strong interferers,  $\eta = .3970$ .

which point a decision-directed MMSE adaptation can be used to obtain faster convergence and better demodulation performance. Note that the convergence is extremely slow taking about 300-400 iterations to get a reasonable SIR, and the final SIR value (approx. 8.25dB) falls well below the MSIR, which is bounded below by 15.23dB using (11). Figure 4 shows results for the same system using a 4-element array. The final SIR is about 7.6dB which is again much less than the MSIR of at least 16dB, which is again a large shortfall in performance. It is illustrative to compare the effort (refer to (12)) for the two cases. The effort for the single element case is 2.0253 and for the four-element case is 1.4991. This is explained by two observations:

1. the near-far resistance is higher in the four-element case (.397 versus .3333).
2. the matched-filter is "more" orthogonal to the interferers in the four-element case.

These two observations also account for the faster convergence of the four-element case.

The convergence of the algorithms depends on the number of users,  $K$ , and on the near-far resistance  $\eta$ . To illustrate this fact we display a case in Figure 5 where we have 6 interferers and a single antenna element (as earlier) but  $\eta = 0.1444$ . We find that even after 600 iterations we do not



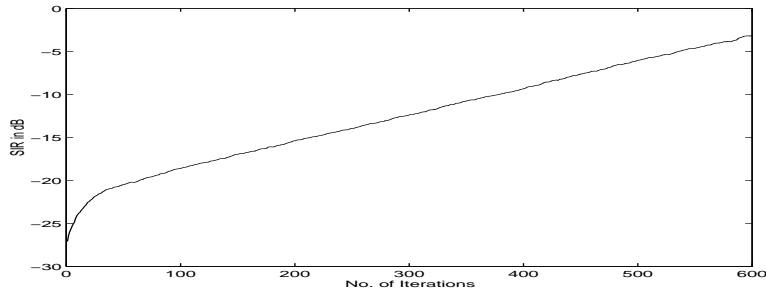


Figure 5: The single element case with 6 strong interferers,  $\eta = .1444$ .

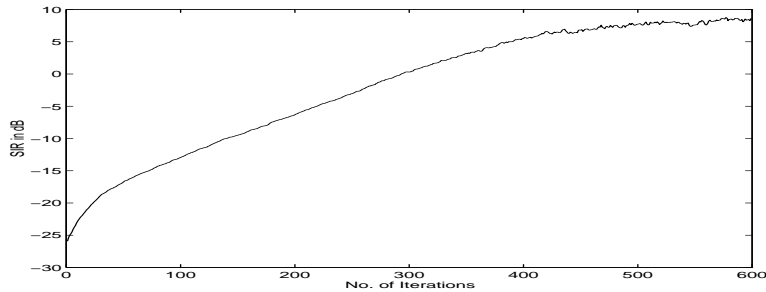


Figure 6: The four element case with 6 strong interferers,  $\eta = .3209$ .

have convergence. The same case with the four-element array exhibits better convergence (refer to Figure 6 ). The near-far resistance now is  $\eta = 0.3209$  and the final SIR is 8.5dB (MSIR is at least 15dB). The effort for the single element case is 4.8150 and for the four-element case is 2.1027.

So far we have discussed the case where the near-far resistance for a single-element antenna is nonzero, so that interference suppression should be achievable if the adaptation is run for long enough. If the number of users is more than the processing gain, we expect the desired signal to be spanned by the interfering signals with high probability, in which case the near-far resistance would be zero. The MOE criterion would in this case suppress both signal and interference, as illustrated in Figure 7. The four-element array in the same case gives a final SIR of approx. 8dB (MSIR is at least 16.71dB) and the effort is 1.1457 (refer to Figure 8 ). The near-far resistance in this case is  $\eta = .4688$ . The convergence is slower than with fewer interfering signals.

We have seen that the single-element case cannot support a number of users larger than the processing gain in a near-far resistant manner. We would now like to investigate the limits on capacity when a four-element antenna-array is used. Figure 9 displays results for a system with 14 interferers. The SIR after a large number of iterations is approximately 6.5dB (MSIR is at

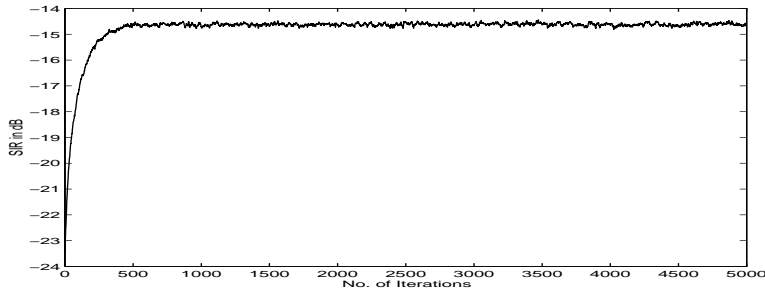


Figure 7: The single element case with 11 strong interferers,  $\eta = 0$ .

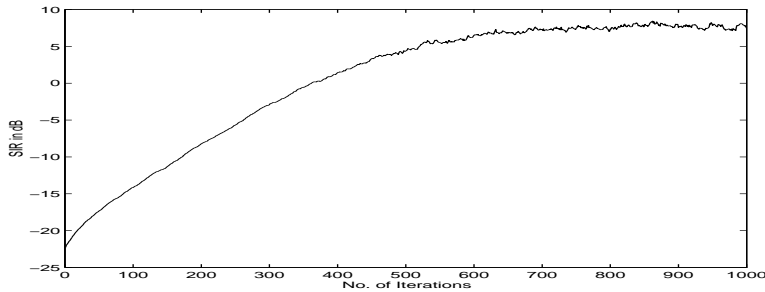


Figure 8: The four element case with 11 strong interferers,  $\eta = .4688$ .

least 15.8dB). The effort used is 1.4727 and  $\eta = 0.3810$ . The convergence is extremely slow. In fact, the SIR curve is yet to show any convergence! In Figure 10 we consider a system with 21 interferers. Here we have even slower convergence. The near-far resistance is  $\eta = 0.2681$ . The effort the algorithm has put in so far amounts to 1.9119. The MSIR is at least 14.283dB. Figure 11 has a slightly different situation. We have the same number of users but the signature sequences are such that  $\eta = 0.0934$ , which is significantly lower. As expected from our discussions above, the SIR curve does not show any convergence even after 8000 iterations. The effort used so far is given by 7.1826 and the MSIR is at least 9.7dB. It is evident that we are operating close to the capacity of the system, i.e., the point at which the desired user's signal is spanned by the interferers with high probability.

### 3.1 Imperfect knowledge of $\mathbf{S}_1$

So far we have assumed that we know  $\mathbf{S}_1$ , i.e., we have exact knowledge about the desired user's signature sequence, direction of arrival and timing. In this subsection we would like to briefly address the issue of imperfect knowledge of these quantities. It has been shown Madhow[12, 13]

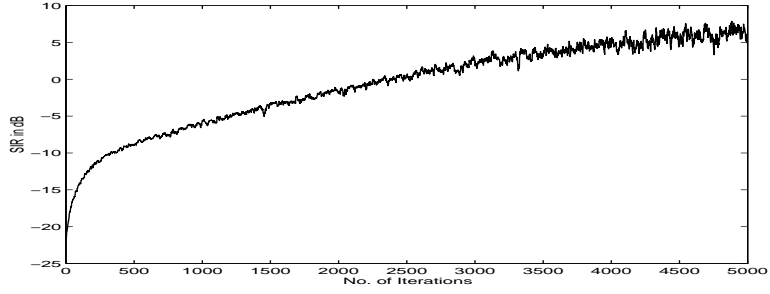


Figure 9: The four element case with 14 strong interferers,  $\eta = .3810$ .

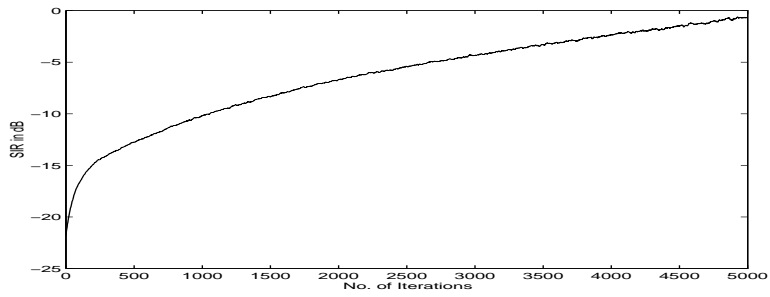


Figure 10: The four element case with 21 strong interferers,  $\eta = .2681$ .

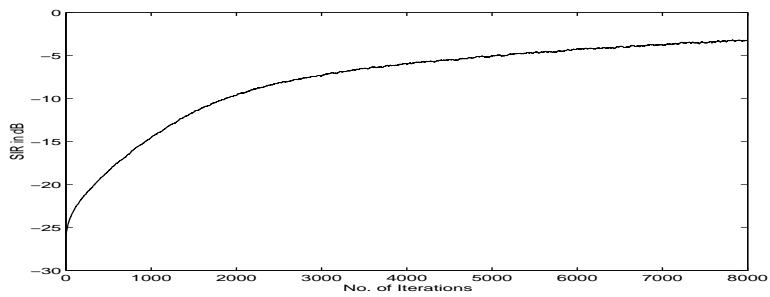


Figure 11: The four element case with 21 strong interferers,  $\eta = .0934$ .

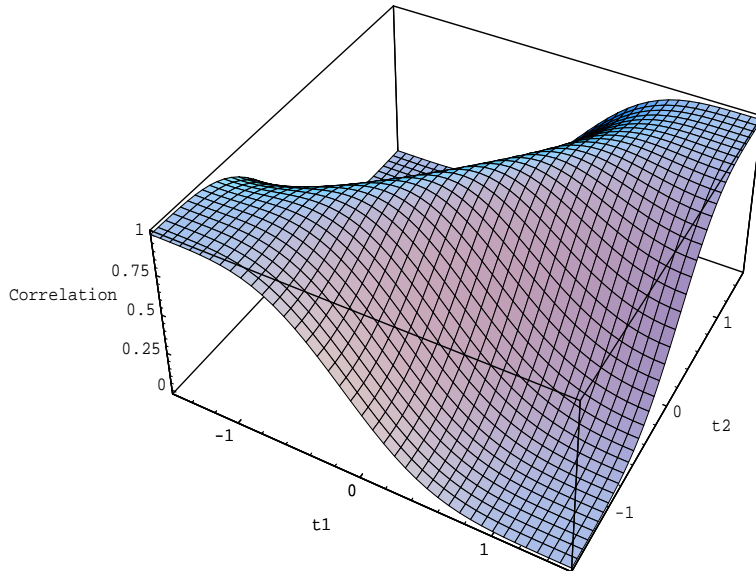


Figure 12: The absolute value squared of the correlation for various directions of arrival (in radians).

that the blind demodulator in [7] can be used as the basis for a scheme for joint timing acquisition and demodulation by running parallel demodulators under a number of timing hypotheses. We therefore concentrate on the issue of imperfect knowledge of the direction of arrival, which can be addressed using similar ideas. In fact, since the correlation between the array response for two different DoAs decays slowly with their difference, it is only necessary to hypothesize a small number of nominal DoAs, run blind demodulators under each hypothesis, and pick the demodulator with the best estimated performance.

Figure 12, which shows the absolute value squared of the correlation (for a four element antenna) for various directions of arrival, illustrates the slow decay in correlation with changes in DoA. Figure 13 shows, as a function of the nominal DoA, the maximum DoA error such that the correlation between the array responses for the true and nominal DoAs is at least 0.9. This is then used to determine the number of DoA nominals, or “sectors”, required.

For the numerical results that follow, we use 5 sectors, with nominal DoAs given by  $\{-90^\circ, -45^\circ, 0^\circ, 45^\circ, 90^\circ\}$ . Blind demodulators are run for each hypothesized DoA in parallel using a least squares implementation [7] (which performs much better than the stochastic gradient method) with 500 symbol-rate iterations. The obvious algorithm of choosing the hypothesis with the largest output energy is found not to work well in all instances. In particular, if the mismatched nominal has a substantial component in the noise subspace, an eigendecomposition of the correla-

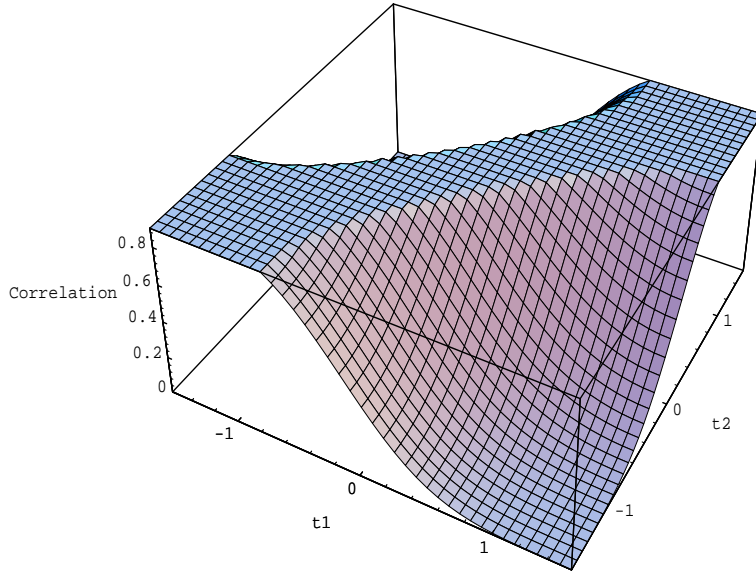


Figure 13: The maximum DoA error such that the absolute value squared of the correlation between the array responses for the true and nominal DoAs is at least 0.9.

tion matrix for the observations can be used to show that the output energy can be large. However, an eigendecomposition also implies that the output energy for such a nominal, when normalized by the energy of the correlator (i.e.,  $\chi + 1$ , where  $\chi$  denotes the effort), will be small. Thus, our algorithm is to choose the nominal with the largest estimated *normalized* output energy, divided by  $\chi + 1$ . The detailed computations in the algorithm are omitted due to lack of space.

The algorithm is simulated for a system with 22 synchronous transmissions with randomly chosen signature sequences. As before each of the 21 interfering transmissions has amplitude 20 times that of the desired signal and the SNR for the desired transmission is 20 dB. The DoAs for the interfering transmissions are chosen at random, while that of the desired signal is fixed at 60 degrees, so that all the DoA hypotheses are mismatched. The mismatch is addressed using a norm constraint as in [7] or [12, 13], which can be implemented by adding a fictitious noise variance  $\nu$ . We expect the best hypotheses to be those corresponding to DoAs of 45 and 90 degrees. In 50 runs of the least squares implementation, we find that one of these hypotheses is chosen every time by the algorithm. This is shown in Figure 14. We run the algorithm for 300 iterations in each case. Interestingly, choosing the largest estimated output energy always leads to the choice of a bad hypothesis corresponding to a DoA of -45 degrees, which shows that normalizing the output energy by  $\chi + 1$  is crucial. The near-far resistance is  $\eta = 0.1914$ . The SIR attained by the corresponding demodulator, averaged over the runs, is plotted in Figure 15.

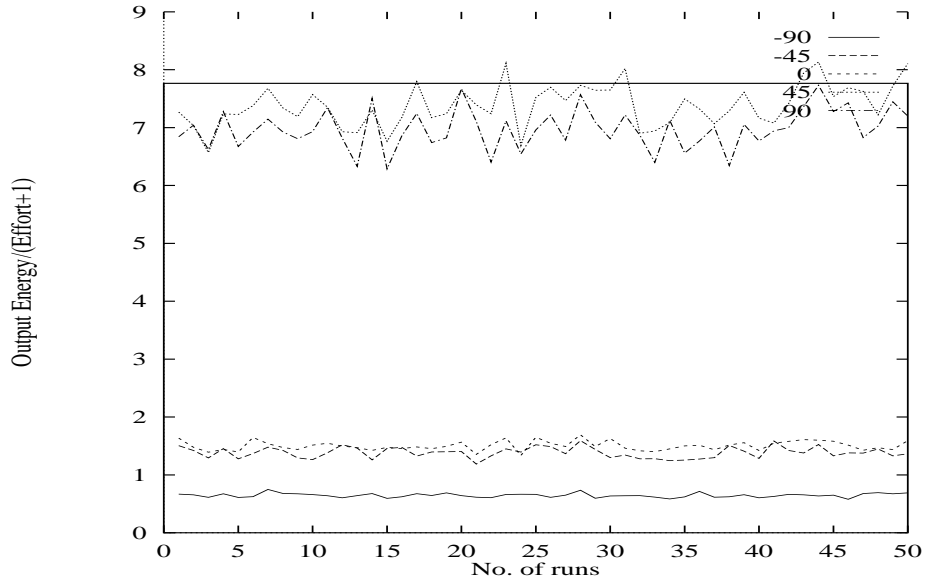


Figure 14: Output Energy/(Effort+1) attained by the demodulators of the different hypotheses at the end of 300 iterations of the algorithms, for each of the 50 runs.

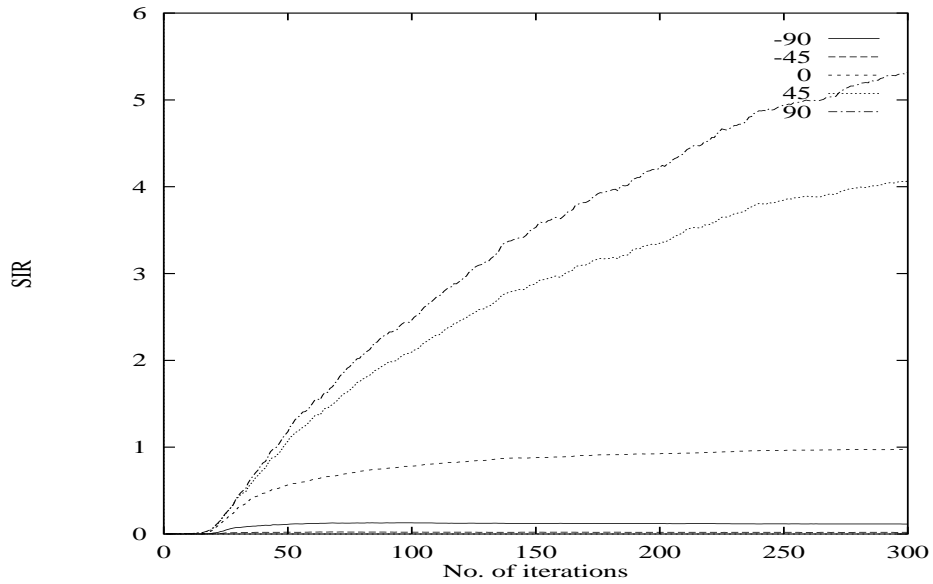


Figure 15: The SIR attained by the demodulators corresponding to different hypotheses.

## 4 Conclusions

We have seen that the use of antenna arrays can yield substantial capacity gains, and that the blind demodulator can be implemented without knowledge of the DoA of the desired transmission by using a sectorization algorithm. However, the large number of taps for the demodulator result in excessively slow convergence for a stochastic gradient adaptive implementation, and high complexity for a least squares implementation. Our current efforts are therefore focused on devising fast adaptive mechanisms which have reasonable complexity. In particular, the mechanism should converge fast even for small near-far resistance and a large number of users, and should aim towards getting closer to the maximum signal-to-interference ratio than the stochastic gradient algorithm. Another direction for further study is the extension of these methods to multipath fading channels.

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