

# **Analysis of Multiuser Diversity in Wireless Data Networks**

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# Outline

Wireless Scheduling

Time Invariant Channel

Weighted Proportionally Fair Allocation

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# Wireless Scheduling

**Problem:** How to dynamically share a wireless link efficiently and fairly amongst various users?

## Wireless scheduling paradigm:

- Wireless link is resource limited - bandwidth, spreading codes, power, interference.
- Time-varying.
- Means to achieve capacity often involves varying rates of transmission based upon channel state. Scheduler not aware of radio-conditions cannot wait till good channel state to transmit.

Great benefits to be had with higher layers knowing the physical layer parameters. Thus, joint approach seems more natural - Collins and Cruz'1999, Zhang and Wasserman'2000, Berry and Gallager'2001, Tse'2000, Holtzman'2000, Jalali *et al.*'2000, Chawla *et al.*2000, Shakkottai and Stolyar'2000, Agrawal, *et al.*'2001, Rangsan L and Agrawal'2001.

Efficiency and fairness compete:

1. In 2-G systems only voice was present. Coverage (fairness) was the only concern. Very good conditions for certain users never exploited - nothing to be gained really. Not very efficient!
2. Consider packet data and the policy of service only to the best user(s). Most efficient system. ONLY on-off nature of traffic would allow other users to transmit. No fairness!

It is clear from (1) that such concerns are important only for packetized services.

## Time-invariant channels

We consider non-realtime applications - specifically rate-adaptive services - and downlink case.

Let user  $j$  get rate  $\bar{R}_j$  at time.

Define  $\rho_j$  to be the fraction of time (over a long time) that the resources are given to user  $j$ .

Throughput of user  $j$  -  $\rho_j \bar{R}_j$ . Choose  $\rho_j$  such that

$$\max_{\rho_j} \sum_{j \in \mathcal{J}} U_j(\rho_j \bar{R}_j P_b M_j) \quad (1)$$

subject to:

$$\begin{aligned} \rho_j &\geq 0 \\ \sum_j \rho_j &\leq 1 \end{aligned}$$

$U_j(\cdot)$  is a utility function belonging to the family

$$U^\alpha(x) = \begin{cases} \text{sgn}(\alpha)x^\alpha & \alpha < 1, \alpha \neq 0 \\ \log(x) & \alpha = 0 \end{cases}$$

The solution to (1) is

$$\rho_j = \frac{\bar{R}_j^{\beta-1}}{\sum_{i \in \mathcal{J}_b} \bar{R}_i^{\beta-1}}, \forall \alpha < 1,$$

where  $\beta = \frac{1}{1-\alpha}$ .

Therefore,  $\rho_j \propto \bar{R}_j^{\beta-1}$  and throughput  $\propto \bar{R}_j^\beta$ .

Define  $C_j = \bar{R}_j^\beta$ .

### Observations:

- $\alpha = 1$ , i.e.,  $\beta = +\infty$  results in  $\rho_j = 1$  for  $j = \arg \max \bar{R}_j$  or the policy that serves the best user.
- $\alpha = 0$ , i.e.,  $\beta = 1$  results in proportionally fair allocation resulting in  $\rho_j$  being equal for all the users - the trade-off assumed in the HDR solution of Tse *et al.*
- $\alpha = -\infty$ , i.e.,  $\beta = 0$  results in users getting a throughput independent of their channels - Equal throughput solution!

# Weighted Proportionally Fair Allocation

$$\max_{\rho_j} \sum_{j \in \mathcal{J}} Wt_j \log(\rho_j \bar{R}_j) \quad (2)$$

subject to the constraints of (1). Solution is

$$\rho_j^* = \frac{Wt_j}{\sum_i Wt_i}.$$

and  $Thput_j^* = \frac{Wt_j \bar{R}_j}{\sum_i Wt_i}$ .

Solution of (1) same as (2) with weights  $Wt_j = \frac{C_j}{R_j}$ .

**Question:** How does one achieve the wpf allocation?

Let  $W_j(t)$   
 = amount of data transmitted by user  $j$  upto  $t$  (present)  
 =  $W_j(t - 1) +$  data transmitted at time  $t$ .

Let  $\bar{W}_j(t) = \frac{W_j(t)}{C_j}$  be the normalized throughput.

**Policy:** At BS  $b$  serve user  $j^*(t) = \arg \min_{j \in \mathcal{J}_b} \bar{W}_j(t)$ .

We have the following

**Proposition 1** *As  $t \rightarrow \infty$  the algorithm outlined gives the wpf allocation, and hence, the optimal allocation.*

## Time-Varying Channels

Assume that user  $j$  gets rate  $R_j(t)$  at time  $t$ .

Try simple approach of using  $C_j = \bar{R}_j^\beta$ .

Then  $Thput_j(t) \rightarrow Thput_j^*(\bar{R})$  (a.s.) under fairly general conditions (Proof for i.i.d. with  $1 + \epsilon$  moment).

**Is this the best that can be done? NO!**

If we transmit to user  $j$  when his rate is better than his average, then we can do better.

If there are many users, then it is very likely that some user will be in a good state - **Multuser Diversity**.

Compute the average effective data rate  $\hat{R}_j^{\text{avg}}(t)$  as follows

$$R_j^{\text{avg}}(t+1) = (1 - \psi)R_j^{\text{avg}}(t) + \psi R_j(t).$$

Define

$$\begin{aligned} C_j(t) &= w_j [R_j^{\text{avg}}(t)]^\beta \left[ \frac{R_j(t)}{R_j^{\text{avg}}(t)} \right]^\gamma & (3) \\ &= w_j [R_j^{\text{avg}}(t)]^{\beta-\gamma} [R_j(t)]^\gamma \\ &= C_j^1(t) C_j^2(t), \end{aligned}$$

where  $0 \leq \gamma \leq \beta$ .



Let  $D_j(t)$  be the amount of data transmitted in frame  $t$  for user  $j$ . Update  $\bar{W}_j$  as follows

$$\bar{W}_j(t+1) = (1 - \phi)\bar{W}_j(t) + \phi \frac{D_j(t)}{C_j^1(t)}. \quad (4)$$

**Policy:** Rank users in increasing order of  $\tilde{W}_j(t) = \bar{W}_j(t)/C_j^2(t)$  and serve user  $j^*$  with minimum  $\tilde{W}_j(t)$ , i.e.,  $D_j(t) = 1_{[j=j^*]}R_j(t)$ .

3 Variations possible:

1. **Variation 1:** Use  $C_j^1(t) = C_j(t)$  and  $C_j^2(t) = 1$ . This resembles the algorithm analysed earlier.
2. **Variation 2:** Use  $C_j^1(t) = w_j[\hat{R}_j^{\text{avg}}(t)]^\beta$  and  $C_j^2(t) = \left[\frac{\hat{R}_j(t)}{\hat{R}_j^{\text{avg}}(t)}\right]^\gamma$ .
3. **Variation 3:** Use  $C_j^1(t) = 1$  and  $C_j^2(t) = C_j(t)$ .

The algorithm in Tse'2000 is similar to **Variation 3** with  $\beta = \gamma = 1$ . We assume that  $\psi = \alpha\phi$ .

Define

$$Th_{put_j}(t+1) = (1 - \phi)Th_{put_j}(t) + \phi D_j(t). \quad (5)$$

# Performance

CDMA case:

25 cells, 15 users per cell

Data rates - 2400, 1800, 1200, 600, 300 bits per frame (10ms).

Max thput per cell - 1.44 Mbps.

$\beta$	Variant 1, $\gamma = 0$	Variant 1	Variant 2	Variant 3
0	294.39, 5.83	294.39, 5.83	294.39, 5.83	294.39, 5.83
1	353.14, 7.97	385.84, 8.85	421.82, 9.27	442.36, 9.77
2	423.43, 10.00	457.74, 10.83	480.86, 10.95	522.15, 11.73

Comparison of average throughput per cell for different values of  $\beta$  and variations of the scheduling algorithm.

In columns 2, 3 and 4: for

- $\beta = 0, \gamma = 0$ .
- $\beta = 1, \gamma = 1$  - HDR proposal.
- $\beta = 2, \gamma = 1$ .

# Stochastic Approximation

[Bucklew, Kurtz, Sethares]:

$$W_{k+1} = W_k + \mu H(W_k, Y_k, U_{k+1}),$$

where  $W_k$  represents the parameter estimation errors,  $Y_k$  some function of inputs,  $U_k = q(W_k, Y_k, \Psi_k)$  is a disturbance process with  $\{\Psi_k\}$  i.i.d. and independent of  $\{Y_k\}$ , and  $W_0$  independent of  $\{(Y_k, \Psi_k)\}$ .

Define

$$\bar{H}(w, y) = \int H(w, y, u) \eta(w, y, du)$$

where  $\eta$  is the conditional distribution of  $U_{k+1}$  given  $\mathcal{F}_k$  and

$$\hat{H}(w) = \int \bar{H}(w, y) \nu_Y(dy).$$

If  $\{Y_k\}$  stationary and ergodic,  $W_\mu(0) \rightarrow w_0$  in probability and  $\bar{H}(w, y)$  continuous in  $(w, y)$ , then as  $\mu \rightarrow 0$ ,  $W_{[t/\mu]}$  converges weakly to

$$W(t) = w_0 + \int_0^t \hat{H}(W(s)) ds.$$

## Variant 1

We have

$$\bar{W}_j(t+1) = \bar{W}_j(t) + \phi \left( \mathbf{1}_{[j=j^*]} \frac{R_j^{1-\gamma}(t)}{(R_j^{\text{avg}}(t))^{\beta-\gamma}} - \bar{W}_j(t) \right), \quad (6)$$

where  $j^* = \arg \min_j \bar{W}_j(t)$ .

As  $\phi \rightarrow 0$  and for large  $t$ ,  $R_j^{\text{avg}}(t) = \bar{R}_j$  and we get a version of the simple algorithm!!! Thus,

$$Thput_j = \frac{E[R_j]^{\beta-\gamma+1} / E[R_j^{1-\gamma}]}{\sum_i E[R_j]^{\beta-\gamma} / E[R_j^{1-\gamma}]}. \quad (7)$$

We do not exploit multiuser diversity (for small enough  $\phi$ ) and for  $\gamma = 1$  we get same performance as  $\gamma = 0$ . In general expect performance to be worse (in terms of sum utility) than  $\gamma = 0$  case.

## Variants 2 and 3

ODE - V2

$$\frac{d\bar{W}_j}{dt}(t) = E_R[1_{[j=j^*]} \frac{R_j}{(R_j^{\text{avg}}(t))^\beta}] - \bar{W}_j(t) \quad (8)$$

$$\frac{dTh\tilde{put}_j}{dt}(t) = E_R[1_{[j=j^*]} R_j] - Th\tilde{put}_j(t). \quad (9)$$

Scale  $\bar{W}_j$  by  $E[R_j]^\beta$ , then equilibrium solution of V2 and V3 the same!!!

Thus, for small  $\phi$  and large  $t$  need to consider only one of them.

## Variant 3

Assume  $R_j(t)$  is  $\chi$ -squared distributed with  $2n$  degrees of freedom and mean  $\frac{1}{\lambda_j}$ .

For 2 users equilibrium point given by solution to

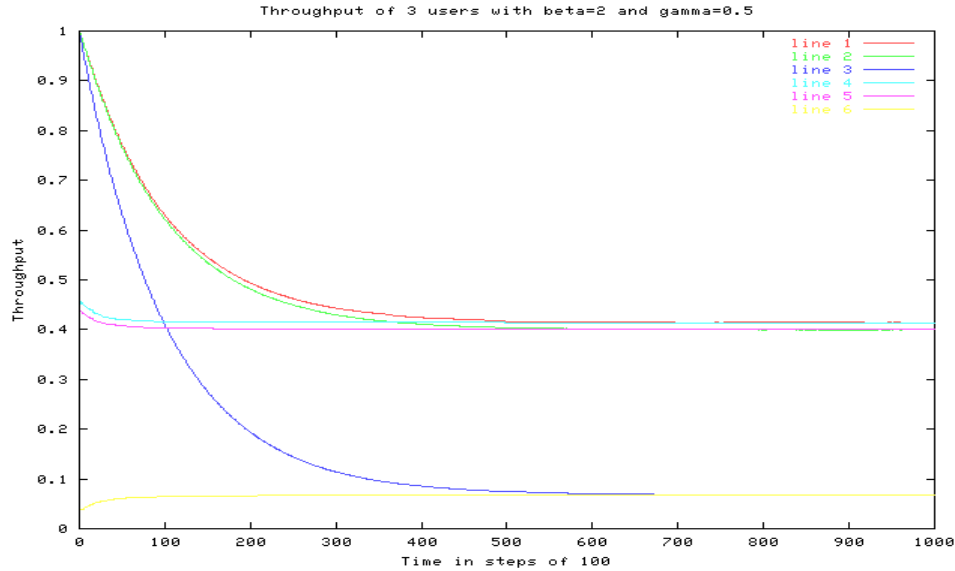
$$\bar{W}_1 = \frac{1}{\lambda_1} \left[ 1 - \left( \sum_{l=0}^{n-1} \frac{n+l!}{n!!} \frac{1}{(1+1/c)^l} \right) \frac{1}{(1+c)^{n+1}} \right],$$

$$\bar{W}_2 = \frac{1}{\lambda_2} \left[ 1 - \left( \sum_{l=0}^{n-1} \frac{n+l!}{n!!} \frac{1}{(1+c)^l} \right) \frac{1}{(1+1/c)^{n+1}} \right],$$

where

$$c = \left( \frac{\bar{W}_2}{\bar{W}_1} \right)^{1/\gamma} \left( \frac{\lambda_2}{\lambda_1} \right)^{\beta/\gamma}.$$

ODE has unique solution, therefore convergence (as  $\phi \rightarrow 0$ ) is also in probability. Equilibrium point is locally stable. Numerical investigations indicate that it might be globally asymptotically stable as well.



Variant 1 throughput curves.

Scenario: 3 users with 2-state Markovian rate process.

$$\Phi = \begin{bmatrix} 8.68e - 4 & 0.885 & 0.535 \\ 0.165 & 0.984 & 0.39 \end{bmatrix}$$

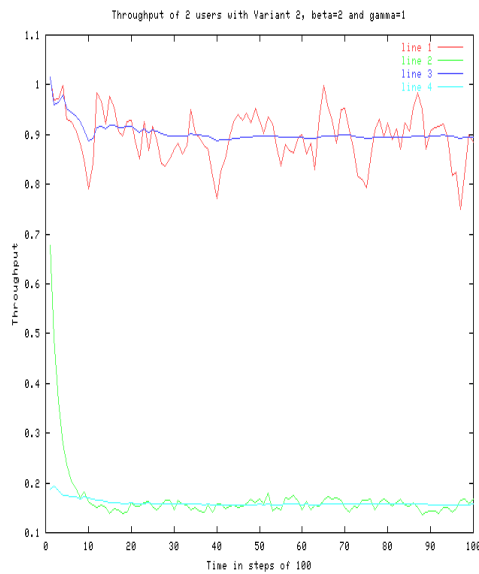
$$R = \begin{bmatrix} 0.994 & 0.972 & 0.235 \\ 0.257 & 0.975 & 0.515 \end{bmatrix}$$

$\alpha = 10, \phi = 0.0001, \text{time} = 100000.$

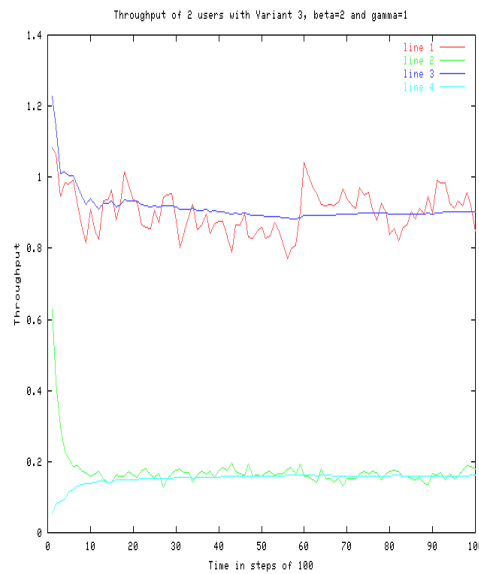
	User 1	User 2	User 3
Theory - $\beta = 2, \gamma = 0.5$	0.414	0.4	0.0678
Sim - $\beta = 2, \gamma = 0.5$	0.414	0.4	0.068
$\beta = 2, \gamma = 0$	0.415	0.401	0.0669

$\bar{W}$ : Theory 0.4223, Sim 0.4228, 0.4227, 0.4229.

Scenario: 2 users i.i.d Exponential rates  
 $R=[0.9905 \ 0.294]$ ,  $\beta = 2$ ,  $\gamma = 1$ ,  $\phi=0.005$ , time=10000.



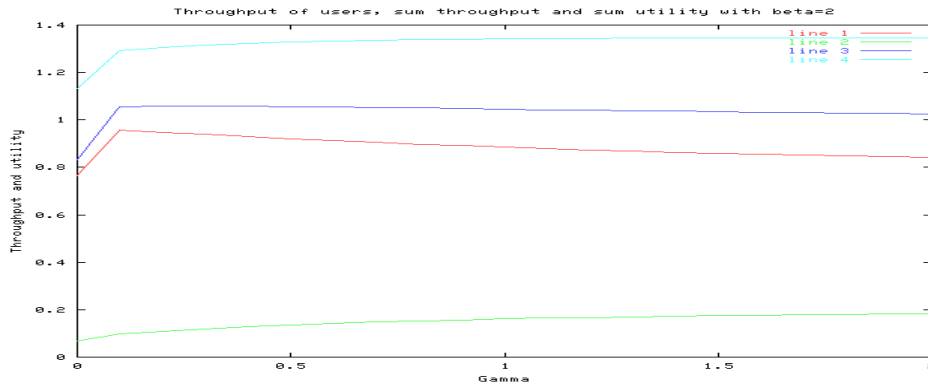
Variant 2 throughput



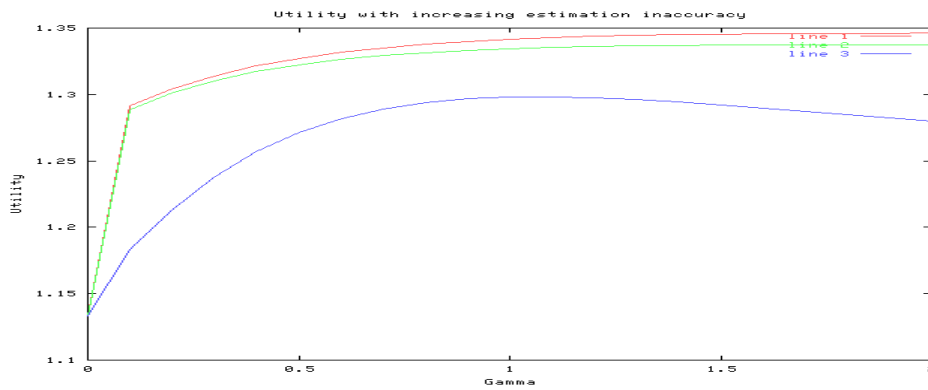
Variant 3 throughput

	User 1	User 2
Theory	0.885	0.161
V2 sim	0.895	0.157
V3 sim	0.901	0.161
$\beta = 2, \gamma = 0$	0.764	0.0673

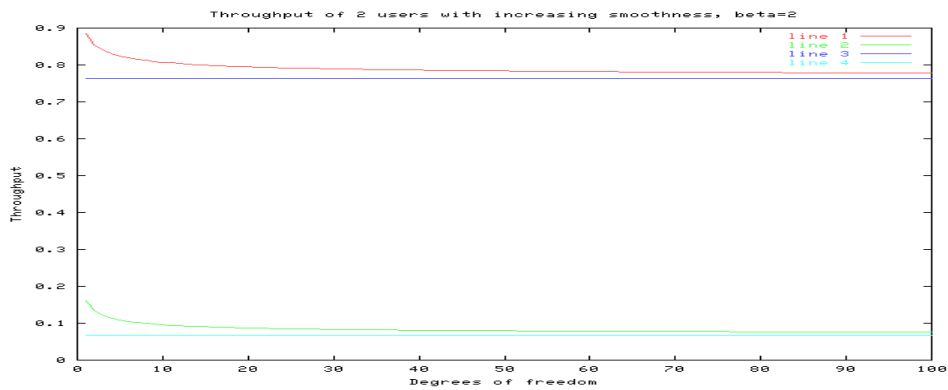




Variant 3 with  $\beta = 2$



Variant 3 with  $\beta = 2$  and different uncertainty in rates.



Variant 3 with  $\beta = 2$  with different distributions.

## Future Work

- Approximate solutions to aid in predicting performance or optimizations.
- Investigate the second-order performance - incorporates effect of the correlation structure of channel rate variations.
- Present analysis based on infinite-backlog assumption. What is the capacity region shaped like with queues and arrival processes?

## Conclusions

- Significant advantages to using current channel condition information. Tails of the distributions of the rate processes impact the gains.
- For small enough  $\phi$  Variant 1 does not exploit inherent multiuser diversity.
- For small enough  $\phi$  Variants 2 and 3 exhibit the same performance.
- For every  $\beta > 0$  there seems to be a best  $\gamma$  the value of which depends on the level of uncertainty of the current rates of the users.