Binary trees

; a database is
;  - false
;  - (make-db-node person database database)
(define-struct db-node (person left right))
Lookup for binary trees

; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible
(define (lookup ssn bt)
  (cond
    [(boolean? bt) false]
    [else
      (cond
        [(ssn-matches? ssn (db-node-person bt))
          (db-node-person bt)]
        [else
          (pick (lookup ssn
            (db-node-left bt))
            (lookup ssn
              (db-node-right bt)))]])])})
Helper function for binary trees

; pick : person-or-false person-or-false → person-or-false
; to pick whichever, p1 or p2, that is a person,
; or false if none are
(define (pick p1 p2)
  (cond
    [(and (boolean? p1) (boolean? p2))
     false]
    [(and (person? p1) (boolean? p2))
     p1]
    [(and (boolean? p1) (person? p2))
     p2]
    [(and (person? p1) (person? p2))
     p1]))

; ssn-matches? : ssn person → boolean
(define (ssn-matches? ssn person)
  (= ssn (person-ssn person)))
(lookup
  5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
(lookup
  5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false))))

=> (pick
    (lookup
      5
      (make-bst
        (make-person 2 'jon)
        (make-bst
          (make-person 1 'anna)
          false
          false)
        (make-bst
          (make-person 3 'ellen)
          false
          false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))
(pick
  (lookup
    5
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    false
    false))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
(pick
  (pick
    (lookup 5
      (make-bst
        (make-person 1 'anna)
        false
        false))
    (lookup 5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup 5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

<= (pick
  (lookup 5
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (lookup 5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))
(pick
 (pick
   (lookup 5
     (make-bst
       (make-person 1 'anna)
       false
       false))
   (lookup 5
     (make-bst
       (make-person 3 'ellen)
       false
       false)))
 (lookup 5
   (make-bst
     (make-person 6 'jane)
     (make-bst
       (make-person 5 'george)
       false
       false)
     (make-bst
       (make-person 7 'emily)
       false
       false))) =>
(pick
  (pick
    (lookup
      5
      (make-bst
        (make-person 1 'anna)
        false
        false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false))))
=> (pick
    (pick
      (pick
        (lookup 5 false)
        (lookup 5 false))
      (lookup
        5
        (make-bst
          (make-person 3 'ellen)
          false
          false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))
\[
\leq \quad (\text{pick}\\ \text{pick}\\ \text{pick}\\ (\text{lookup } 5 \ false)\\ (\text{lookup } 5 \ false)\\ (\text{lookup}\\ 5\\ (\text{make-bst}\\ \text{make-person } 3 \ 'ellen)\\ \false\\ \false))\\ (\text{lookup}\\ 5\\ (\text{make-bst}\\ \text{make-person } 6 \ 'jane)\\ (\text{make-bst}\\ \text{make-person } 5 \ 'george)\\ \false\\ \false)\\ (\text{make-bst}\\ \text{make-person } 7 \ 'emily)\\ \false\\ \false)))
\]
(pick
 (pick
   (pick false (lookup 5 false))
 (lookup
   5
   (make-bst
    (make-person 3 'ellen)
    false
    false)))
(lookup
  5
  (make-bst
   (make-person 6 'jane)
   (make-bst
    (make-person 5 'george)
    false
    false)
   (make-bst
    (make-person 7 'emily)
    false
    false))))

<= (pick
 (pick
   (lookup 5 false)
   (lookup 5 false))
 (lookup
   5
   (make-bst
    (make-person 3 'ellen)
    false
    false)))
 (lookup
  5
  (make-bst
   (make-person 6 'jane)
   (make-bst
    (make-person 5 'george)
    false
    false)
   (make-bst
    (make-person 7 'emily)
    false
    false)))))
(pick
(pick
  (pick false (lookup 5 false))
(lookup
  5
  (make-bst
    (make-person 3 'ellen)
    false
    false)))
(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false))))
(pick
  (pick
    (pick false (lookup 5 false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))) =>

(pick
  (pick false false false)
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))

(pick
  (pick false false false)
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))

(pick
  (pick false false false)
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))

(pick
  (pick false false false)
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))
(pick
  (pick
    (pick false false)
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)
    )
  )
(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false)))))
(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))))

(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false))))

<= (pick
  (pick
    (pick false false)
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))))

(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false)))
(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))) => 16
(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
=> (pick
    (pick
      false
      (pick
        (lookup 5 false)
        (lookup 5 false))
      (lookup
        5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
            false
            false)
          (make-bst
            (make-person 7 'emily)
            false
            false))))
(pick
  (pick false false
    (pick
      (lookup 5 false)
      (lookup 5 false)))
  (lookup 5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george) false false)
      (make-bst
        (make-person 7 'emily) false false))))
(pick
  (pick
   false
   (pick false (lookup 5 false)))
(lookup
  5
  (make-bst
   (make-person 6 'jane)
   (make-bst
    (make-person 5 'george)
    false
    false)
    (make-bst
     (make-person 7 'emily)
     false
     false))))

<=
(pick
  false
  (pick
   (lookup 5 false)
   (lookup 5 false))
(lookup
  5
  (make-bst
   (make-person 6 'jane)
   (make-bst
    (make-person 5 'george)
    false
    false)
    (make-bst
     (make-person 7 'emily)
     false
     false))))
(pick
  (pick false (lookup 5 false)))
(lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    false))
(=>) 20
(pick
  (pick
    false
    (pick false (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))
=>
(pick
  (pick false (pick false false))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))
<= (pick
  (pick false (pick false false))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))
)
(pick
  (pick false false)
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))))

<= (pick
    (pick false (pick false false))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false)))))
(pick (pick false false) =>
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false))))
(pick
  (pick false false)
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))))

=> (pick
    false
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))
(pick false (lookup 5
(make-bst
 (make-person 6 'jane)
(make-bst
 (make-person 5 'george)
 false
 false)
(make-bst
 (make-person 7 'emily)
 false
 false))))
(pick false (pick (lookup 5 (make-bst (make-person 5 'george) false false)) (lookup 5 (make-bst (make-person 7 'emily) false false)))))

<= (pick false (lookup 5 (make-bst (make-person 6 'jane) (make-bst (make-person 5 'george) false false) (make-bst (make-person 7 'emily) false false))))
(pick false
  (pick
    (lookup 5
      (make-bst
        (make-person 5 'george)
        false
        false)
    )
    (lookup 5
      (make-bst
        (make-person 7 'emily)
        false
        false)))
)) => 28
(pick false
   (pick
      (lookup 5
         (make-bst
            (make-person 5 'george)
            false
            false))
      (lookup 5
         (make-bst
            (make-person 7 'emily)
            false
            false))))
=> (pick false
    (pick
       (make-person 5 'george)
       (lookup 5
         (make-bst
            (make-person 7 'emily)
            false
            false))))
<= (pick false
  (pick
    (make-person 5 'george)
    (lookup
      5
      (make-bst
        (make-person 7 'emily)
        false
        false))))
(pick false
 (pick
   (make-person 5 'george)
   (pick
     (lookup 5 false)
     (lookup 5 false))))

<= (pick false
   (pick
     (make-person 5 'george)
     (lookup
      5
      (make-bst
       (make-person 7 'emily)
       false
       false))))
(pick false =>
  (pick
    (make-person 5 'george)
    (pick
      (lookup 5 false)
      (lookup 5 false))))
(pick false
  (pick
    (make-person 5 'george)
    (pick
      (lookup 5 false)
      (lookup 5 false))))

=> (pick false
    (pick
      (make-person 5 'george)
      (pick false (lookup 5 false))))
(pick false
 (pick
  (make-person 5 'george)
  (pick false (lookup 5 false))))
(pick false
  (pick
    (make-person 5 'george)
    (pick false false)))

<= (pick false
    (pick
      (make-person 5 'george)
      (pick false (lookup 5 false))))
(pick
false
(pick
 (make-person 5 'george)
(pick false false)))

=>
36
(pick false
  (pick
    (make-person 5 'george)
    (pick false false))))

=> (pick false
    (pick
      (make-person 5 'george)
      false))
\[
\begin{align*}
&\leq \quad (\text{pick} \\
&\quad \text{false} \\
&\quad (\text{pick} \\
&\quad \quad (\text{make-person} \ 5 \ '\text{george}) \\
&\quad \quad \text{false}))
\end{align*}
\]
(pick
false
(make-person 5 'george))

<= (pick
false
(pick
 (make-person 5 'george)
false))
(pick  
false  
(make-person 5 'george))
(pick false (make-person 5 'george))
(that took forever!)
Binary search trees

; a database is
; - false
; - (make-db-node person database[left] database[right])
; where all people in left have smaller ssns than person, and
; all people in right have larger ssns than person.
(define-struct db-node (person left right))
Lookup for binary search trees

; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible
(define (lookup ssn bt)
  (cond
   [(boolean? bt) false]
   [else
    (cond
     [(ssn-matches? ssn (db-node-person bt))
      (db-node-person bt)]
     [(ssn-less? ssn (db-node-person bt))
      (lookup ssn (db-node-left bt))]
     [(ssn-more? ssn (db-node-person bt))
      (lookup ssn (db-node-right bt)))]))))
Helper functions for binary search trees

; ssn-less? : ssn person → boolean
(define (ssn-less? ssn person)
  (< ssn (person-ssn person)))

; ssn-more? : ssn person → boolean
(define (ssn-more? ssn person)
  (> ssn (person-ssn person)))
(lookup 5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
(lookup 5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
    false
    false)
  false
  false))

=> (lookup 5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    false
    false)
  false
  false)))
\[
\texttt{\textless = (lookup}
\texttt{ 5}
\texttt{ (make-bst}
\texttt{ (make-person 6 'jane)}
\texttt{ (make-bst}
\texttt{ (make-person 5 'george)}
\texttt{ false}
\texttt{ false)}
\texttt{ (make-bst}
\texttt{ (make-person 7 'emily)}
\texttt{ false}
\texttt{ false)))}
\]
(lookup 5
  (make-bst
   (make-person 5 'george)
   false
   false)) <= (lookup 5
  (make-bst
   (make-person 6 'jane)
   (make-bst
    (make-person 5 'george)
    false
    false)
   (make-bst
    (make-person 7 'emily)
    false
    false)))
(lookup 5 =>
  (make-bst
   (make-person 5 'george)
   false
   false))
(lookup 5
  (make-bst
    (make-person 5 'george)
    false
    false)) => (make-person 5 'george)
(much quicker that time, no?)