Graphs

A node is:
(make-node symbol list-of-symbols)

A graph is either
empty
(cons node graph)
(define-struct node (name neighbors))

(define a-graph
  (list
    (make-node 'a (list 'b))
    (make-node 'b (list 'd 'e))
    (make-node 'c (list 'e 'f))
    (make-node 'd (list 'g))
    (make-node 'e (list 'g))
    (make-node 'f (list 'g))
    (make-node 'g empty)))
Graphs

; find-path : graph symbol symbol -> (listof symbol) or false
(define (find-path graph origin dest)
  (cond
   [(symbol=? origin dest) (list dest)]
   [else
    (maybe-cons origin
      (find-path/list
        graph
        (neighbors graph origin)
        dest))]))
;; find-path/list : graph (listof symbol) symbol
;; -> (listof symbol) or false
(define (find-path/list graph origins dest)
  (cond
   [(empty? origins) false]
   [else
    (pick-one
     (find-path graph
      (first origins) dest)
     (find-path/list graph
      (rest origins) dest))])))
Graphs

;; neighbors : graph symbol -> (listof symbol)
(define (neighbors graph s)
  (foldl (lambda (xsofar)
    (cond
      [(symbol=? s (node-name x))
       (node-neighbors x)]
      [elsesofar]))
    false
    graph))
Graphs

;; maybe-cons : symbol (listof symbol) or false
;; -> list-of-symbol or false
(define (maybe-cons a b)
  (cond
    [(boolean? b) b]
    [else (cons a b)]))

;; pick-one : (listof symbol) or false (listof symbol) or false
;; -> (listof symbol) or false
(define (pick-one a b)
  (cond
    [(boolean? a) b]
    [else a])))
(find a-graph 'b 'g) =>
(find a-graph 'b 'g) => (maybe-cons 'b
  (find/list a-graph
    (neighbors a-graph 'b)
    'g))
(\leq (\text{maybe-cons} 'b
(f\text{ind/list} a-\text{graph}
(\text{neighbors} a-\text{graph} 'b)
'g))
}
\( (\text{maybe-\text{return}} \ 'b \ (\text{find/list} \ \text{a-graph} \ 'd \ 'e) \ 'g)) \leq (\text{maybe-\text{return}} \ 'b \ (\text{find/list} \ \text{a-graph} \ \text{neighbors} \ 'b) \ 'g)) \)
(maybe-cons
  'b
  (find/list
    a-graph
    '(d e)
    'g)) =>
\[(\text{maybe-\text{cons}} \quad 'b \quad (\text{find/list} \quad \text{a-graph} \quad 'd \quad e) \quad 'g))\] 

\[\Rightarrow \quad (\text{maybe-\text{cons}} \quad 'b \quad (\text{pick-one} \quad (\text{find} \quad \text{a-graph} \quad 'd \quad 'g) \quad (\text{find/list} \quad \text{a-graph} \quad 'e \quad 'g)))\]
<=> (maybe-cons
   'b
   (pick-one
    (find
     a-graph
     'd
     'g)
    (find/list
     a-graph
     '(e)
     'g))))
(maybe-cons 'b
 (pick-one
   (maybe-cons 'd
     (find/list a-graph
       (neighbors a-graph 'd)
       'g))
   (find/list a-graph 'e 'g)))

<= (maybe-cons 'b
 (pick-one
   (find a-graph 'd
     'g)
   (find/list a-graph 'e 'g)))
(maybe-cons 'b =>
  (pick-one
   (maybe-cons 'd
     (find/list a-graph
       (neighbors a-graph 'd)
       'g))
   (find/list a-graph
     'e
     'g)))
\[
\begin{align*}
\text{(maybe-cons)} & \quad \text{=>} \quad \text{(maybe-cons)} \\
'\text{b} & \\
\text{(pick-one)} & \\
\text{(maybe-cons)} & \\
'\text{d} & \\
\text{(find/list a-graph)} & \\
\text{(neighbors a-graph)} & \\
'\text{d} & \\
'\text{g}) & \\
\text{(find/list a-graph)} & \\
'(\text{e}) & \\
'\text{g})) & \\
\end{align*}
\]
\[ \leq (\text{maybe-cons} \ \\
    'b \ \\
    (\text{pick-one} \ \\
      (\text{maybe-cons} \ \\
        'd \ \\
        (\text{find/list} \ \\
          \text{a-graph} \ \\
          '(g) \ \\
          'g)) \ \\
        (\text{find/list} \ \\
          \text{a-graph} \ \\
          '(e) \ \\
          'g)))) \]
(maybe-cons
  'b
  (pick-one
    (maybe-cons
      'd
      (pick-one
        (find
          a-graph
          'g
          'g)
        (find/list
          a-graph
          '(e)
          'g)))))
  (find/list
    a-graph
    '('g))
(maybe-cons  
  'b  
  (pick-one  
    (maybe-cons  
      'd  
      (pick-one  
        (find  
          a-graph  
          'g  
          'g)  
        (find/list  
          a-graph  
          '('  
            '('  
              'g)))  
      (find/list  
        a-graph  
        '('  
          '(e)  
          '(')))  
  =>}
(maybe-cons
 'b
 (pick-one
  (maybe-cons
   'd
   (pick-one
    (find
     a-graph
     'g
     'g)
    (find/list
     a-graph
     '() 'g))
   (find/list
    a-graph
    '(e) 'g))
 (find/list
 a-graph
 '(e) 'g))
 => (maybe-cons
 'b
 (pick-one
  (maybe-cons
   'd
   (pick-one
    (find
     a-graph
     'g
     'g)
    (find/list
     a-graph
     '() 'g)))
   (find/list
    a-graph
    '(e) 'g)))
\[ \leq (\text{maybe-cons} \ 'b \ (\text{pick-one} \ (\text{maybe-cons} \ 'd \ (\text{pick-one} \ ' (g) \ (\text{find/list} \ \text{a-graph} \ ' () \ ' (e) \ ' (g)))))) \]
(maybe-cons
  'b
  (pick-one
    (maybe-cons
      'd
      (pick-one
        '(g)
        false))
    (find/list a-graph
      '(e)
      'g)))
  (find/list a-graph
    'g)))

<= (maybe-cons
  'b
  (pick-one
    (maybe-cons
      'd
      (pick-one
        '(g)
        false))
    (find/list a-graph
      '(e)
      'g)))
  (find/list a-graph
    'g)))
(maybe-cons
  'b
  (pick-one
    (maybe-cons
      'd
      (pick-one
        '(g)
        false))
    (find/list
      a-graph
      '(e)
      'g)))
\[
\begin{align*}
&(\text{maybe-\text{cons}} \\
&\quad \text{'b} \\
&\quad (\text{pick-one} \\
&\quad \quad (\text{maybe-\text{cons}} \\
&\quad \quad \quad \text{'d} \\
&\quad \quad \quad (\text{pick-one} \\
&\quad \quad \quad \quad \text{'(g) } \\
&\quad \quad \quad \quad \text{false}) \\
&\quad \quad \text{(find/list} \\
&\quad \quad \quad \text{a-graph} \\
&\quad \quad \quad \quad \text{'(e) } \\
&\quad \quad \quad \quad \text{'g})) \\
&\quad = \quad (\text{maybe-\text{cons}} \\
&\quad \quad \text{'b} \\
&\quad \quad (\text{pick-one} \\
&\quad \quad \quad (\text{maybe-\text{cons}} \\
&\quad \quad \quad \quad \text{'d} \\
&\quad \quad \quad \quad \quad \text{'(g) } \\
&\quad \quad \quad \quad \quad (\text{find/list} \\
&\quad \quad \quad \quad \quad \text{a-graph} \\
&\quad \quad \quad \quad \quad \quad \text{'(e) } \\
&\quad \quad \quad \quad \quad \quad \quad \text{'g}))
\end{align*}
\]
\texttt{<= (maybe-cons 'b
   (pick-one
      (maybe-cons 'd
         '(g))
      (find/list a-graph
         '(e
            'g))\}))}
(maybe-cons
 'b
 (pick-one
   '(d g)
   (find/list
    a-graph
    '(e)
    'g))))

<= (maybe-cons
 'b
 (pick-one
   (maybe-cons
     'd
     '(g))
   (find/list
    a-graph
    '(e)
    'g))))
(maybe-cons
  'b
  (pick-one
    '(d g)
    (find/list
      a-graph
      '(e)
      'g))))
(maybe-cons
  'b
  (pick-one
    '(d g)
    (find/list
      a-graph
      'e
      'g))))

=> (maybe-cons
    'b
    (pick-one
      '(d g)
      (pick-one
        (find
          a-graph
          'e
          'g)
        (find/list
          a-graph
          '()
          'g)))))
\[ \leq (\text{maybe-cons}
\begin{array}{l}
'b \\
(pick-one
\begin{array}{l}
'(d\;g) \\
(pick-one
\begin{array}{l}
(find
\begin{array}{l}
a\text{-graph}
'e
'g)
(find/list
\begin{array}{l}
a\text{-graph}
'()
'g))))
\end{array}
\end{array}
\end{array}
\end{array}
\end{array} \]
(maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (find/list
          a-graph
          (neighbors
            a-graph
            'e)
            'g)))
      (find/list
        a-graph
        'g)))
  (find/list
    a-graph
    'g)))
<= (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (find
        a-graph
        'e
        'g)
      (find/list
        a-graph
        '() 'g))))
(maybe-cons
   'b
   (pick-one
     '(d g)
     (pick-one
       (maybe-cons
         'e
         (find/list
           a-graph
           (neighbors
             a-graph
             'e)
             'g))
         (find/list
           a-graph
           '()
           'g)))))
```scheme
(maybe-cons
 'b
 (pick-one
   '(d g)
   (pick-one
    (maybe-cons
     'e
     (find/list
      a-graph
      (neighbors
       a-graph
       'e)
       'g))
    (find/list
     a-graph
     '('g))))
 (find/list
  a-graph
  '()
  'g))))

=> (maybe-cons
  'b
  (pick-one
   '(d g)
   (pick-one
    (maybe-cons
     'e
     (find/list
      a-graph
      (neighbors
       a-graph
       'e)
       'g))
    (find/list
     a-graph
     '()
     'g)))))
```
<= (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (find/list a-graph
          '(g)
          'g)))
      (find/list a-graph
        '()
        'g))))
(maybe-cons =>
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (pick-one
          (pick-one
            (find
              a-graph
              'g
              'g)
            (find/list
              a-graph
              '() 'g)))
          (find/list
            (find
              a-graph
              'g
              'g))))
    (find/list
      (find
        a-graph
        'g
        'g)))
  (find/list
    (find
      a-graph
      '() 'g)))
  (find/list
    ...
(maybe-cons 'b
   (pick-one '(d g)
     (pick-one (maybe-cons 'e
       (pick-one (find a-graph 'g 'g)
         (find/list a-graph '() 'g))
       (find/list a-graph '() 'g))))
   (find/list a-graph '() 'g))) => (maybe-cons 'b
   (pick-one '(d g)
     (pick-one (maybe-cons 'e
       (pick-one (find a-graph 'g 'g)
         (find/list a-graph '() 'g))
       (find/list a-graph '() 'g)))))
<= (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (pick-one
          '(g)
          (find/list
            a-graph
            '()
            'g)))))
  (find/list
    a-graph
    '()
    'g)))))
(maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (pick-one
          '(g)
          false))
      (find/list a-graph
        '()
        'g))))
  (find/list a-graph
    '()
    'g))))
<= (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (pick-one
          '(g)
          false))
      (find/list a-graph
        '()
        'g)))
  (find/list a-graph
    '()
    'g))))
(maybe-cons =>
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        (pick-one
          '(g
            false))
        (find/list
          a-graph
          '()
          'g)))))
\[(\text{maybe-cons} \ 'b) (\text{pick-one} (\ 'd \ 'g)) (\text{pick-one} (\text{maybe-cons} 'e (\text{pick-one} 'g) \text{false})) (\text{find/list} \text{a-graph} (\ 'g)))) \]

\[\Rightarrow (\text{maybe-cons} \ 'b) (\text{pick-one} (\ 'd \ 'g)) (\text{pick-one} (\text{maybe-cons} 'e (\ 'g)) (\text{find/list} \text{a-graph} (\ () 'g))))\]
<= (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      (maybe-cons
        'e
        '(g))
      (find/list
        a-graph
        '()
        'g))))
(maybe-cons 'b
  (pick-one '(d g)
    (pick-one '(e g)
      (find/list a-graph '
        ()
        'g))))

<= (maybe-cons 'b
  (pick-one '(d g)
    (pick-one (maybe-cons 'e
        '(g))
      (find/list a-graph '
        ()
        'g)))))
(maybe-cons =>
  'b
  (pick-one
    '(d g)
    (pick-one
      '(e g)
      (find/list
        a-graph
        '()
        'g))))}
(maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      '(e g)
      (find/list
        a-graph
        '()
        'g)))))
=> (maybe-cons
  'b
  (pick-one
    '(d g)
    (pick-one
      '(e g)
      false)))
\[ \leq (\text{maybe-cons}
\begin{array}{ll}
'b \\
(pick-one
\begin{array}{ll}
'(d \ g) \\
(pick-one
\begin{array}{ll}
'(e \ g) \\
\text{false})
\end{array}
\end{array}
\end{array}
\]
(maybe-cons 'b
  (pick-one
    '(d g)
    '(e g)))) <= (maybe-cons 'b
  (pick-one
    '(d g)
    (pick-one
      '(e g)
      false)))
(maybe-cons
   'b
   (pick-one
      '(d g)
      '(e g))))
(maybe-cons
  'b
  (pick-one
    '(d g)
    '(e g))))

=> (maybe-cons 'b '(d g))
\( \leq (\text{maybe-cons} \ 'b \ ' (d \ g)) \)
' (b d g) <= (maybe-cons 'b ' (d g))
(find a-graph 'd 'e) =>
(find a-graph 'd 'e) => (maybe-cons
  'd
  (find/list
    a-graph
    (neighbors
      a-graph
      'd)
    'e))
\[ \leq \ (\text{maybe-cons} \ 'd \ (\text{find/list} \ a\text{-graph} \ (\text{neighbors} \ a\text{-graph} \ 'd) \ 'e)) \]
(maybe-cons
 'd
 (find/list
  a-graph
 ' (g)
 'e))

<= (maybe-cons
 'd
 (find/list
  a-graph
  (neighbors
   a-graph
    'd)
 'e))
(maybe-cons 'd =>
  (find/list a-graph
    '(g
     'e)))
\[(\text{maybe-cons} \ 'd \ (\text{find/list} \ a\text{-graph} \ 'g \ 'e)) \Rightarrow (\text{maybe-cons} \ 'd \ (\text{pick-one} \ (\text{find} \ a\text{-graph} \ 'g \ 'e) \ (\text{find/list} \ a\text{-graph} \ '() \ 'e)))\]
<= (maybe-cons
d
(pick-one
 (find
  a-graph
  'g
  'e)
 (find/list
  a-graph
  '()
  'e)))
(maybe-cons
  'd
  (pick-one
    (maybe-cons
      'g
      (find/list
        a-graph
        (neighbors
          a-graph
          'g)
          'e)))
    (find/list
      a-graph
      '() 'e)))
  (maybe-cons
  'd
  (pick-one
    (find
      a-graph
      'g
      'e)
    (find/list
      a-graph
      '() 'e)))
(maybe-cons 'd =>
  (pick-one
   (maybe-cons
    'g
    (find/list
     a-graph
     (neighbors
      a-graph
      'g)
    'e))
   (find/list
    a-graph
    '()
    'e))))
\[
\begin{align*}
\text{(maybe-cons} & \quad \text{find/list} \\
\quad \quad \text{'d} & \quad \text{a-graph} \\
\quad \quad \quad \text{(pick-one} & \quad \text{neighbors} \\
\quad \quad \quad \quad \text{(maybe-cons} & \quad \text{'g} \\
\quad \quad \quad \quad \quad \text{(find/list} & \quad \text{a-graph} \\
\quad \quad \quad \quad \quad \quad \text{('g)} & \quad \text{a-graph} \\
\quad \quad \quad \quad \quad \quad \quad \text{('e))} & \quad \text{a-graph} \\
\quad \quad \quad \quad \text{(find/list} & \quad \text{('()} & \quad \text{'e))} \\
\quad \quad \quad \quad \quad \text{('e))} & \quad \text{('e))}) \\
\end{align*}
\]
<= (maybe-cons
   'd
   (pick-one
    (maybe-cons
     'g
     (find/list
      a-graph
      '('()
      'e))
    (find/list
     a-graph
     '('()
     'e)))
   (find/list
    a-graph
    '('()
    'e)))
(maybe-cons
  'd
  (pick-one
    (maybe-cons
      'g
      false)
    (find/list
      a-graph
      '()
      'e)))

<= (maybe-cons
    'd
    (pick-one
      (maybe-cons
        'g
        false)
      (find/list
        a-graph
        '()
        'e)))
(maybe-cons  
  'd  
  (pick-one  
    (maybe-cons  
      'g  
      false)  
    (find/list  
      a-graph  
      '(')  
      '(')  
      'e))))
(maybe-cons
d
(pick-one
 (maybe-cons
g
false)
(find/list
 a-graph
 ()
 'e)))

=> (maybe-cons
d
(pick-one
false
(find/list
 a-graph
 ()
 'e)))
\leq (\texttt{maybe-\textup{cons}}
\begin{array}{r}
\texttt{'d} \\
(\texttt{pick-one}
\begin{array}{r}
\texttt{false} \\
(\texttt{find/list}
\begin{array}{r}
\texttt{a-graph} \\
\begin{array}{r}
\texttt{'}() \\
\texttt{'e}))
\end{array}
\end{array}
\end{array}
\end{array})
(maybe-cons 'd (pick-one false false))

<= (maybe-cons 'd (pick-one false false (find/list a-graph '() 'e) )))
(maybe-cons =>
  'd
  (pick-one
   false
   false))
(maybe-cons 'd
  (pick-one
    false
    false
    false)) => (maybe-cons 'd false)
\leq (\texttt{maybe-cons 'd false})
false  <= (maybe-cons 'd false)