Max Starting from either exponential version of \( \text{max} \):

\[
\text{max} : \text{non-empty-list-of-numbers} \rightarrow \text{number}
\]

\[
\text{to determine the largest number in alon}
\]

(\text{define} (\text{max} alon)
  (\text{cond}
    [(null? (cdr alon)) (first alon)]
    [\text{else} (\text{cond}
         [(< (car alon) (\text{max} (cdr alon)))
          (\text{max} (cdr alon))]
         [\text{else}
          (\text{car alon})])]))
\]

\[
\text{max} : \text{list-of-positive-numbers} \rightarrow \text{number}
\]

\[
\text{to determine the largest number in alon}
\]

(\text{define} (\text{max} alon)
  (\text{cond}
    [(null? alon) 0]
    [\text{else} (\text{cond}
         [(< (car alon) (\text{max} (car alon)))
          (\text{max} (cdr alon))]
         [\text{else}
          (\text{car alon})])]))
\]

use \text{let} to formulate an alternate version of that only uses a linear number of recursive calls.

\textbf{Solution}

\[
\text{max} : \text{non-empty list of numbers} \rightarrow \text{number}
\]

(\text{define} (\text{max} alon)
  (\text{cond}
    [(null? (cdr alon)) (car alon)]
    [\text{else} (\text{let} ([rst (\text{max} (cdr alon))])
         (\text{cond}
           [(< (car alon) rst)
            rst]
           [\text{else}
            (\text{car alon})]))]))
\]

\[
\text{max} : \text{list of positive numbers} \rightarrow \text{number}
\]

(\text{define} (\text{max} alon)
  (\text{cond}
    [(null? alon) 0]
    [\text{else} (\text{let} ([rst (\text{max} (cdr alon))])
         (\text{cond}
           [(< (car alon) rst)
            rst]
           [\text{else}
            (\text{car alon})]))]))
\]
**Best** Abstract over the comparison operator in `max` and produce a function `best` that accepts a function and returns the best element, according to that function.

Here’s the header and purpose statement for `best`:

```scheme
;; best : (integer integer -> boolean) list-of-numbers -> number
;; computes the best element in a list, according to metric
(define (best metric a-lon) ...
```

**Solution**

```scheme
(define (best metric a-lon)
  (cond
    [(null? a-lon) 0]
    [else (let ([rst (best metric (cdr a-lon))])
      (if (metric (car a-lon) rst)
          (car a-lon)
          rst)]))
```

Write both `max` and `min` in terms of `best`.

**Solution**

```scheme
(define (max l) (best >= l))
(define (min l) (best <= l))
```

**Filter** Write a function called `filter-lt7` that returns all of the elements in a list that are less than 7.

**Solution**

```scheme
;; filter-lt7 : list-of-numbers -> list-of-numbers
(define (filter-lt7 a-lon)
  (cond
    [(null? a-lon) '()]
    [else (if (<= (car a-lon) 7)
            (cons (car a-lon) (filter-lt7 (cdr a-lon)))
                   (filter-lt7 (cdr a-lon)))]))
```

Write the function `filter` that accepts a predicate on numbers and returns all of the elements in the list that satisfy the predicate. Here is the header and purpose statement for the function:

```scheme
;; filter : (number -> boolean) list-of-numbers -> list-of-numbers
;; returns a list of the elements in a-lon that satisfy p.
(define (filter p a-lon) ...
```

**Solution**

```scheme
;; filter : (number -> boolean) list-of-numbers -> list-of-numbers
;; returns a list of the elements in a-lon that satisfy p.
(define (filter p a-lon)
  (cond
    [(null? a-lon) '()]
    [else (if (p (car a-lon))
            (cons (car a-lon) (filter p (cdr a-lon)))
                   (filter p (cdr a-lon))))])
```
Using filter, re-define filter-lt7. Also, define filter-odd, a function that accepts a list and returns a list containing the odd numbers in its input.

Solution

\[
\text{(define (filter-lt7 l)} \\
\text{ (filter (lambda (x) (< x 7)) l))}
\]

\[
\text{(define (filter-odd l)} \\
\text{ (filter odd? l))}
\]

Building Lists Define a function called build-consecutive-list that accepts a number and returns a list of the number from that number down to 1.

Solution

\[
\begin{align*}
\text{;; build-consecutive-list : number \rightarrow (listof numbers)} \\
\text{(define (build-consecutive-list n)} \\
\text{ (cond} \\
\text{ \[(zero? n) \text{ '()}\]} \\
\text{ \[else} \\
\text{ \text{ (cons n (build-consecutive-list (- n 1)))})\]}
\end{align*}
\]

Define a function called build-list that accepts a number \(n\) and a function \(f\) from numbers to numbers and returns a list consisting of the results of calling \(f\) on the numbers from \(n\) down to 1.

Solution

\[
\begin{align*}
\text{;; build-list : number \rightarrow (listof numbers)} \\
\text{(define (build-list f n)} \\
\text{ (cond} \\
\text{ \[(zero? n) \text{ '()}\]} \\
\text{ \[else} \\
\text{ \text{ (cons (f n) (build-list f (- n 1)))})\]}
\end{align*}
\]

Use build-list to redefine build-consecutive-list and to define build-perfect-squares, a function that returns the first \(n\) perfect squares.

Solution

\[
\begin{align*}
\text{(define (build-consecutive-list l)} \\
\text{ (build-list (lambda (x) x) l))}
\end{align*}
\]

\[
\begin{align*}
\text{(define (build-perfect-squares l)} \\
\text{ (build-list (lambda (x) (\* x x)) l))}
\end{align*}
\]

Binding Structure Draw arrows to indicate the binding structure of these programs [note: DrScheme’s Check Syntax will tell you the answers to this question, but you will not have Check Syntax available to you on the exam. Be sure you can figure these out by yourself]:

\[
\begin{align*}
\text{(define (f x)} \\
\text{ x)} \\
\text{(f 1)}
\end{align*}
\]

\[
\begin{align*}
\text{(define (len l)} \\
\text{ (cond} \\
\text{ \[(null? l) 0\]} \\
\text{ \[else (+ (len (cdr l)) 1))])}
\end{align*}
\]
\begin{verbatim}
(define (quad x)
  (let ([sq (* x x)])
    (* sq sq)))

(define (g x)
  (let ([x x])
    (+ x x)))

(define (g x)
  (let ([x x] [y x])
    (+ x x)))
\end{verbatim}

\textbf{Solution} Use Check Syntax in DrScheme.