Finger Exercises  Determine the value of the following expressions in DrScheme.

\[
(+ (+ (+ 1 2) 3) 4)
\]
\[
(+ 1 2 3 4)
\]
\[
(/ 1 (* 2 3))
\]
\[
(sqrt 9)
\]
\[
(sqrt 10)
\]
\[
(define (f2c f)
  (* (- f 32) 5/9))
\]
\[
(f2c 32)
\]
\[
(f2c 212)
\]
\[
(f2c −40)
\]

Solution

10
10
1/6
3
3.1622776601683795
0
100
−40

Hand Evaluation  Determine the value of this expression by hand. Show all of the steps that you took. (There is more than one way to do this problem.)

\[
(− (* (sqrt 36) (/ 1 2)) (+ 1 2))
\]

Solution  There is more than one path to take to solve this problem. Here are all of them, represented with arrows and boxes. Any of these paths is okay.
Algebraic Notation vs Scheme Notation  Translate the following expressions from algebraic notation to prefix parenthesized notation. Evaluate them in DrScheme. (Do any of these have more than one translation?)

1 + 2 * 3 + 4

\( f(x) = 1 - x^2 \)

\( f(f(\frac{1}{2})) \)

\( g(x) = f(x) * f(x) \)

\( g(12) \)

Solution

\[ (+ (+ 1 (* 2 3)) 4) \]

\[ (+ 1 (+ (* 2 3) 4)) \]

\[ (+ 1 (* 2 3) 4) \]
Auxiliary Function Definitions  Imagine the owner of a movie theater who has complete freedom in setting ticket prices. The more he charges, the fewer the people who can afford tickets. In a recent experiment the owner determined a precise relationship between the price of a ticket and average attendance. At a price of $5.00 per ticket, 120 people attend a performance. Decreasing the price by a dime ($0.10) increases attendance by 15. Unfortunately, the increased attendance also comes at an increased cost. Every performance costs the owner $180.00. Each attendee costs another four cents ($0.04). The owner would like to know the exact relationship between profit and ticket price so that he can determine the price at which he can make the highest profit.

Being a careful student of program design, the movie owner writes a series of helper functions to describe the precise relationships between the profit and the ticket price:

```
;;; profit-first : number → number
;;; determines the profit of the program
(define (profit ticket-price)
  (− (revenue ticket-price)
      (cost ticket-price)))

;;; revenue : number → number
;;; determines the revenue for a particular ticket price.
(define (revenue ticket-price)
  (* (attendees ticket-price)
     ticket-price))

;;; cost : number → number
;;; determines the cost for a particular ticket price.
(define (cost ticket-price)
  (+ 180
     (* .04 (attendees ticket-price)))))

;;; attendees : number → number
;;; determines the number of attendees for a particular ticket price.
(define (attendees ticket-price)
  (+ 120
     (* (/ 15 .10)
        (− 5.00 ticket-price)))))
```

The owner’s neighbor also got interested in this problem and wrote a program to calculate the relationship between the profit and the ticket price:

```
(define (profit ticket-price)
  (− (* (+ 120
          (* (/ 15 .10)
             (− 5.00 ticket-price)))))
```

```
Verify that both programs produce the same output for ticket prices of $4.00, $5.00, and $6.00.

\[ (\neg 5.00 \text{ticket-price})) \]
\[ (+ 180) \]
\[ (* .04) \]
\[ (+ 120) \]
\[ (* (/ 15 .10) \]
\[ (- 5.00 \text{ticket-price}))))) \]

Auxiliary Function Definitions and Program Maintenance  After studying the cost structure of his show, the owner discovered several ways of lowering the cost. As a result of his improvements, he no longer has a fixed cost. He now simply pays $1.50 per attendee.

Modify both programs to reflect this change. Verify that they still produce the same results for $4.00, $5.00, and $6.00.

Solution

\[ ;; \text{profit-first} : \text{number} \rightarrow \text{number} \]
\[ ;; \text{determines the profit of the program} \]
\[ \text{(define) (profit-first ticket-price)} \]
\[ (- (\text{revenue ticket-price)) \]
\[ (\text{cost ticket-price})) \]

\[ ;; \text{revenue} : \text{number} \rightarrow \text{number} \]
\[ ;; \text{determines the revenue for a particular ticket price.} \]
\[ \text{(define) (revenue ticket-price)} \]
\[ (* (\text{attendees ticket-price)} \text{ticket-price})) \]

\[ ;; \text{cost} : \text{number} \rightarrow \text{number} \]
\[ ;; \text{determines the cost for a particular ticket price.} \]
\[ \text{(define) (cost ticket-price)} \]
\[ (* 1.50 (\text{attendees ticket-price})) \]

\[ ;; \text{attendees} : \text{number} \rightarrow \text{number} \]
\[ ;; \text{determines the number of attendees for a particular ticket price.} \]
\[ \text{(define) (attendees ticket-price)} \]
\[ (+ 120) \]
\[ (* (/ 15 .10) (- 5.00 \text{ticket-price}))))) \]

\[ ;; \text{profit-second} : \text{number} \rightarrow \text{number} \]
\[ ;; \text{determines the profit of the program} \]
\[ \text{(define) (profit-second ticket-price)} \]
\[ (- (+ 120) \]
\[ (* (/ 15 .10) \]
\[ (- 5.00 \text{ticket-price}))))) \]
\[ \text{ticket-price}) \]
\[ (* 1.50 (+ 120) \]
\[ (* (/ 15 .10) \]
\[ (- 5.00 \text{ticket-price}))))) \]

(profit-first 3.00)
(profit-second 3.00)
<table>
<thead>
<tr>
<th></th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit-First</td>
<td>4.00</td>
</tr>
<tr>
<td>Profit-Second</td>
<td>4.00</td>
</tr>
<tr>
<td>Profit-First</td>
<td>5.00</td>
</tr>
<tr>
<td>Profit-Second</td>
<td>5.00</td>
</tr>
</tbody>
</table>