Performance Bounds for Computational Imaging

Oliver Cossairt
Assistant Professor
Northwestern University

Collaborators: Mohit Gupta\textsuperscript{1}, Changyin Zhou\textsuperscript{1}, Daniel Miau\textsuperscript{1}, Shree Nayar\textsuperscript{1}, Kaushik Mitra\textsuperscript{2}, Ashok Veeraraghavan\textsuperscript{2}

\textsuperscript{1}Columbia University
\textsuperscript{2}Rice University
Computational Imaging: Increased Functionality

- Take multiple pictures and computationally combine

- HDR Imaging
- Panoramic Stitching
- Light Field Capture
- Image-Based Lighting
- Digital Holography

Others
- Multiview Stereo
- Depth from Focus/Defocus
- Tomography
- Structured Light
- Deconvolution microscopy
- etc.

[Debevec et al. ’00]
[Greenbum et al. ’12]
[Wilburn et al. ’04]
Computational Imaging: Increased Performance

Coded image capture for increased performance

Coded Aperture
- [Mertz ’65]
- [Gottesman ’89]

Defocus Blur
- [Hausler ’72]
- [Nagahara ’08]
- [Dowski, Cathey ’96]
- [Levin et al. ’07]
- [Zhou, Nayar ’08]

Motion Blur
- [Raskar ’06]
- [Levin ’08]
- [Cho ’10]

Multi/Hyper-Spectral
- [Sloane ’79]
- [Hanley ’99]
- [Baer ’99]
- [Wetzstein et al., ’12]

Light Field Capture
- [Lanman ’08]
- [Veeraraghavan ’07]
- [Liang ’08]

Reflectance
- [Schechner ’03]
- [Ratner ’07]
- [Ratner ’08]
Computational Imaging Performance

Short Exposure Vs. Long Exposure

- Short Exposure: 50 millisec
- Long Exposure: 50 millisec
Computational Imaging Performance

When does computational imaging improve performance?

[Raskar '06]
Measuring Computational Imaging Performance
Image Formation Model

Assumption:

A) Linear model of incoherent image formation
Affine Noise Model

Noise Variance at \( k^{\text{th}} \) Pixel:

\[
\sigma^2_k = J_k + \sigma^2_r
\]

- Signal dependent / independent noise
- Ignore Dark current, fixed pattern

Noise PDF:

\[
n \sim N(\overline{J}, \sigma^2 I)
\]

\[
\overline{J} = \sum_k J_k \quad \sigma^2 = \overline{J} + \sigma^2_r
\]

- Photon noise modeled as Gaussian (ok for more than 10 photons)
- Photon noise spatially averaged

Assumption:

B) Affine noise model (photon noise is Gaussian)
Lighting Conditions

Signal-level and photon noise depend on illumination

\[
\bar{J} \approx 10^{15} \cdot I_{\text{src}} \cdot R \cdot (F / \#)^{-2} \cdot t \cdot q \cdot p^2
\]

Average Signal (e^-) \hspace{1cm} Illumination (lux) \hspace{1cm} Reflectivity \hspace{1cm} Aperture \hspace{1cm} Exposure Time (s) \hspace{1cm} Quantum Efficiency \hspace{1cm} Pixel Size (m)

Ex) \( q = 0.5, \ R = 0.5, \ F/8, \ t = 6\text{ms}, \ p = 6\text{um} \)

<table>
<thead>
<tr>
<th>Illumination ( I_{\text{src}} ) (lux)</th>
<th>Quarter moon</th>
<th>Full moon</th>
<th>Twilight</th>
<th>Indoor lighting</th>
<th>Cloudy day</th>
<th>Sunny day</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{-2}</td>
<td>10^{-2}</td>
<td>10</td>
<td>10^2</td>
<td>10^3</td>
<td>10^4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>8\times10^{-3}</td>
<td>0.8</td>
<td>8.1</td>
<td>81.4</td>
<td>814.3</td>
<td>8143</td>
</tr>
</tbody>
</table>

Assumption:

C) Naturally occurring light conditions for photography

[Cossairt et al. TIP ‘12]
For Gaussian noise, Mean-Squared-Error (MSE) can be computed analytically

\[ \text{MSE}(H) = \sigma^2 \sqrt{\frac{N}{\text{Tr}(H^{-1} H^{-1})}} \]

Ex) Coded Motion Deblurring

**Observation:**

1) Multiplexing performance depends on coding matrix
Observation:

2) Multiplexing increases signal-dependent noise
Observation:

3) Performance depends on multiplexing and signal prior.
Image Prior Models

Assume we have a PDF for images, e.g.

\[ P(x) = \exp \left( \| B \cdot x \|^{\alpha} \right) \]

Other priors
- Total Variation (TV)
- Wavelet/sparsity prior
- Learned priors (K-SVD)

Compute the Maximum A Posteriori (MAP) estimate

\[ x^* = \underset{x}{\text{argmax}} \left( \| y - H \cdot x \|^2 + \| B \cdot x \|^\alpha \right) \]

MSE difficult to express analytically when \( \alpha \neq 2 \)

**Assumption:**

\( D) \) Signal prior models naturally occurring images
Image Priors and Noise

**Observation:**

4) Signal priors help more at low light levels
Observations:
1) Multiplexing performance depends on coding matrix
2) Multiplexing helps most in low light
3) Performance depends on both multiplexing and signal prior
4) Signal priors help most in low light

Assumptions
A) Incoherent imaging
B) Affine noise model
C) Natural lighting conditions
D) Natural image prior

When does computational imaging improve performance?
Example:
Motion Deblurring
Motion Deblurring vs. Impulse Imaging

What is the best possible coding performance we can get?

$$MSE > \sigma^2 \frac{(N-C) + C(N-1)^2}{(N-C)NC^2}$$

[Rate '07]
When Does Motion Deblurring Improve Performance?

Upper Bound on SNR Gain:

\[ G < \sqrt{2 \cdot \left( 1 + \frac{\sigma_0^2}{J} \right)} \]

Performance depends only on lighting conditions!

\[ q=.5, R = .5, F/2.1, p = 1um, \sigma_r = 4e^- \]

[Cossairt et al. TIP '12]
Flutter Shutter Simulation

$q = .5$, $R = .5$, $F/2.1$, pixel size = 1um, read noise $\sigma_r = 16\epsilon$

Twilight (10 lux)

- PSNR = -7.2 dB

Cloudy Day ($10^3$ lux)

- PSNR = 12.4 dB

Flutter Shutter (180ms)

- PSNR = -3.0 dB

Deblurred

- PSNR = 10.1 dB
Example: Extended DOF Imaging
Depth of Field and Noise

Small apertures have large depth of field and low SNR
Focal Sweep

Sensor  Lens

400  600  900  1200  1500  1700  2000  (depth)

Point Spread Function (PSF)

[Hausler ‘72, Nagahara et al. ‘08]
Focal Sweep

Sensor  

Lens  

\[ \text{Integrated PSF} \]

\[ \text{[Hausler '72, Nagahara et al. '08]} \]
Quasi Depth Invariant PSF

Extended depth of field with a single deconvolution
Extended Depth of Field Telescope

Meade LX200 8” Telescope
2000mm FL

Focal Sweep: Processed

Traditional Image

75 m
50 m
Focal Sweep Simulation

Pixel size = 5um
Read noise $\sigma_r = 16e$

**Twilight**
(10 lux)

- **Traditional** (F/2.0)
  - PSNR = 5.5 dB

- **Traditional** (F/20.0)
  - PSNR = 18.5 dB

- **Focal Sweep** (F/2.0)
  - PSNR = 35 dB
  - PSNR = 38.5 dB

**Daylight**
($10^5$ lux)
Focal Sweep Simulation (with Prior)

Pixel size = 5um
Read noise $\sigma_r = 16e^-$

Twilight
(10 lux)

Daylight
($10^5$ lux)

<table>
<thead>
<tr>
<th>Traditional (F/2.0)</th>
<th>Traditional (F/20.0)</th>
<th>Focal Sweep (F/2.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR = 16.4 dB / 5.5 dB</td>
<td>PSNR = 22.8 dB / 18.5 dB</td>
<td></td>
</tr>
<tr>
<td>PSNR = 35.9 dB / 35 dB</td>
<td>PSNR = 39.6 dB / 38.5 dB</td>
<td></td>
</tr>
</tbody>
</table>

BM3D Algorithm: [Dabov et al. '06]
A Universal Image Prior

State-of-the-art priors are hard to analyze!

Gaussian Mixture Model (GMM) prior

\[ P(x) \sim \sum_{k=1}^{K} \alpha_k N(\mu_k, \Sigma_k) \]

- GMM parameters learned from database of 30 Million image patches
- Analytic expression for MSE depending only on

\[ MSE(H, \sigma^2, \alpha_k, \mu_k, \Sigma_k) \]

[Mitra et al., submitted to PAMI ’13]
Focal Sweep Performance with GMM Prior

Impulse camera: \( F/11 \), Focal Sweep: \( F/1 \)

\[ q = .5, \quad R = .5, \quad t = \text{6ms}, \quad p = \text{1um}, \quad \sigma_r = 4e^- \]

![Graph showing SNR gain vs. illuminance for different lighting conditions with and without GMM prior.](image)
Defocus Deblurring Performance with GMM Prior

Defocus Deblurring gain as high as 8dB for Cubic Phase Plate

[Mitra et al., submitted to TIP '13]
Conclusions

• Results for Motion Deblurring, EDOF also applicable to many other computational cameras

• Computational imaging performance should always be measured relative to *impulse imaging*

• Computational imaging performance depends jointly on *multiplexing, noise, and signal priors*

• Important question: “How much performance improvement from multiplexing *above and beyond* use of signal priors?”
Depth of Field

Microscope

Tachinid Fly

Small DOF

http://en.wikipedia.org/wiki/Focus_stacking
Depth of Field

Microscope → Tachinid Fly

Large DOF

http://en.wikipedia.org/wiki/Focus_stacking
Telephoto Focal Sweep with Deformable Optics

Canon 800mm EFL Lens

Sensor

Deformable Lens

[Miau et al. ICCP '13]
Video Quality Comparison

Conventional

EDOF (Deformable Lens)
Focal Sweep Performance

Noise Variance:

\[ \sigma_i^2 = \bar{J} + \sigma_r^2 \]

Mean-Squared Error:

\[ MSE = \sigma_i^2 \]

Noise Variance:

\[ \sigma_m^2 = C^2 \cdot \bar{J} + \sigma_r^2 \]

Mean-Squared Error:

\[ MSE \approx \frac{\sigma_m^2}{2C^2} \]

[Cossairt et al. TIP '12]
When Does Defocus Deblurring Improve Performance?

Focal sweep multiplexing gain can be expressed analytically

\[
G < \sqrt{2 \cdot \left(1 + \frac{\sigma_0^2}{J}\right)}
\]

Performance depends only on lighting conditions!

\[q = .5, R = .5, t = 20ms, p = 5\mu m, \sigma_f = 4e^-\]
Focal Sweep Without Moving Parts

Focal Sweep

Image → Lens

Diffusion Coding (No Moving Parts)

Image → Lens → Radial Diffuser

[Cossairt et al. Siggraph '10]

500 x 3 micron
What is the Optimal Aperture Size?

Impulse Camera
F/16

- Too Noisy
- Pro: 64x Light (SNR ↑↑)
- Con: Deblur (SNR ↓↓)

Focal Sweep

- 2° diffuser

Performance is independent of aperture size
Diffusion Coding vs. Traditional Camera

Traditional F/1.8

Traditional F/18 (Normalized)

Diffusion Coding F/1.8 (Captured)

Diffusion Coding F/1.8 (Deblurred)
Example:
Deblurring vs. Denoising
Example:
Computational Aberration Removal
Computational Gigapixel Camera

Also See: MOSAIC Program, Duke, UCSD, Distant Focus
Resolution vs. Lens Scale

PSF size increases linearly
Resolution vs. Lens Scale

PSF size increases linearly

Deblurring Error is sub-linear

\[ \sigma_d \propto M^{2/3} \]

[Cossairt et al. JOSA '11]