Performance Analysis of MMSE Receivers for DS-CDMA in Frequency-Selective Fading Channels

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***Abstract*—The performance of the minimum mean-squared error (MMSE) receiver for detection of direct sequence code division multiple access is considered in various fading channel models. Several modifications to the basic MMSE receiver structure which have been recently proposed for use on nonselective fading channels are reviewed and shown to represent different approximations to a single common form. The performance of this general structure is analyzed as well as various extensions suitable for frequency-selective fading channels. Particular attention is given to the performance advantage gained through knowledge of the fading parameters of the various transmission paths of each user's signal. It is shown that having this knowledge is not particularly useful on a flat fading channel unless the loading is very heavy and even then the difference in performance is only minimal. On the other hand, having this knowledge is crucial in a multipath fading channel and the inability to learn the fading channel parameters will lead to substantial degradation in capacity. A heuristic explanation to support this result based on a dimensionality argument is also presented.**

***Index Terms*—Code-division multiple access, fading channels, multiuser channels, signal detection, spread-spectrum communication.**

I. Introduction

THE MINIMUM mean-squared error (MMSE) receiver for detection of direct-sequence code-division multiple-access (DS-CDMA) is receiving significant attention as it offers an attractive tradeoff between performance, complexity, and the need for side information. The MMSE receiver was first developed by Xie *et al.* [1] as a nonadaptive receiver. This was followed by various adaptive implementations which operated in a decision-directed mode [2]–[5]. Later it was shown in [6] that the MMSE receiver can be operated in a blind mode, alleviating the need for training. Most of the work up to this point dealing with the MMSE receiver and its adaptive implementations has assumed a Gaussian noise channel model. A few exceptions are the work in [7], [8], and [10], where a flat fading channel is considered, and [11], where a slowly time-varying multipath fading channel (single user) is considered. Also, [12]–[14] have considered multiple users in a frequency-selective fading model.

Our goal in this paper is to evaluate the theoretical performance of the MMSE receiver in a general fading channel that may be either frequency-selective or -nonselective. While it is well known that the MMSE receiver is theoretically capable of performing RAKE-like multipath combining on a frequency-selective channel, current adaptive implementations have not been able to achieve this type of performance except when the channel fade rate is very slow. Several authors (including ourselves) [12], [14]–[16] have suggested using a multiple adaptive filter structure where there is a separate adaptive filter for each resolvable transmission. The outputs of these filters could then be combined coherently using explicit channel tracking for each path of the desired user, or they could be combined differentially with equal gains. Doing either relieves the MMSE receiver from having to track the multipath fading for the desired user which is often too fast for typical adaptive algorithms to track. One of the main results of this paper shows that while explicitly tracking the fading parameters for each transmission path of the desired user allows the receiver to retain the diversity advantage of a RAKE receiver, multipath fading can still cause significant degradation in performance of an MMSE receiver. To obtain the full benefits of the MMSE receiver, the receiver must have knowledge of the fading parameters from each path of *all* user's transmissions. A standard MMSE receiver will implicitly learn this information if the fading rate is sufficiently slow, but current adaptive implementations do not seem to be able to achieve this kind of performance at fading rates that are of practical interest. In this paper, we demonstrate through analytical performance analysis the substantial difference in the performance of the MMSE receiver when operated on a slow versus a fast fading channel. It is hoped that the thorough analysis presented of the MMSE receiver in a frequency-selective fading channel will help researchers to develop better techniques to practically achieve the full potential promised by the MMSE receiver.

The paper is organized as follows. Section II lays out the system model for DS-CDMA in a frequency-selective fading channel. The MMSE receiver is briefly described in Section III along with a review of the various modifications that have been proposed for operation in fading channels. It is shown that the blind minimum output energy receiver of Honig et al. [6], the modified MMSE receiver of Barbosa and Miller [7], [8], and the differential RLS receivers, proposed by Honig *et al.* [10] and Zhu and Madhow [17], all represent approximations to a common ideal form. Possible extensions of this ideal form are then presented for a frequency-selective fading channel. The performance analysis of the modified MMSE receivers

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presented in Section III is given in Section IV and Appendix A. Numerical results based on this performance analysis are given in Section V and a heuristic explanation for the results observed are given in Section VI.

II. System Model

A standard model for asynchronous DS-CDMA is assumed. The kth user transmits a signal of the form (complex baseband)

$$S_k(t) = \sqrt{2}\sum_{m=0}^{M} d_k(m)c_k(t-mT_s)\,\exp(j\phi_k') \tag{1}$$

where ϕ_k' and $d_k(m)$ are the carrier phase and mth differentially encoded[1] data bit respectively, for the kth user. The unspread symbol duration is T_s and hence the baud rate is $1/T_s$. The spreading waveform is given by

$$c_k(t) = \sum_{n=0}^{N-1} c_{k,n}\psi(t-nT_c) \tag{2}$$

where $\psi(t)$ is the chip pulse shape, and T_c is the chip duration. For simplicity, in the remainder of this work, we take $\psi(t)$ to be a square pulse on the interval [0, T_c), but there is no fundamental reason why a different chip pulse shape could not be used. In order to allow for linear MMSE detection, the period of the spreading sequence is taken to be equal to the number of chips per bit. That is, $N = T_s/T_c$.

The received signal is taken to be of the form

$$R(t) = \sum_{k=1}^{K} R_k(t) + N_w(t) \tag{3}$$

where $R_k(t)$ is the received signal from the kth user and $N_w(t)$ is complex white Gaussian noise with $E[N_w(t)N_w^*(s)] = 2N_o\delta(t-s)$. Assuming a fading multipath channel, each received signal takes on the form

$$R_k(t) = \sum_{r=1}^{Q_k}\sqrt{P_{k,r}}\gamma_{k,r}(t)S_k(t-\tau_{k,r}) \tag{4}$$

where Q_k is the number of paths for the kth signal, and $P_{k,r}$, $\gamma_{k,r}(t)$, and $\tau_{k,r}$ are the received power, the complex fading process (normalized such that $E[|\gamma_{k,r}(t)|^2] = 1$]), and the relative delay for the rth received path of the kth user's signal, respectively. The fading processes are taken to be zero mean Gaussian random processes with autocorrelation functions given by $R_\gamma(\tau) = J_o(2\pi f_{D,k}\tau)$, where $f_{D,k}$ is the maximum Doppler frequency of the kth user's signal and is dependent on that user's vehicle speed relative to that of the receiver. For simplicity, it is assumed that $f_{D,k} = f_D$ for all k. Furthermore, the fading process for each path is taken to be statistically independent of the fading process for all other paths. User number one is assumed to be the desired user and it is further assumed that the receiver's clock is synchronized with the reception of the first path of the desired user. That is, $\tau_{1,1}$ is taken to be zero. Without loss of generality, $\tau_{k,1}$ is taken to be uniformly distributed over $[0, T_s)$. At this point, the delays of the secondary paths are left unspecified, however, we will operate on the assumption throughout the rest of this paper that the delay spread is constrained to be greater than one chip but less than one symbol interval, (i.e., $T_c < \tau_{1,Q_1} < T_s$). This assumption keeps the intersymbol interference (ISI) to a minimum and greatly simplifies our analytical results. The reader is directed to [13] for recent work dealing with channels with a longer delay spread.

III. The MMSE Receiver and Fading Channels

In order to clarify some of the approaches taken in later sections, a brief review is given here of the MMSE receiver and some modifications necessary for operation in fading channels. The MMSE receiver takes the signal at complex baseband and passes it through a chip matched filter and samples the output of that filter at the chip rate and synchronous with the reception of the desired user's first path. N chip samples are stored for each symbol received and together these chip samples form the "received vector" for the mth symbol

$$\boldsymbol{r}(m) = (r_{mN},\, r_{mN+1},\, \ldots\, r_{(m+1)N-1})^T \tag{5a}$$

$$r_i = \int \psi(t-iT_c)R(t)\,dt. \tag{5b}$$

The MMSE receiver filters this received vector with a finite impulse response, possibly time-varying, discrete filter characterized by the N-element tap weight vector $\boldsymbol{w}(m)$. That is, during each symbol interval, the MMSE receiver forms $z(m) = \boldsymbol{w}^H(m)\boldsymbol{r}(m)$. The data symbol decision is then based on the output of this filter, $z(m)$.

Traditionally, the MMSE receiver has been operated in a coherent manner, in which case the decisions are made according to $\hat{d}_1(m) = \text{sgn}(Re[z(m)])$. If the transmitted data bits are differentially encoded, then the coherently detected data can be differentially decoded to form $\hat{b}_1(m) = \hat{d}_1(m)\hat{d}_1(m-1)$. For reasons to be explained later, this approach can lead to difficulties on a fading channel and so it is also possible to use differential detection on the output of the MMSE filter. In which case, the data decisions are formed according to $\hat{b}_1(m) = \text{sgn}(Re[z(m)z^*(m-1)])$.

The tap weights of the MMSE filter are chosen to minimize the mean-squared error

$$J(m) = E\left[|e(m)|^2\right] = E\left[|d_1(m) - z(m)|^2\right]. \tag{6}$$

It is well known that the tap weight vector which minimizes this mean squared error is given by $\boldsymbol{w}(m) = \boldsymbol{R}^{-1}(m)\boldsymbol{p}(m)$, where $\boldsymbol{R}(m) = E[\boldsymbol{r}(m)\boldsymbol{r}^H(m)]$ and $\boldsymbol{p}(m) = E[d_1^*(m)\boldsymbol{r}(m)]$. In order to study the characteristics of this tap weight vector, the form of the received vector is specified

$$\begin{aligned}\boldsymbol{r}(m) = \sum_{k=1}^{K}\sum_{r=1}^{Q_k}\sqrt{\frac{P_{k,r}}{P_1}}\,\gamma_{k,r}(m) \\ \cdot\left[d_k(m-L_{k,r}-1)\boldsymbol{c}_k^L(NT_c-\mu_{k,r})\right. \\ \left. + d_k(m-L_{k,r})\boldsymbol{c}_k^R(\mu_{k,r})\right] + \boldsymbol{n}(m)\end{aligned} \tag{7}$$

where

$$L_{k,r} = \lfloor \tau_{k,r}/T_s \rfloor$$
$$\mu_{k,r} = \tau_{k,r} - L_{k,r}T_s$$

[1]The actual data are given by $b_k(m)$ where $d_k(m) = b_k(m)d_k(m-1)$.

and

$$P_1 = \sum_{r=1}^{Q_1} P_{1,\,r}$$

is the total power received from the desired signal. In the above expression, it is assumed that the fading processes do not substantially change over the duration of a symbol and hence the time dependences of the fading processes have been dropped. The received vector has been scaled by $\sqrt{2P_1}T_c$ and, as a result, the noise component of the received vector has a covariance matrix given by $E[\boldsymbol{n}(m)\boldsymbol{n}^H(m)] = \sigma^2 \boldsymbol{I}$ where $\boldsymbol{I}$ is an identity matrix and $\sigma^2 = N/(E_b/N_o)$. Also the carrier phase ϕ'_k has been absorbed into the fading processes, $\gamma_{k,\,r}(t)$, without loss of generality. Finally, the left and right acyclic shifted code vectors have been introduced.[2]

In order to write this in a more compact form, define $\Lambda_k(l) = \{r\colon L_{k,\,r} = l\}$, and let

$$\boldsymbol{h}_{k,\,l}(m) = \sum_{r\in\Lambda_k(l)} \sqrt{\frac{P_{k,\,r}}{P_1}}\,\gamma_{k,\,r}(m)\boldsymbol{c}_k^R(\mu_{k,\,r}) + \sum_{r\in\Lambda_k(l-1)} \sqrt{\frac{P_{k,\,r}}{P_1}}\,\gamma_{k,\,r}(m)\boldsymbol{c}_k^L(NT_c - \mu_{k,\,r}). \tag{8}$$

Then

$$\boldsymbol{r}(m) = \sum_{k=1}^{K} \sum_{l=0}^{L_{k,\,\max}+1} d_k(m-l)\boldsymbol{h}_{k,\,l}(m) + \boldsymbol{n}(m) \tag{9}$$

where $L_{k,\,\max}$ is the maximum value (with respect to r) that $L_{k,\,r}$ takes on. If we assume that the delay spread of the channel is less than one symbol interval (i.e., $\tau_{1,\,r} < T_s,\ \forall\, r$), then $L_{1,\,\max} = 0$ and the received vector can be written as

$$\boldsymbol{r}(m) = d_1(m)\boldsymbol{h}_{1,\,0}(m) + d_1(m-1)\boldsymbol{h}_{1,\,1}(m) + \tilde{\boldsymbol{r}}(m) \tag{10}$$

where the first term represents the desired signal, the second term is intersymbol interference (ISI), and $\tilde{\boldsymbol{r}}(m)$ is the combination of multiple access interference and noise.

A. Adaptive Implementations of the MMSE Receiver

In practice, the true MMSE receiver cannot be implemented because the ideal forms of the autocorrelation and steering vectors are not known to the receiver. Various adaptive algorithms to recursively update the filter tap weights are used, the most common of which are the least mean squares (LMS) and the recursive least squares (RLS) algorithm. In order to explain some important behaviors of these adaptive approximations to the MMSE receiver, consider a block least squares approach, where the tap weight vector for a block of M bit intervals is calculated according to

$$\hat{\boldsymbol{w}} = \arg\min_{\boldsymbol{w}} \sum_{m=1}^{M} \left| d_1(m) - \boldsymbol{w}^H r(m) \right|^2 = \hat{\boldsymbol{R}}^{-1}\hat{\boldsymbol{p}} \tag{11}$$

[2]If $\boldsymbol{c} = (c_1, c_2, \ldots, c_N)^T$, and p is an integer, then $\boldsymbol{c}^R(pT_c) = (0, 0, \ldots, 0, c_1, c_2, \ldots, c_{N-p})^T$. Also if p is an integer and $\delta \in [0, 1)$, then $\boldsymbol{c}^R((p+\delta)T_c) = (1-\delta)\boldsymbol{c}^R(pT_c) + \delta\boldsymbol{c}^R((p+1)T_c)$. Similar definitions apply for $\boldsymbol{c}^L$.

where $\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{p}}$ are the sample autocorrelation matrix and sample steering vector, respectively, given by

$$\hat{\boldsymbol{R}} = \frac{1}{M} \sum_{m=1}^{M} \boldsymbol{r}(m)\boldsymbol{r}^H(m) \tag{12a}$$

$$\hat{\boldsymbol{p}} = \frac{1}{M} \sum_{m=1}^{M} d_1^*(m)\boldsymbol{r}(m). \tag{12b}$$

The nature of the resulting tap weight vector depends on how the fading channel is characterized.

B. Adaptive MMSE Receivers in Flat Fading Channels

Consider first, a flat Rayleigh fading channel model. In that case, (10) reduces to

$$\boldsymbol{r}(m) = d_1(m)\gamma_1(m)\,\boldsymbol{c}_1 + \tilde{\boldsymbol{r}}(m). \tag{13}$$

The sample steering vector then takes the form

$$\hat{\boldsymbol{p}} = \boldsymbol{c}_1\left(\frac{1}{M}\sum_{m=1}^{M}\gamma_1(m)\right) + \frac{1}{M}\sum_{m=1}^{M} d_1^*(m)\tilde{\boldsymbol{r}}(m). \tag{14}$$

Since $\tilde{\boldsymbol{r}}(m)$ has no terms containing $d_1(m)$, the second term represents noise due to a finite observation interval. The first term is the desired part. Neglecting the second term, the steering vector can be written as $\hat{\boldsymbol{p}} = \boldsymbol{c}_1\overline{\gamma}_1$, where $\overline{\gamma}_1$ represents the time average of the fading process over the window of observation.

Two extreme cases are considered here. The first we refer to as the (very) slow fading model. In the slow fading model, it is assumed that the fading rate is so slow that the fading processes remain essentially unchanged over the entire observation window of M bits. In this case, the steering vector becomes $\hat{\boldsymbol{p}} = \boldsymbol{c}_1\gamma_1$. The MMSE receiver forms

$$\begin{aligned} z(m) &= \boldsymbol{w}^H\boldsymbol{r}(m) \\ &= \gamma_1^*\boldsymbol{c}_1^T\hat{\boldsymbol{R}}^{-1}\boldsymbol{r}(m) \\ &= d_1(m)|\gamma_1|^2\boldsymbol{c}_1^T\hat{\boldsymbol{R}}^{-1}\boldsymbol{c}_1 + \tilde{\boldsymbol{n}}(m) \end{aligned} \tag{15}$$

where $\tilde{\boldsymbol{n}}(m)$ is the residual MAI plus noise out of the MMSE filter. Note that the unknown phase on the desired signal induced by the Rayleigh fading has been automatically accounted for by the steering vector.

In the other extreme, which we refer to as (very) fast fading, the behavior of the receiver is not as pleasant. In the fast fading model, we assume that the fading rate is sufficiently high that the fading processes go through many cycles during the window of observation. Quantitatively, we are assuming $Mf_DT_s \gg 1$. In this case, the time average of the fading process over the window of observation is essentially zero. This results in the degenerate steering vector, $\hat{\boldsymbol{p}} = \boldsymbol{0}$ and hence the receiver is useless since $\boldsymbol{w} = \boldsymbol{0}$. Note that the fading rate does not have to be particularly high for this scenario to occur if the observation window is long.

Several modifications to the basic MMSE receiver have been proposed to avoid the problem outlined above. One option is to use the blind minimum output energy (MOE) receiver of [6] which essentially uses a tap weight vector of $\boldsymbol{w} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{c}_1$. That is, the steering vector is taken to be $\boldsymbol{p} = \boldsymbol{c}_1$. Since this does not account for the phase on the desired signal, differential detection is necessary on the output of the MOE filter. Other approaches require an estimate of the fading process for the desired user in one form or another.

Barbosa and Miller [7], [8] used an explicit linear predictive channel estimator to remove the phase of the fading process of the desired user from the received signal before passing it through the MMSE filter. In other words, let $\theta_1(m)$ be the phase of the fading process of the desired user. The modified receiver in [7], [8] forms $\boldsymbol{y}(m) = e^{j\theta_1(m)}\boldsymbol{r}(m)$ and uses $\boldsymbol{y}(m)$ as the input to the MMSE filter. The sample autocorrelation matrix is clearly unchanged since $\boldsymbol{y}(m)\boldsymbol{y}^H(m) = \boldsymbol{r}(m)\boldsymbol{r}^H(m)$. The new sample steering vector works out to be $\hat{\boldsymbol{p}} = \overline{|\gamma_1(m)|}\boldsymbol{c}_1$. Hence the MMSE tap weight vector in this case is the same as in the MOE receiver (to within a constant of proportionality).

Given knowledge of the desired user's fading, $\gamma_1(m)$, another approach is to incorporate that knowledge into the error signal and redefine the error as

$$e(m) = d_1(m)\gamma_1(m) - \boldsymbol{w}^H\boldsymbol{r}(m). \tag{16}$$

This approach has been considered in [9]. Keeping to the block least squares approach, the MMSE tap weights in this case become, $\boldsymbol{w} = \hat{\boldsymbol{R}}^{-1}\hat{\boldsymbol{p}}$, where $\hat{\boldsymbol{R}}$ is the sample autocorrelation matrix as before, but now

$$\hat{\boldsymbol{p}} = \frac{1}{M}\sum_{m=1}^{M} d_1^*(m)\gamma_1^*(m)\boldsymbol{r}(m) \approx \boldsymbol{c}_1\left(\frac{1}{M}\sum_{m=1}^{M}|\gamma_1(m)|^2\right). \tag{17}$$

In this case, the steering vector is proportional to $\boldsymbol{c}_1$ regardless of the fade rate. Under the slow fading assumption, $\hat{\boldsymbol{p}}(m) = |\gamma_1(m)|^2\boldsymbol{c}_1$ while for the fast fading case,[3] $\hat{\boldsymbol{p}} = \overline{|\gamma_1|^2}\boldsymbol{c}_1 = \boldsymbol{c}_1$. Several papers (e.g., [10], [17]) have appeared recently using this idea to develop modified MMSE receivers and the resulting adaptive algorithms we will refer to as differential least squares (DLS). Similar to what is done with standard differential detection of PSK, the output of the MMSE filter can be used to give a rough estimate of the combination $d_1(m)\gamma_1(m)$ required by (17). To see this, note that the output of the MMSE filter [with the modified definition of error in (16)] can be given by $z(m) \approx d_1(m)\gamma_1(m)$ (neglecting residual MAI and noise). Hence by forming

$$\begin{aligned} b_1(m)z(m-1) &\approx b_1(m)d_1(m-1)\gamma_1(m-1) \\ &= d_1(m)\gamma_1(m-1) \\ &\approx d_1(m)\gamma_1(m) \end{aligned} \tag{18}$$

we get an expression that is available to the receiver which can be used as an approximation to $d_1(m)\gamma_1(m)$ for the purposes of forming a steering vector. That is, in DLS we form the steering vector according to

$$\hat{\boldsymbol{p}} = \frac{1}{M}\sum_{m=1}^{M} b_1^*(m)z^*(m-1)\boldsymbol{r}(m). \tag{19}$$

The point of the above discussion is that the various modifications to the basic MMSE receiver that have been presented in the literature recently all represent different approximations to the same basic receiver. That is, they try to adaptively approximate a receiver which generates a tap weight vector according to $\boldsymbol{r} \propto \boldsymbol{R}^{-1}\boldsymbol{c}_1$. With differential detection, the constant of proportionality does not affect the performance. In this paper we will not focus on the implementation differences of these various approaches, but rather focus on the performance of the basic MMSE receiver and how it changes as a result of the assumptions on the speed of the fading processes. This is studied through the two extreme cases described previously as the slow and fast fading models.

To this point, discussion has focused on how the different fading models affect the steering vector. Essentially the result can be summarized as follows:

$$\boldsymbol{p} = E[d_1(m)\boldsymbol{r}(m)] = E[\gamma_1(m)\boldsymbol{c}_1]. \tag{20}$$

For the slow fading model, $\gamma_1(m)$ is treated as a fixed constant, resulting in $\boldsymbol{p} = \gamma_1(m)\boldsymbol{c}_1$, while for the fast fading model, $\gamma_1(m)$ is treated as a zero-mean complex Gaussian random variable, resulting in $\boldsymbol{p} = \boldsymbol{0}$. This led to the need of the modified MMSE structures which force $\boldsymbol{p} \propto \boldsymbol{c}_1$ in the fast fading case. In the performance analysis which follows, the effect of the fading model on the autocorrelation matrix will also need to be considered and it is treated in a similar manner. In fact, it is this difference in the form of this matrix as a function of the channel model which produces a difference in performance for the MMSE receivers.

C. MMSE Receivers in Frequency-Selective Fading Channels

Before moving on to the performance analysis, we first present a few possible extensions to the MMSE receiver for frequency-selective fading channels. For the slow fading case, the MMSE tap weights are given by $\boldsymbol{w}(m) = \boldsymbol{R}^{-1}(m)\boldsymbol{p}(m)$ where [from (10) treating all the $\gamma_{k,r}$ as fixed constants]

$$\boldsymbol{p}(m) = \boldsymbol{h}_{1,0}(m) \tag{21a}$$

$$\boldsymbol{R}(m) = \boldsymbol{h}_{1,0}(m)\boldsymbol{h}_{1,0}^H(m) + \boldsymbol{h}_{1,1}(m)\boldsymbol{h}_{1,1}^H(m) + \tilde{\boldsymbol{R}}(m) \tag{21b}$$

$$\tilde{\boldsymbol{R}}(m) = E\left[\tilde{\boldsymbol{r}}(m)\tilde{\boldsymbol{r}}^H(m)\right] = \sum_{k=2}^{K}\sum_{l=0}^{L_{k,\max}+1}\boldsymbol{h}_{k,l}(m)\boldsymbol{h}_{k,l}^H(m) + \sigma^2\boldsymbol{I}. \tag{21c}$$

For the fast fading model

$$\boldsymbol{p} = \boldsymbol{0} \tag{22}$$

$$\begin{aligned} \hat{\boldsymbol{R}} &= \boldsymbol{R}_{\text{fast}} \\ &= \sum_{k=1}^{K}\sum_{l=0}^{L_{k,\max}+1} E\left[\boldsymbol{h}_{k,l}(m)\boldsymbol{h}_{k,l}^H(m)\right] + \sigma^2\boldsymbol{I} \\ &= \sum_{k=1}^{K}\sum_{r=1}^{Q_k}\frac{P_{k,r}}{P_1}\Big[\boldsymbol{c}_k^L(NT_c-\mu_{k,r})\boldsymbol{c}_k^{L^T}(NT_c-\mu_{k,r}) \\ &\quad + \boldsymbol{c}_k^R(\mu_{k,r})\boldsymbol{c}_k^{R^T}(\mu_{k,r})\Big] + \sigma^2\boldsymbol{I}. \end{aligned} \tag{23}$$

Again, the result in (22) requires that the MMSE receiver be modified for the frequency-selective channel as well. Toward that end, we consider extensions of the same sort of approaches that were used in the flat fading case.

[3] Note that this case produces a receiver filter which is equivalent to the MOE receiver.

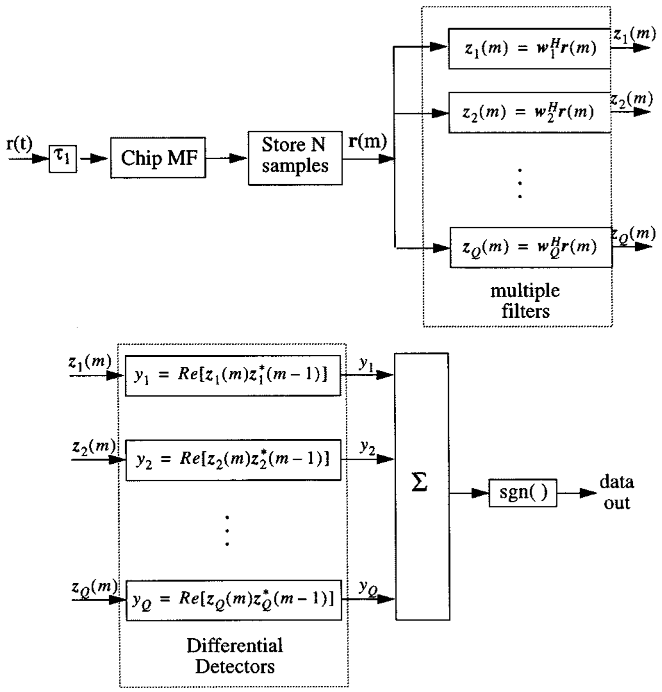

Fig. 1. Multiple filter receivers with equal gain differential combining.

Suppose we were able to somehow externally track the fading processes for all paths of the desired user only (i.e., the $\gamma_{1,r}$ are known). Then the steering vector of (21a) can be formed explicitly according to (8). In doing so we could form a modified MMSE receiver for the fast fading channel which uses as tap weights, $\boldsymbol{w}(m) = \boldsymbol{R}_{\text{fast}}^{-1}\boldsymbol{h}_{1,0}$. This will be referred to as the fast single filter MMSE receiver. Another approach would be to create a separate filter for each path of the desired user. The general structure is illustrated in Fig. 1. Specifically, we can create Q_1 MMSE filters with the modified error signals

$$e_q(m) = d_1(m)\gamma_{1,q}(m) - \boldsymbol{w}_q^H(m)\boldsymbol{r}(m) \tag{24}$$

which results in the MMSE tap weight vectors given by

$$\boldsymbol{w}_q = \hat{\boldsymbol{R}}^{-1}\boldsymbol{c}_1^R(\mu_{1,q}). \tag{25}$$

This filter is referred to as the multiple filter modified MMSE receiver. Note that maximal ratio combining of the filter outputs is equivalent to a single coherent MMSE filter. For the MOE approach, the qth filter's tap weights are decomposed as

$$\boldsymbol{w}_q = \boldsymbol{c}_1^R(\mu_{1,q}) + \boldsymbol{x}_q, \qquad q = 1, 2, \ldots, Q_1 \tag{26}$$

where $\boldsymbol{x}_q$ is chosen to minimize the variance of the output of the qth filter subject to the constraint that $\boldsymbol{x}_q$ is orthogonal to an "anchor" vector or subspace [6], [12]. In this case, the anchor can be either $\boldsymbol{c}_1^R(\mu_{1,q})$ or the subspace spanned by all of the $\boldsymbol{c}_1^R(\mu_{1,q})$, (i.e., $\boldsymbol{x}_q^H\boldsymbol{C}_R = \boldsymbol{0}$). The former approach gives the multiple filter modified MMSE receiver specified in (25) while the latter constraint leads to the solution

$$\boldsymbol{w}_q = \hat{\boldsymbol{R}}^{-1}\boldsymbol{C}_R\left(\boldsymbol{C}_R^T\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}_R\right)^{-1}\boldsymbol{C}_R^T\boldsymbol{c}_1^R(\mu_{1,q}). \tag{27}$$

In the following, this filter is referred to as the multiple filter MOE receiver. This technique isolates each path of the desired user, which is appropriate for noncoherent combining. Note that maximal ratio combining of the filter outputs in this case is not equivalent to the coherent MMSE filter.

IV. Performance Analysis of MMSE Receivers in a Frequency-Selective Fading Channel

In this section, we present a performance analysis of the MMSE receiver in a slow frequency-selective fading channel. Given the form of the received vector in (10), the output of the MMSE filter then becomes

$$z(m) = v_0 d_1(m) + v_1 d_1(m-1) + \tilde{n}(m) \tag{28}$$

where $v_i = \boldsymbol{w}^H(m)\boldsymbol{h}_{1,i}(m)$ and $\tilde{n}(m) = \boldsymbol{w}^H(m)\tilde{\boldsymbol{r}}(m)$ is the residual MAI plus noise at the output of the MMSE filter. Note the form of the tap weight vector is specified by the autocorrelation matrix and steering vector given in (21).

Let $\boldsymbol{q}$ be the vector containing all the signal parameters (i.e., amplitude, phase, timing) of each path of each user's signal. The probability of error for a differential detector, conditioned on knowing $\boldsymbol{q}$, is given by[4]

$$\begin{aligned} P_{e|\boldsymbol{q}} = &\frac{1}{8}\exp\left(-\frac{|v_0+v_1|^2}{\tilde{\sigma}^2}\right) + \frac{1}{8}\exp\left(-\frac{|v_0-v_1|^2}{\tilde{\sigma}^2}\right) \\ &+ \frac{1}{4}\left[1 - Q\left(\frac{2|v_0|}{\tilde{\sigma}}, \frac{2|v_1|}{\tilde{\sigma}}\right) + Q\left(\frac{2|v_1|}{\tilde{\sigma}}, \frac{2|v_0|}{\tilde{\sigma}}\right)\right]. \end{aligned} \tag{29}$$

The quantity, $\tilde{\sigma}^2$, is the variance of the residual noise plus MAI term and can be written as

$$\begin{aligned} \tilde{\sigma}^2 &= E\left[|\tilde{n}(m)|^2\right] \\ &= \boldsymbol{w}^H(m)E\left[\tilde{\boldsymbol{r}}(m)\tilde{\boldsymbol{r}}^H(m)\right]\boldsymbol{w}(m) \\ &= \boldsymbol{w}^H(m)\tilde{\boldsymbol{R}}(m)\boldsymbol{w}(m) \\ &= \boldsymbol{p}^H(m)\boldsymbol{R}^{-1}(m)\tilde{\boldsymbol{R}}(m)\boldsymbol{R}^{-1}(m)\boldsymbol{p}(m). \end{aligned} \tag{30}$$

The conditional probability of error given in (29) is found using general formulas for Rician variates found in [18] and is based on approximating the residual noise plus MAI out of the MMSE filter as a Gaussian random variable. This Gaussian approximation generally turns out to be quite good for the MMSE receiver [19] and is commonly used in the analysis of matched filter based CDMA systems as well, even though the approximation is not as good in that case.

The average probability of error can be found by evaluating (29) for several realizations of $\boldsymbol{q}$ and then taking a sample average of the result. Unfortunately, this averaging must be performed over many realizations of $\boldsymbol{q}$ in order to obtain accurate results. This problem can be circumvented by neglecting the ISI term in (29). For typical delay spreads, the contribution of ISI to the output MSE is quite small, so we assume that $|v_0| \gg |v_1|$.

[4]Marcum's Q-function is defined as

$$Q(\alpha, \beta) = \int_\beta^\infty x\,\exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x)\,dx.$$

Taking v_1 to be zero, (29) reduces to the analytically more convenient form[5]

$$P_{e|\boldsymbol{q}} = \frac{1}{2}\exp\left(-\frac{|v_0|^2}{\tilde{\sigma}^2}\right). \tag{31}$$

At this point, we note that $v_0 = \boldsymbol{p}^h(m)\boldsymbol{R}^{-1}(m)\boldsymbol{p}(m)$. Using the expression in (10) (and neglecting the middle term which will contribute the insignificant ISI) together with the matrix inversion lemma it can be shown that

$$v_0 = \frac{\boldsymbol{p}^H(m)\tilde{\boldsymbol{R}}^{-1}(m)\boldsymbol{p}(m)}{1+\boldsymbol{p}^H(m)\tilde{\boldsymbol{R}}^{-1}(m)\boldsymbol{p}(m)}. \tag{32}$$

Furthermore, the expression for $\tilde{\sigma}^2$ can be simplified to

$$\tilde{\sigma}^2 = \frac{\boldsymbol{p}^H(m)\tilde{\boldsymbol{R}}^{-1}(m)\boldsymbol{p}(m)}{(1+\boldsymbol{p}^H(m)\tilde{\boldsymbol{R}}^{-1}(m)\boldsymbol{p}(m))^2}. \tag{33}$$

As a result of the two simplifications presented in (32) and (33), the error probability conditioned on $\boldsymbol{q}$ becomes

$$\begin{aligned} P_{e|\boldsymbol{q}} &= \frac{1}{2}\exp\left(-\boldsymbol{p}^H\tilde{\boldsymbol{R}}^{-1}\boldsymbol{p}\right)\\ &= \frac{1}{2}\exp\left(-\boldsymbol{h}_{1,0}^H\tilde{\boldsymbol{R}}^{-1}\boldsymbol{h}_{1,0}\right)\\ &= \frac{1}{2}\exp\left(-\boldsymbol{g}^H\boldsymbol{P}\boldsymbol{C}_R^T\tilde{\boldsymbol{R}}^{-1}\boldsymbol{C}_R\boldsymbol{P}\boldsymbol{g}\right) \end{aligned} \tag{34}$$

where

$$\boldsymbol{g} = [\gamma_{1,1}, \gamma_{1,2}, \ldots, \gamma_{1,Q_1}]^T,$$
$$\boldsymbol{P} = \text{diag}\left(\sqrt{P_{1,1}/P_1}, \sqrt{P_{1,2}/P_1}, \ldots, \sqrt{P_{1,Q_1}/P_1}\right)$$

and

$$\boldsymbol{C}_R = \left[\boldsymbol{c}_1^R(\mu_{1,1}), \ldots, \boldsymbol{c}_1^R(\mu_{1,Q_1})\right].$$

As mentioned before, the average probability of error is then found by averaging the above expression over the distributions of the random quantities in $\boldsymbol{q}$. Since most of the random quantities are in the term $\tilde{\boldsymbol{R}}^{-1}$, it seems unlikely that an analytical form for the average probability of error can be found. However, averaging over the complex Gaussian random vector $\boldsymbol{g}$ is straightforward. Hence, we write $\boldsymbol{q} = [\boldsymbol{g}, \tilde{\boldsymbol{q}}^T]^T$, where $\tilde{\boldsymbol{q}}$ contains all the unknown quantities corresponding to the interfering users. Then

$$\begin{aligned} P_{e|\tilde{\boldsymbol{q}}} &= E_{\boldsymbol{g}}\left[\tfrac{1}{2}\exp\left(-\boldsymbol{g}^H\boldsymbol{P}\boldsymbol{C}_R^T\tilde{\boldsymbol{R}}^{-1}\boldsymbol{C}_R\boldsymbol{P}\boldsymbol{g}\right)\right]\\ &= \frac{1}{2\det(\boldsymbol{I}+\boldsymbol{P}\boldsymbol{C}_R^T\tilde{\boldsymbol{R}}^{-1}\boldsymbol{C}_R^T\boldsymbol{P})}. \end{aligned} \tag{35}$$

Fortunately, most of the variations in (34) are due to the randomness in $\boldsymbol{g}$ and not due to the randomness in $\tilde{\boldsymbol{q}}$. Hence, by randomly selecting values for the components of $\tilde{\boldsymbol{q}}$, we can get a good estimate of the average probability of error. We have taken a semi-analytical approach to evaluating the average probability of error by evaluating (35) for several different realizations of the random vector $\tilde{\boldsymbol{q}}$ and then taking a sample average of those realizations. That is, let $\tilde{\boldsymbol{q}}_i$, $i = 1, 2, \ldots, S$ be independently generated realizations of $\tilde{\boldsymbol{q}}$. Then

$$\hat{P}_e = \frac{1}{S}\sum_{i=1}^{S} P_{e|\tilde{\boldsymbol{q}}_i} \tag{36}$$

where $P_{e|\tilde{\boldsymbol{q}}_i}$ is evaluated according to (35). It seems that about $S = 100$ samples is enough to give an accurate approximation to the average probability of error for the examples considered.

Before turning attention to fast fading channels, we consider briefly the special case when the channel is frequency nonselective (i.e., flat fading). In that case, there is a single path for all users ($Q_k = 1$). As a result, the matrix $\boldsymbol{P}$ is just the scalar, 1, and the matrix $\boldsymbol{C}_R$ becomes the column vector consisting of the code sequence of the desired user, $\boldsymbol{c}_1$. Using these simplifications, the error probability conditioned on $\tilde{\boldsymbol{q}}$ reduces to

$$P_{e|\tilde{\boldsymbol{q}}} = \frac{1}{2+2\boldsymbol{c}_1^H\tilde{\boldsymbol{R}}^{-1}\boldsymbol{c}_1}. \tag{37}$$

In both (35) and (37), the expression for $\tilde{\boldsymbol{R}}$ given in (21c) should be used.

To analyze the single filter MMSE in a fast frequency-selective environment, the results developed in Section III can be used with a minor modification. Equation (35) can still be used to calculate the probability of error conditioned on the parameter vector, $\tilde{\boldsymbol{q}}$, however the form of the matrix $\tilde{\boldsymbol{R}}$ must now be replaced with

$$\begin{aligned} \tilde{\boldsymbol{R}} &= \tilde{\boldsymbol{R}}_{\text{fast}}\\ &= \sum_{k=2}^{K}\sum_{r=1}^{Q_k}\frac{P_{k,r}}{P_1}\Big[\boldsymbol{c}_k^L(NT_c-\mu_{k,r})\boldsymbol{c}_k^{L^T}(NT_c-\mu_{k,r})\\ &\quad + \boldsymbol{c}_k^R(\mu_{k,r})\boldsymbol{c}_k^{R^T}(\mu_{k,r})\Big] + \sigma^2\boldsymbol{I}. \end{aligned} \tag{38}$$

Note that this expression is identical to that in (23) except the lower limit of the outer sum has been changed to remove the contribution from the desired user. The performance analysis of the multiple filter receivers is rather lengthy and is delegated to the Appendix.

V. Numerical Results

The performance of the various receiver structures presented in Section III are next evaluated according to the equations derived in Section IV. In summary, we consider the performance of the following four different receiver structure/channel model combinations.

- *MMSE single filter receiver in slow fading*—This is the ideal MMSE receiver where the autocorrelation matrix and steering vector are given by (21).
- *MMSE single filter receiver in fast fading*—In this case the tap weights are given by $\boldsymbol{w}(m) = \boldsymbol{R}_{\text{fast}}^{-1}\boldsymbol{h}_{1,0}$, where the steering vector $\boldsymbol{p}(m) = \boldsymbol{h}_{1,0}(m)$ is formed explicitly according to (8) using knowledge of the channel fading processes for the desired user. The autocorrelation matrix $\boldsymbol{R} = \boldsymbol{R}_{\text{fast}}$ is given in (23).
- *Multiple filter MOE receiver in fast fading*—This is the structure depicted in Fig. 1 where the tap weights for each

[5]The approximation in (31) has been numerically compared to the exact expression in (29) and found to be in close agreement provided that the delay spread of the channel is small compared to a bit interval.

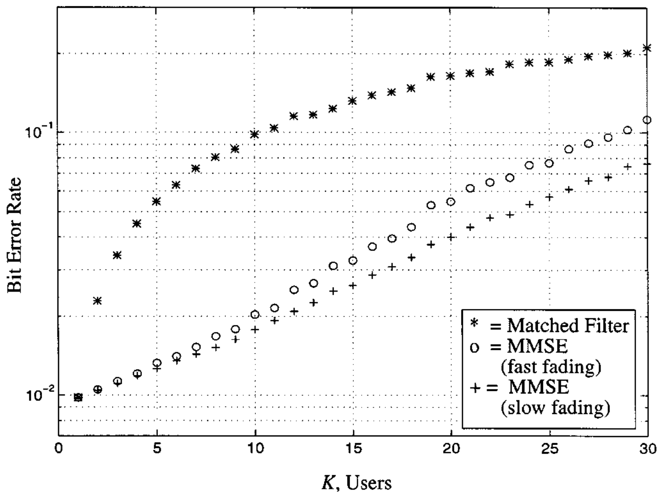

Fig. 2. Bit-error rate of the MMSE and MF on a flat fading channel; $E_b/N_0 = 17$ dB, $N = 31$ (Gold codes), Rayleigh fading channel, log-normally distributed interference power with 1.5-dB standard deviation.

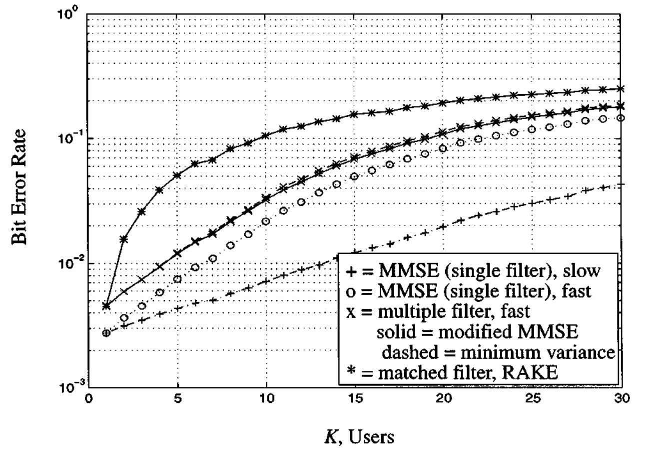

Fig. 3. Bit-error rate of the various MMSE receivers and the matched filter receiver with equal gain combining on a frequency-selective fading channel; $E_b/N_0 = 15$ dB, $N = 31$ (Gold codes), two-path (equal strength) Rayliegh fading channel, log-normally distributed interference power with 1.5-dB standard deviation.

filter are chosen according to (27). Equal gain differential combing is used as illustrated in Fig. 1.

- *Multiple filter modified MMSE receiver in fast fading*—This is the structure depicted in Fig. 1 where the tap weights for each filter are chosen according to (25). Again, equal gain differential combing is used.
- *Conventional (RAKE) receiver*—This is the classical RAKE receiver with equal weight differential combining. This receiver can be formulated using the structure in Fig. 1 with the tap weights chosen according to $\boldsymbol{w}_q = \boldsymbol{c}_1^R(\mu_{1,q})$.

Numerical results are presented for four different environments to demonstrate how the performance of the various receivers vary according to different channel conditions. The cases presented are as follows.

- *Flat Rayleigh Fading (Fig. 2)*—In this case the single and multiple filter MMSE receivers for fast fading are the same thing, so only three curves are presented. The received power from each interfering user is taken to be a log-normally distributed random variable whose mean (long term) is the same as the desired signal and whose standard deviation is 1.5 dB. Instantaneous values of the received powers vary due to the fading processes.
- *Two-Path Frequency-Selective Rayleigh Fading (Fig. 3)*—This is the same environment as in Fig. 2 except each user's signal is now received via a two-path fading channel. The value for the energy per bit quoted is the total energy received on both paths which is equally distributed between the two paths. The delay between the two paths is a random variable uniformly distributed over the interval [1, 6) chips.
- *Four-Path Frequency-Selective Rayleigh Fading (Fig. 4)*—This is the same environment as in Fig. 3 except there are now four equal strength paths instead of two.
- *Two-Path Frequency-Selective Rayleigh Fading with Near–Far Effect (Fig. 5)*—This is the same environment of Fig. 3 except now the standard deviation of the log-normally distributed interference powers has been increased from 1.5 dB to 8 dB. While the former value might be typical of closed loop power control, the latter is chosen to represent open loop power control.

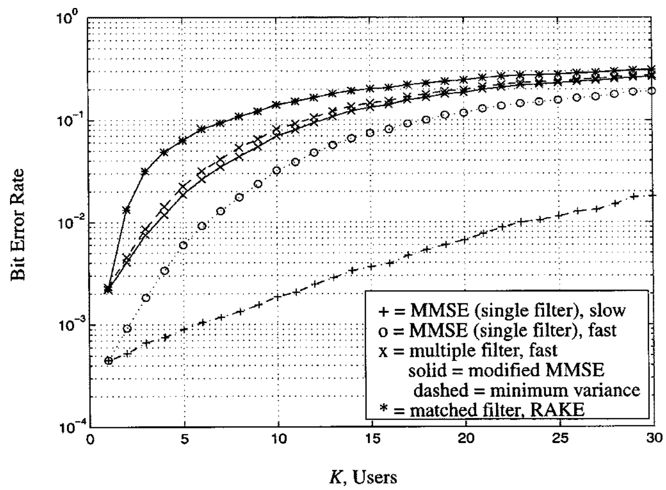

Fig. 4. Bit-error rate of the various MMSE receivers and the matched filter receiver with equal gain combining on a frequency-selective fading channel; $E_b/N_0 = 15$ dB, $N = 31$ (Gold codes), four-path (equal strength) Rayliegh fading channel, log-normally distributed interference power with 1.5-dB standard deviation.

The main conclusion to be drawn from the results displayed in Figs. 2–5 deals with the difference between the performance of the various MMSE structures in the flat fading and frequency-selective environments. In Fig. 2 it is seen that in a flat fading channel, there is little performance difference for the MMSE receiver structures in the fast and slow fading channel models. However, Figs. 3–5 demonstrate that the MMSE receivers which have been modified for the fast fading environment degrade substantially in the presence of fast multipath fading. The results for the four-path model in Fig. 4 show that the degradation is more severe as the signal energy is distributed among more paths. Some other observations are as follows.

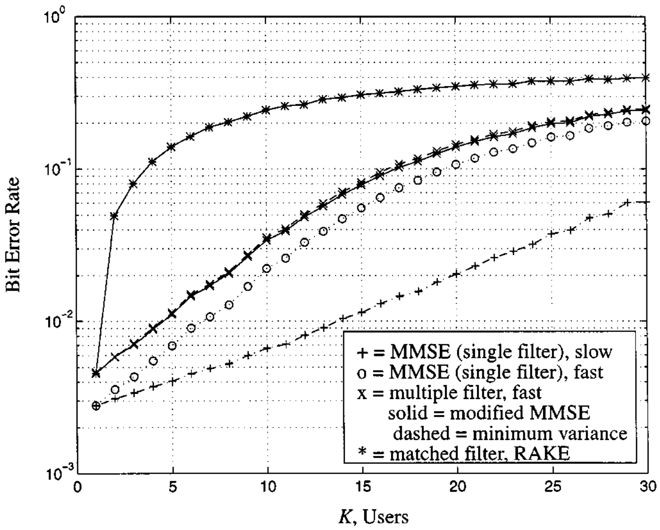

Fig. 5. Bit-error rate of the various MMSE receivers and the matched filter receiver with equal gain combining on a frequency-selective fading channel; $E_b/N_0 = 15$ dB, $N = 31$ (Gold codes), two-path (equal strength) Rayliegh fading channel, log-normally distributed interference power with 8-dB standard deviation.

- For the MMSE receiver in slow fading, the presence of frequency selectivity provides a substantial increase in the capacity of the system. This is due to the fact that the MMSE receiver can suppress a substantial amount of the MAI, even when the system is heavily loaded. The fading then becomes the dominant error mechanism and hence the diversity provided by the frequency-selective channel is very beneficial.
- There is only a small difference in performance between the single filter and multiple filter MMSE structures in a fast fading channel. Furthermore, the single filter approach is less complex.
- All of the MMSE results showed robustness to a near-far environment, even in a frequency-selective environment, whereas the RAKE filter degraded rapidly, as expected.

VI. Discussion

The difference in performance of the MMSE receiver in selective and nonselective fading channels can be explained heuristically using a "dimensionality" argument. Consider the contribution of user k to the received vector given by (9)

$$\boldsymbol{r}_k(m) = \sum_{l=0}^{L_{k,\max}+1} d_k(m-l)\boldsymbol{h}_{k,l}(m). \tag{39}$$

Recall that $L_{k,\max} = \lfloor \tau_{k,Q_k}/T_s \rfloor$. If the delay spread of the channel is small compared to the bit interval (but not necessarily small compared to the chip interval) then the quantity $L_{k,\max}$ will be equal to 0 most of the time and occasionally equal to 1. Hence the dimensionality of the contribution from the kth user is only 2, or sometimes 3 as illustrated in Fig. 6. This is independent of the number of paths present. The MMSE receiver has essentially N degrees of freedom which it can use to suppress the strong interference components, and hence it must usually use up only 2 degrees of freedom per each interfering user, regardless of the number of resolvable paths in the channel. In particular, the MMSE receiver adjusts its weights to suppress the vectors $\boldsymbol{h}_{k,l}$ for $l = 0, \ldots, L_{k,\max}+1$. Naturally, the receiver can only do this if it has (implicit) knowledge of $\boldsymbol{h}_{k,l}$. From (8), it is clear that this requires knowledge of $\{\gamma_{k,r}\}$, the fading coefficients of each path for each user. If the fading is too rapid to track, the receiver must treat the $\{\gamma_{k,r}\}$ as unknowns. In that case, the contribution from each user to the received vector is better viewed in the form

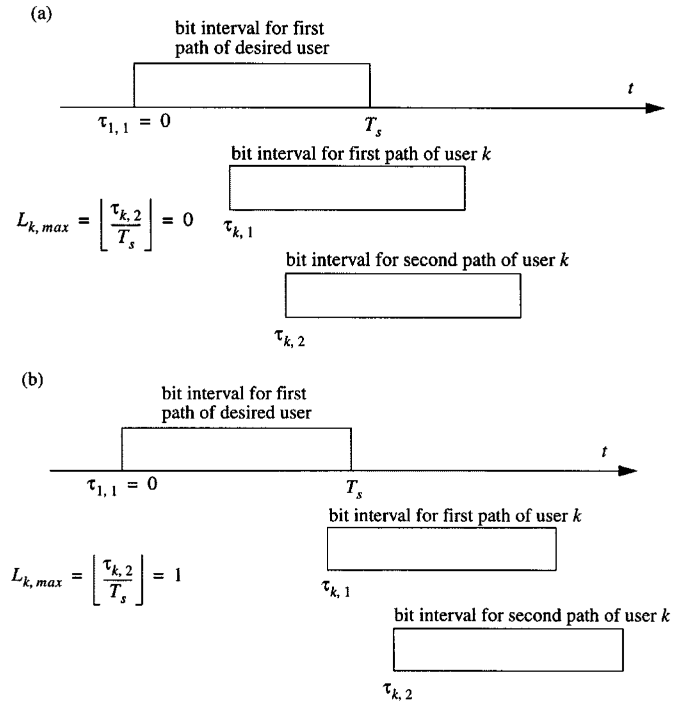

Fig. 6. Timing configurations for an example two-path frequency-selective channel. (a) Two-dimensional interference space per user. (b) Three-dimensional interference space per user.

$$\boldsymbol{r}_k(m) = \sum_{r=1}^{Q_k} \gamma_{k,r}(m)[d_k(m-L_{k,r}-1)\boldsymbol{c}_{k,L}(NT_c-\mu_{k,r}) + d_k(m-L_{k,r})\boldsymbol{c}_{k,R}(\mu_{k,r})]. \tag{40}$$

With both $d_k(m)$ and $\gamma_{k,r}$ being unknown, the MMSE must adjust its weights to suppress $\boldsymbol{c}_{k,L}(NT_c-\mu_{k,r})$ and $\boldsymbol{c}_{k,R}(\mu_{k,r})$ for $r = 1, \ldots, Q_k$. Thus, the MMSE must use up as many as $2Q_k$ degrees of freedom to suppress the kth user's interfering signal. If $Q_k = 1$ (flat fading), then the dimensionality of the interference is the same (equal to 2) regardless of whether the fading channel coefficient is known or unknown. On the other hand, if $Q_k > 1$, the dimensionality of the interfering signal is higher when the fading channel coefficients are unknown. For the example shown in Fig. 3, $Q_k = 2$, and so for the known channel case each interfering signal typically spans a two dimensional space. With N degrees of freedom, the MMSE receiver can then suppress roughly $N/2$ strong interfering signals. On the other hand, when the channel is unknown, each interfering signal spans a four dimensional space. Thus the MMSE receiver can suppress roughly $N/4$ strong interferers. Hence, we expect the capacity in the known channel case to be roughly twice the capacity when the channel is unknown. This seems to be consistent with the results in Fig. 3.

The above argument is intended to be only a crude explanation for the results shown in Figs. 2–5. In fact, that argument applies to a zero-forcing (e.g., decorrelator) type of a receiver, rather than an MMSE receiver. If the interfering components are all strong, then the preceding argument also applies to the MMSE receiver. However, if some of the multipath components are weak, then the capacity reduction due to the frequency selectivity of the channel will not be as severe. It should be noted that a similar explanation was given in [17], although no analytical results were presented there.

In this paper we have demonstrated through analytical means the inability of adaptive implementations of MMSE receivers to perform up to the potential promised by the ideal MMSE receiver in fast frequency-selective fading channels. It is not our intention here to simulate the performance of various adaptive implementations, rather that has been done in a companion paper [16]. In that paper, it is seen that all the adaptive implementations known at this point start to degrade at fairly low vehicle speeds in frequency-selective channels. One way to achieve the performance promised by the ideal MMSE receiver is to explicitly track the channel parameters of all users and then form the autocorrelation matrix and steering vectors directly according to (21). This has been done in [20] and [21], for example, but leads to a very complicated multi-user receiver. Also, forming the tap weights for the MMSE receiver directly sacrifices the robustness of the adaptive implementations. The receiver will now be more sensitive to synchronization errors, narrowband interference, etc. In our opinion, further work is needed to develop adaptive algorithms which can better track rapidly fading environments.

APPENDIX

The performance analysis of the multiple filter receiver shown in Fig. 1 is sketched in this appendix. The starting point of this analysis is the received vector as given in (9), (10) which is repeated here

$$\begin{aligned}\boldsymbol{r}(m) &= \sum_{k=1}^{K}\sum_{l=0}^{L_{k,\max}+1} d_k(m-1)\boldsymbol{h}_{k,l}(m) + \boldsymbol{n}(m)\\ &= d_1(m)\boldsymbol{h}_{1,0}(m) + d_1(m-1)\boldsymbol{h}_{1,1}(m) + \tilde{\boldsymbol{r}}(m). \end{aligned}\tag{A.1}$$

This signal is then passed through a number of filters (one for each path of the received signal) resulting in the output signals

$$z_q(m) = v_{0,q}d_1(m) + v_{1,q}d_1(m-1) + \tilde{n}_q(m) \tag{A.2}$$

where $v_{i,q} = \boldsymbol{w}_q^H\boldsymbol{h}_{1,i}(m)$ and $\tilde{n}_q(m) = \boldsymbol{w}_q^H\tilde{\boldsymbol{r}}(m)$. These outputs of the filters are then differentially combined to form the ultimate decision statistic

$$D = \sum_{q=1}^{Q_1}\mathrm{Re}[z_q(m)z_q^*(m-1)] = \boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z} \tag{A.3a}$$

where

$$\begin{aligned}\boldsymbol{z} = [&z_1(m), z_2(m), \ldots, z_{Q_1}(m),\\ &z_1(m-1), z_2(m-1), \ldots, z_{Q_1}(m-1)]^T\end{aligned}\tag{A.3b}$$

$$\boldsymbol{A} = \frac{1}{2}\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{0}\end{bmatrix}. \tag{A.3c}$$

To make the analysis simpler, the ISI terms are neglected and the case where two consecutive transmitted bits are both +1 is considered so that

$$z_q(m) = \boldsymbol{w}_q^H\boldsymbol{h}_{1,0}(m) + \tilde{\boldsymbol{n}}_q(m). \tag{A.4}$$

Conditioned on $d_1(m) = d_1(m-1) = 1$, the probability of error is given by

$$\begin{aligned}&\Pr(\text{error}|d_1(m) = d_1(m-1) = 1)\\ &\quad = \Pr(D<0) = \Pr\left(\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z} < 0\right).\end{aligned}\tag{A.5}$$

We define the moment generating function of D, as

$$M_D(s) = E\left[e^{-sD}\right]. \tag{A.6}$$

The error probability can then be written in terms of the residues of the moment generating function in the right half plane

$$\Pr(\text{error}|d_1(m) = d_1(m-1) = 1) = -\sum_{RHP}\mathrm{Res}\left\{\frac{M_D(s)}{s}\right\}. \tag{A.7}$$

Because of the presence of the Gaussian random processes, the statistic D is a Gaussian quadratic form. Hence, its moment generating function can be expressed as

$$M_D(s) = [\det(\boldsymbol{I} + s\boldsymbol{A}\boldsymbol{\Phi}_z)]^{-1} = \prod_{i=1}^{Q_1}(1+s\lambda_i)^{-1} \tag{A.8}$$

where $\boldsymbol{\Phi}_z = E[\boldsymbol{z}\boldsymbol{z}^H|d_1(m) = d_1(m-1) = 1]$, and the λ_i are the eigenvalues of the matrix $\boldsymbol{A}\boldsymbol{\Phi}_z$. Assuming that the eigenvalues are all distinct, the moment generating function consists of Q_1 simple poles, and the required residues are easily calculated. The resulting error probability is then

$$\Pr(\text{error}|d_1(m) = d_1(m-1) = 1) = \sum_{i:\lambda_i<0}\prod_{j\neq i}\frac{\lambda_i}{\lambda_i - \lambda_j}. \tag{A.9}$$

It is noted that the probability of error will be the same regardless of the sequence of data symbols assumed, hence (A.9) represents the probability of error (unconditioned) for the multiple filter structure with equal gain differential combining. The autocorrelation matrix required to calculate this expression is given in block matrix form by

$$\boldsymbol{\Phi}_z = \begin{bmatrix}\boldsymbol{S} + \boldsymbol{Q}_0 + \sigma^2\boldsymbol{W}^H\boldsymbol{W} & \boldsymbol{S} + \boldsymbol{Q}_1\\ \boldsymbol{S} + \boldsymbol{Q}_1^H & \boldsymbol{S} + \boldsymbol{Q}_0 + \sigma^2\boldsymbol{W}^H\boldsymbol{W}\end{bmatrix} \tag{A.10a}$$

where

$$\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_{Q_1}] \tag{A.10b}$$

$$\boldsymbol{S} = \boldsymbol{W}^H E\left[\boldsymbol{h}_{1,0}\boldsymbol{h}_{1,0}^H\right]\boldsymbol{W} \tag{A.10c}$$

$$\boldsymbol{Q}_i = \sum_{k=2}^{K}\sum_{l=0}^{L_{k,\max}+1}\boldsymbol{W}^H E\left[\boldsymbol{h}_{k,l}\boldsymbol{h}_{k,l-i}^H\right]\boldsymbol{W}, \quad i = 0, 1. \tag{A.10d}$$

The expected values indicated in the above expressions work out to be

$$E\Big[\boldsymbol{h}_{k,l}\boldsymbol{h}_{k,l}^H\Big] = \sum_{r\in\Lambda_k(l)} \frac{P_{k,r}}{P_1}\boldsymbol{c}_k^R(\mu_{k,r})\boldsymbol{c}_k^{R^T}(\mu_{k,r}) + \sum_{r\in\Lambda_k(l-1)} \frac{P_{k,r}}{P_1}\boldsymbol{c}_k^L(NT_c-\mu_{k,r}) \cdot \boldsymbol{c}_k^{L^T}(NT_c-\mu_{k,r}) \tag{A.11a}$$

$$E\Big[\boldsymbol{h}_{k,l}\boldsymbol{h}_{k,l-1}^H\Big] = \sum_{r\in\Lambda_k(l-1)} \frac{P_{k,r}}{P_1}\boldsymbol{c}_k^L(NT_c-\mu_{k,r}) \cdot \boldsymbol{c}_k^{R^T}(\mu_{k,r}). \tag{A.11b}$$

The results of this analysis hold regardless of the specific choice of the filters used, as long as the values of the filters do not depend on the values of the fading processes on each of the paths. Three different forms will be considered in this work. The first is the MOE filters specified in (27) in which case $\boldsymbol{W} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{C}_R(\boldsymbol{C}_R^T\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}_R)^{-1}\boldsymbol{C}_R^T\boldsymbol{C}_R$. We also consider the modified MMSE filters specified by (25) so that $\boldsymbol{W} = \hat{\boldsymbol{R}}^{-1}\boldsymbol{C}_R$. Finally to put the results of these adaptive filters in a proper context, we also consider the conventional matched filter approach whereby $\boldsymbol{w}_q = \boldsymbol{c}_1^R(\mu_{1,q})$ and hence $\boldsymbol{W} = \boldsymbol{C}_R$. The value of $\hat{\boldsymbol{R}} = \boldsymbol{R}_{\text{fast}}$ specified by (23) is used to generate the numerical results for the fast fading channel.

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