Sharp quantum Hall edges: Experimental realizations of edge states without incompressible strips

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Experimental results of sharp edge potentials in the quantum Hall (QH) regime of a two-dimensional electron system are presented. First the general QH edge state picture is reviewed, including recent theory which explains the importance of incompressible strips at the sample edge. Then two sharp edge geometries are presented where incompressible strips are predicted to vanish below the scale of the magnetic length: T-junction edge tunneling devices and L-junction bent quantum wells. Tunnel spectroscopy in the T-junction device directly measures the real space position of the edge state orbit centers, verifying an almost ideal sharp edge confinement and providing evidence that incompressible strips vanish in such a potential. Conduction along the junction of the L-sample is presented, revealing unconventional temperature and voltage dependence along a unique type of QH boundary. Hartree calculations explain how this behavior can result from the sharp non-planar confinement potential which is not possible to realize either in planar systems or at soft-confinement boundaries.

1 Introduction
The quantum Hall effect (QH) occurs when a quantizing magnetic field $B$ induces a mobility gap near the Fermi energy of a two-dimensional electron systems (2DES) [1–4]. This gap arises at electron densities $n$ commensurate with the density of flux quanta $n_e = eB/h$, such that the filling factor ratio $\nu = n/n_e$ is near certain integers or simple fractions $N$. The Hall resistance is quantized $\rho_{xy} = (1/N)h/e^2$ and the longitudinal resistance vanishes $\rho_{xx} = 0$ for a finite range of magnetic field $B$ around the condition $\nu = N$.

The $N = 1, 2, 3 \ldots$ integer QH effect is induced by either Landau quantization gaps, $\Delta_E = \hbar eB/m^* \nu$, or Zeeman gaps $E_z = g^* \mu_B B$, where $m^*$ is the effective mass, $g^*$ is the effective Landé g-factor, and $\mu_B$ is the Bohr magneton. For integers $N > 1$ these gaps appear also in the density of states below the Fermi energy, and towards the edge of a sample each gap will be lifted across the Fermi energy in succession as the electron density depletes [5, 6]. Chklovskii et al. [7] explain how for smooth-edge potentials each gap in the bulk results in an incompressible strip at the edge, separated from its neighbor by a metallic compressible strip. When the Hall resistance is quantized, the incompressible strips form electrostatic dipoles that carry the lossless Hall current along the sample edge transverse to the dipole electric field, and the compressible strips completely screen the electric field of the edge potential and carry no current.

Recent work suggests that the incompressible strips play a defining role for the QH effect in the limit of narrow samples with high-mobility [9]. Such samples are smaller than the correlation length of long-range disorder potential fluctuations, so the sample can be approximated as a flat potential with only short range potential scatterers. Provided that incompressible strips are wider than the magnetic length, the Hall effect is quantized; but when the real space width of these strips is smaller than a magnetic
length, disorder scatters the overlapping compressible regions into the bulk, and the Hall effect is no longer quantized. The detailed theory by Siddiki et al. [9] treats the entire sample with a local conductivity model in a Thomas–Fermi Poisson approach to locate the positions and widths of the incompressible strips, to correlate the existence of incompressible strips with the observation of Hall plateaus.

Experimental evidence of this picture has been seen in the scanning-probe experiments of Ahlswede et al. [8], which directly measured the edge potential in narrow samples under bias and show how the the Hall voltage is dropped across the innermost incompressible strip. As soon as the incompressible strip vanishes, the Hall effect becomes unquantized. The above theory also motivated subsequent experiments in narrow high-mobility Hall bars by Horas et al. [10] which show strongly asymmetric plateau structures as predicted.

At a sharp edge, the depletion of electrons at the edge is too rapid to allow the formation of incompressible strips wider than the magnetic length, so according to Ref. [9] one would actually expect that the QH plateaus could vanish at such an edge. To date, however, fabrication of a narrow, sharp-edged Hall sample has not been technically possible. The only means of fabricating such sharp edges has been with cleave-regrowth techniques which typically couple a second electron system to the sharp edge through a tunnel barrier [11–14]. The unusual power-law correlations observed in certain edge-tunneling experiments [11, 12] may be explainable if incompressible strips do, in fact, vanish at such a sharp edge, meaning that this “edge” tunneling experiment is actually probing the bulk compressibility. It is important therefore to devise new geometries which can probe and better understand sharp edges to determine whether incompressible strips vanish, what other characteristics sharp edges have, and how they can be combined in new geometries to discover novel QH phenomena.

This paper will review experimental realizations of sharp edge samples. First, T-junctions of quantum wells permit the dispersion of edge states near an atomically abrupt boundary potential to be probed, and evidence is presented that no incompressible strips wider than the magnetic length exist. Next, L-junctions of quantum wells explore the new kinds of boundaries possible when counter-propagating sharp edges are combined in non-planar geometries. Temperature and voltage dependence demonstrate novel behaviors.

2 T-junction samples

We first introduce the edge-tunneling geometry fabricated by cleaved-edge overgrowth, consisting of two orthogonal quantum wells in a T-shaped junction joined at a tunnel barrier [15–17]. Magnetotunneling measurements at this sharp quantum Hall edge are made. Peaks in the tunnel conductance are shown to arise from momentum conserved tunneling into individual quantum Hall edge states. Knowledge of the Fermi momentum of the tunneling electrons determines the real-space distance of the edge states from the tunnel barrier. The results can be interpreted as a quantifiable description of edge states in the sharp quantum Hall edge limit, and incompressible strips are shown to vanish below the magnetic length scale at this sharp edge.

2.1 Fabrication

The samples consist of two separately contacted perpendicular high mobility quantum wells (QW^- and QW^+) forming a T-shaped structure (Fig. 1), where ^- and ^+ are defined relative to the quantizing magnetic field, B. QW^- is the quantum Hall effect system under study, and QW^+ functions as the probe quantum well. The QWs consist of GaAs embedded in Al_{0.32}Ga_{0.68}As. Using cleaved-edge overgrowth (CEO), a 150 Å thick (001)-quantum well (QW^-) is cleaved along the perpendicular (110)-plane and overgrown with a w = 200 Å thick (110)-quantum well (QW^+) in a second epitaxial growth step. The quantum wells are separated from each other by a b = 50 Å thick A_{0.32}Ga_{0.68}As tunnel barrier. Both QWs are modulation doped with a Si-δ layer 500 Å and 400 Å away from the respective QWs. The electron sheet density in the bulk of QW^- and QW^+ after illumination is n^- = n^+ = 2 × 10^{11} cm^-2 for both and they are 5000 Å and 3600 Å below the surface, respectively. The low temperature mobility in QW^- is μ = 2 × 10^5 cm^2/Vs while for QW^+ we estimate μ = 2 × 10^5 cm^2/Vs. In the y-direction the sample geometry is translationally invariant and the tunnel junction extends about 20 μm in width. The QWs are separately contacted with ohmic indium contacts and the tunnel conductance dU/dV is studied in a 3He cryostat at temperatures of 400 mK.

Figure 1 Samples are fabricated by cleaved edge overgrowth. Two quantum wells (QW^- and QW^+) are arranged in a T-shape separated by a 50 Å thick tunnel barrier. A magnetic field B creates quantum Hall edge currents in QW^- propagating parallel to the extended tunnel junction. QW^+ acts as probe contact.
3 Electronic structure of a sharp edge

Figure 2 shows the differential tunnel conductance $dI/dV$ at zero dc-bias while sweeping the $B$-field. We observe well developed peaks in $dI/dV$ at certain values of the magnetic field. The four resonances are denoted with $n=0, 1, 2, 3$ from right to left. Their height and width is larger for those at higher $B$-field. The peaks show a slightly asymmetric shape with a steeper slope at the high $B$ side. Above 5 T we observe a strong suppression of the tunnel current.

3.1 Dispersion $E$ vs. $k_x$, translational momentum

In the following we explain the special tunnel selection rules based on momentum conserved tunneling. In the presence of a magnetic field the electronic states in $QW^2$ are quantized to Landau levels with energy gaps proportional to $B$. The confining edge potential $V(x)$ enters into the full Schroedinger equation as:

$$\left[ \left( \frac{p - eA}{2m^*} \right)^2 + V(x) \right] \Psi(x, y) = E\Psi(x, y).$$

Choosing the Landau gauge $A(x, y) = xB_y$ where $x=0$ in the center of $QW^2$, Eq. (1) becomes translationally invariant in the $y$-direction. Expressing $\Psi(x, y) = \Psi_{n, k_y}(x) e^{i k_y y}$, we can solve for the orbital solutions in $x$ as well as the dispersion $E_n^\pm(k_y)$:

$$\left[ \frac{p_x^2}{2m^*} + \frac{1}{2} m^* \omega_c^2 (x - k_y l_0^2)^2 + V(x) \right] \Psi_{n, k_y}(x) = E_n^\pm(k_y) \Psi_{n, k_y}(x),$$

where $n$ is the Landau index, $\omega_c = eB/m^*$ is the cyclotron frequency for mass $m^*$, and $l_0 = \hbar/eB$ is the squared magnetic length.

For $QW^2$ the 2D $k$-space dispersion projects onto the $k_y$-axis with the parabolic contour:

$$E(k_y) = \frac{\hbar^2 k_y^2}{2m^*}.$$  \hspace{1cm} (3)

From our choice of Landau gauge, the parabolic dispersion will always be centered at $k_y = 0$. We neglect spin splitting since the bare Zeeman energy is below our experimental resolution, and we neglect the influence of the parallel magnetic field in Eq. (3) since orbital effects in $QW^2$ from the in-plane field can be shown to be negligible\(^1\).

3.2 Dispersion $E$ vs. $X$, orbit center coordinate

If we scale the momentum in $y$ by the square of the magnetic length, we can describe the tunnel selection rules also

\(^1\) Calculations showed that the $k$-space position for the Fermi point $FP^0$ and the dispersion velocity there change less than 5% even up to 10 T where $l_0 < w$, so the parabolic dispersion assumption is justified.
in terms of the conservation of orbit center coordinate in \(x\), \(X = \frac{\hbar}{m} k_x\). This coordinate system will give us the best intuition for a real-space picture of the quantum Hall edge.

Equation (2) then takes the form:

\[
\frac{p_i^2}{2m} + \frac{1}{2} m \omega^2 (x - X)^2 + V(x) \psi_{n,x}(x) = E_n(X) \psi_{n,x}(x) .
\]

We can write \(E_n^1\) as a function of \(X\) as well.

\[
E_n^1(X) = \frac{\hbar^2 X^2}{2m a_0^4} .
\]

Since both energy \(E\) and orbit center \(X\) are conserved upon tunneling, states can be plotted according to their \(E\) vs. \(X\) eigenvalues (Fig. 2), and tunneling is only allowed where the two curves \(E_n^1(X)\) and \(E_n^1(X)\) intersect. \(dE/dV\) features a maximum at magnetic fields whenever the Fermi point \(FP\) of the probe \(QW^d\) crosses the QH dispersion curves. In real space, the orbit center coordinate associated with this Fermi point is \(X_f = \frac{\hbar^2 k_x}{\hbar^2 / eB, \text{ tunable with magnetic field given a constant } |k_\parallel| = (1.2 \pm 0.1) \times 10^4 \text{ m}^{-1}\). We note that the orbit center tunnel distance \(X_e\) is the same as the cyclotron radius in \(QW^d\) for an orthogonal magnetic field.

To illustrate the resulting system, the dispersion curve \(E_n(X)\) is calculated for an ideally sharp step function edge potential \(V(x)\) at the barrier position for zero bias \([18]\). The result is plotted in Fig. 2 for magnetic fields consistent with a resonance condition for the three most prominent tunnel conduction peaks.

### 3.3 Real-space position of sharp edge channels

In the following we study the peak positions in magnetic field \(B_x\) to determine the real space position of the edge states at the Fermi energy. For this purpose, we define a new length scale \(X_{e} (n) = |X_{e} (B_x)| - b - w/2\), the distance of the \(n\)-th experimentally observed orbit center position at resonance from the hard wall barrier. Table 1 shows these experimental positions \(X_e\) for the four observed edge channels. Note that in all cases the distance of the \(n\)-th level from the edge is of order the magnetic length \(l_0\).

At zero voltage bias, the theoretical edge channel position can be deduced from an analytical picture that is simpler than the Hartree method outlined above, provided screening is neglected. First we recall that the orbital wavefunctions in the Landau levels are given by Hermite polynomials multiplied by a gaussian. When the bulk Fermi energy is pinned within the \(N\)-th bulk Landau level, the position where the \(n\)-th Landau band crosses the Fermi energy is simply given by \(X_{e,n} (N, n) = \xi_{m} / \sqrt{\hbar v_F m}\), where \(\xi_{m}\) is the \(m\)-th zero of the \(N\)-th order Hermite polynomial and \(n + 1 \leq N\). This model matches the experimental trend very well, consistently predicting an edge position about one magnetic length closer to the barrier than the experimental value. We attribute this small difference to a slight band bending towards the edge of \(QW^d\), arising from the detailed electrostatics of this non-planar tunnel geometry \([19]\).

For all values of \(n\), the length scale between edge channels is found to be of order or smaller than both the magnetic length \(l_0\) and the Bohr radius \(a_0 = 10 \text{ nm in GaAs}\). As a consequence, the formation of compressible and incompressible strips as predicted by Chklovskii et al. \([7]\) cannot occur here since the electrostatic screening central to the model must take place on length scales larger than both of these. Nor is the integer QH edge reconstruction proposed for soft edges expected to occur here since the observed depletion length scales are significantly less than the requisite \(8 l_0\) \([20]\). Looking at the upper panels of Fig. 2, one sees a pot of the wavefunction associated with the various orbit centers at the Fermi energy. From these it is clear that the states associated with the compressible Landau bands overlap in real space, so that in the presence of short-range disorder electrons will freely scatter charge from the edge all the way to the bulk. Thus the observed resonant tunneling peak positions verify that a sharp edge limit has been observed wherein the compressible edge states overlap, and incompressible strips are too narrow to carry a lossless edge current.

### 4 L-junction samples

A non-planar geometry for the quantum Hall (QH) effect is studied, whereby two quantum Hall (QH) systems are joined at a sharp right angle. When both facets are at equal filling factor \(\nu\) the junction hosts a channel with non-quantized conductance, dependent on \(\nu\). The state is metallic at \(\nu = 1/3\), with conductance along the junction increasing as the temperature \(T\) drops. At \(\nu = 1/2\) it is strongly insulating, and at \(\nu = 3/4\) it shows only weak \(T\) dependence. Upon applying a dc voltage bias along the junction, the differential conductance again shows three different behaviors. Hartree calculations of the dispersion at the junction illustrate possible explanations, and differences from planar QH structures are highlighted.

#### 4.1 Fabrication

This device is named the corner quantum-well heterojunction (CQW), fabricated by over-growing a standard GaAs/AlGaAs heterojunction structure on an ex-situ cleaved corner \([22]\). To achieve high-quality

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**Table 1** Measured distance \(X_e (n)\) of the \(n\)-th orbit guiding center from the tunnel barrier for the edge channels \(n = 0, 1, 2, 3\) at various \(B_x\)-fields, compared to the magnetic length \(l_0\) and to the analytical sharp edge limit \(X_{e,n} (n)\) \((n^2 = 1.9 \times 10^5 \text{ cm}^2)\).

<table>
<thead>
<tr>
<th>(n)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_x) (T)</td>
<td>3.44</td>
<td>1.9</td>
<td>1.33</td>
<td>~1.02</td>
</tr>
<tr>
<td>(\nu)</td>
<td>2.3</td>
<td>4.1</td>
<td>5.9</td>
<td>7.7</td>
</tr>
<tr>
<td>(l_0) (nm)</td>
<td>14</td>
<td>19</td>
<td>22</td>
<td>25</td>
</tr>
<tr>
<td>(X_{e,n} (n)) (nm)</td>
<td>0</td>
<td>13</td>
<td>27</td>
<td>42</td>
</tr>
<tr>
<td>(X_e (n)) (nm)</td>
<td>8 ± 2</td>
<td>26 ± 3</td>
<td>44 ± 5</td>
<td>62 ± 6</td>
</tr>
</tbody>
</table>
growth across the corner, both facets must have the same Miller index class. To expose orthogonal \(\{110\}\)-class cleave-planes, a \(\{110\}\)-GaAs substrate wafer is cleaved once in the orthogonal \(\{110\}\) plane. The shared corner between the \(\{110\}\) and \(\{110\}\) planes serves as the overgrowth corner and is mounted facing the molecular flux. For shorthand, we refer to \(\{110\}\) as the s-facet and \(\{110\}\) as the p-facet (corresponding to ‘substrate’ and ‘precleave’, respectively).

The samples were grown in a Epi Gen-II molecular beam epitaxy (MBE) system with an ultra-high vacuum ambient \((P = 10^{-11} \text{ mbar})\). The \(\hat{z}\)-axis in Fig. 3 is the rotation axis of the substrate as well as the axis around which the molecular sources are symmetrically arranged at an azimuthal angle \(\theta _s = 33^\circ \). The molecular flux \(\Phi _s\) is first calibrated with RHEED on a standard flat substrate. The corner-substrate is then mounted in its place with both s- and p-facets at a nominal \(\theta _p = 90^\circ - \theta _s = 45^\circ \) angle. Under continuous rotation of angle \(\alpha \) around the \(\hat{z}\) axis, the flux incident on the tilted s-facet will consist of an oscillating component which averages to zero, and a constant component equal to the RHEED calibrated flux \(\Phi _p\) projected onto the tilted surface, \(\Phi _p = \Phi _s \cos (\theta _p)\) [21]. The complementary p-facet will correspondingly see an average flux of \(\Phi _s = \Phi _p \cos (\theta _p) = \Phi _p \sin (\theta _p)\). In spite of fluctuations in the molecular flux due to rotation, we nonetheless achieve high quality crystal growth on the tilted facets of the corner substrate. After oxide desorption, the sample is overgrown under rotation \((dz/d\alpha = 7 \text{ rpm})\) at a substrate temperature of \(T_{\text{sub}} = 460^\circ \text{C}\) with all molecular fluxes increased by a factor of \(1/\cos(45^\circ) = \sqrt{2}\) to compensate for the geometrically reduced flux on the tilted substrate. For a typical growth, the beam equivalent pressure \(\text{BEPP} = 5 \times 10^{-4} \text{ mbar}, \text{ and } \text{BEPP} = 2.2 \times 10^{-7} \text{ mbar, corresponding to a RHEED calibrated Ga growth rate of 2.4 Å/s on a flat substrate.}

In the SEM picture in Fig. 3, we confirm the sharpness of the corner growth morphology in a test corner-overgrown superlattice structure. Using the RHEED calibrated flux values, the three dark bands on the left p-(right s-) side of the sample are \(810 \text{ Å} \) (560 Å) AlAs separated by lighter GaAs/AlGaAs layers of varying thickness \(1620 \text{ Å}, 1620 \text{ Å}, 2430 \text{ Å} \) (1120 Å, 1120 Å, and 1680 Å) from the surface downwards. The corner junction clearly maintains a sharp 90° corner profile even under growth. The slightly thicker layers on the left result from a growth angle of \(\theta _p = 56^\circ \) with the ratio of thicknesses \(d\) on the two sides equal to \(d_s/d_p = \tan (\theta _p)\).

**4.2 Transport precharacterization** By measuring the 4-point longitudinal resistance of the two facets (Fig. 4), the densities of the two facets can be independently measured with the result: \(n_s = 1.07 \times 10^{11} \text{ cm}^{-2}\) and \(n_p = 1.30 \times 10^{10} \text{ cm}^{-2}\). Observation of fractional quantum Hall effect minima forming at this strength at filling factor \(\nu = 2/3\) at 350 mK attests that both facets have a transport mobility of order \(\mu = 5 \times 10^5 \text{ cm}^2/\text{Vs}\). We explain the dissimilarity in the two densities by considering that the thickness of growth on the two facets will not be equal if the corner is not mounted at exactly 45° during growth.

To demonstrate continuity of the 2D electron system, we first measure the 2-point resistance across the corner at zero \(B\). The cross-corner resistance is \(\sim 1 \Omega\) comparable to the 2-point resistivities within a given facet, our first indication of a continuous 2D system. For a more rigorous demonstration of 2D continuity, we apply a magnetic field and tilt the sample to an angle where the filling factors on the two facets make simple integer ratios [23]. Figure 5

![Figure 3](image_url) **Figure 3** Left: SEM micrograph of corner overgrowth test structure. Right: Schematic of growth geometry: rotation axis \(\hat{z}\), source angle \(\theta _s\), substrate angle \(\theta _p\), and precleave angle \(\theta _p\).
shows the case $\nu_x/\nu_y = 2$ ($\theta = 31.3^\circ$), and the inset shows the edge-state diagram for the $\nu_x/\nu_y = 2 : 1$ case. With a current supplied from contact Y to contact Z, the 4-point resistance across the corner between voltage contacts A and B, $R_{AB}$, is zero ($0 \pm 0.5 \Omega$) whenever both 2D systems are gapped ($\nu_x/\nu_y = 2; 4:2, 6:3$). From the Landauer–Buttiker diagram, this means the outermost edge channel is transmitted across the corner with no backscattering, demonstrating the continuity of the 2D electron system across the corner. For voltage contacts C–D in the same gapped regions, the Landauer–Buttiker equations predict the observed $R_{CD}$ when both facets are in QH plateaus, as predicted by the Landauer–Buttiker formalism (inset).

For the two samples studied in what follows, the facets have near-equal densities $n_1 = 1.10 \times 10^{11}$ cm$^{-2}$ and $n_2 = 1.28 \times 10^{11}$ cm$^{-2}$, and a junction length $L = 2$ mm for sample A ($n_1 = 1.11 \times 10^{11}$ cm$^{-2}$, $n_2 = 1.45 \times 10^{11}$ cm$^{-2}$, and $L = 4.5$ mm for sample B) [1] with a mobility estimated at around $\mu \sim 5 \times 10^5$ cm$^2$/Vs. Additional samples showed the same behavior which was reproducible in multiple cool-downs.

5.2 Dispersion $E$ vs. $k_y$ and $E$ vs. $X$ We consider now the case of equal $\nu$ on both facets. In analogy with Section 3, we calculate the dispersion at finite $B$, but now using the Hartree potential $V_{II}(x,z)$ solved at $B = 0$ for a sharp corner confinement:

$$\left[ \frac{(p + eA)^2}{2m^*} + V_{II}(x,z) \right] \psi = E\psi .$$

(6)

Figure 7(b) shows the Cartesian coordinates, and the calculations assume equal charge density on both facets,
n_1 = n_2 = 1 \times 10^{11} \text{ cm}^{-2}, \text{ neglecting spin for simplicity. By}
choosing the Landau gauge \mathbf{A} = (0, B x, 0), \text{ momentum } k_y
is a good quantum number, \text{ and the dispersion } E_m(k_y) \text{ results}
from the eigenvalue problem of Eq. (6) for \psi_m(k_y) = \phi_m(x, z) e^{ik_y x}, \text{ where } m \text{ is the Landau index.}

Figure 7 shows the dispersion versus projected orbit centre \( X = k_x l_0 \) for the lowest energy levels. \text{ For comparison, the dispersion of a single sharp QH edge [25] is overlaid on the left and its mirror image overlaid on the right. These two hard-wall-like dispersions arise because the sudden 90° bend in the heterojunction serves as a hard wall for the incident skipping orbits within the opposing facet. Unlike the planar antwire of Ref. [14], there is no tunnel barrier separating the two systems, and the edge states from the two orthogonal facets interpenetrate at the corner. The third subsystem is a deeply bound wire seen previously in Hartree calculations as a 1D accumulation of charge at the corner [22] and indicated here by the central parabolic dispersion. This accumulation wire adds two spin-degenerate 1D modes to the edge in each direction and serves as an additional 1D channel for scattering. Together with the 1D edge modes from the QH systems, the model thus predicts as many as 2N_\nu + 1D modes in each direction, whose dispersions anticross in the Hartree solution.

6 Bent quantum-Hall junction conductance characterization

We now examine the transport data at the bent QH junction. Zero-resistance minima in the longitudinal resistance \( R_x \) for both facets in Fig. 8 (grey) identify the well-formed QH states: \( \nu = 1/3, 2/3, 1, 2, 3, 4, 5, 6. \)
The conductance along the junction is measured using the following 4-point geometry [32, 33]: a current is driven across the corner to ground with an applied bias \( V_x \), and the resultant voltage \( V_{cc} \) or \( V_{cc}^{\prime} \) is measured between two contacts, one on each facet (Fig. 8, inset). The cryostat maintained 30 mK base temperature up to 18 T, rising to 60 mK at 20–23 T.

The cross-corner voltages \( V_{cc} \) (solid) and \( V_{cc}^{\prime} \) (dashed) are plotted in Fig. 8. \text{ With the facets in a QH state, the current along the junction can be calculated [26]}

\[
I_{\text{int}} = j \frac{e}{\hbar} \frac{V_{cc}}{V_{cc}^{\prime}} = \frac{e^2}{\hbar} V_{cc}^{\prime}.
\]

The conductance \( G \) along the junction is

\[
G = \frac{I_{\text{int}}}{V_x} = \frac{e^2}{\hbar} \frac{V_{cc}}{V_{cc}^{\prime}} = \frac{e^2}{\hbar} \frac{V_{cc}^{\prime}}{V_x}.
\]

Figure 8 (Sample A) Longitudinal resistance \( R_x \) within each facet (grey), and cross-corner voltages \( V_{cc} \) (solid black) and \( V_{cc}^{\prime} \) (dashed black). Non-zero \( V_{cc} \) minima indicate finite conduction along the junction. Inset: schematic of edge states and backscattering of current along the junction.
The junction conductance for sample A is plotted versus $\nu$ in Fig. 9(a) for filling factors where $R_{xx} = 0$. At 30 mK, the junction does not conduct for either $\nu = 1$ or 2. At $\nu = 3, 4, 5$, and 6 the junction conductance falls within the range $0.01 - 0.04 \, e^2/h$. The fractional QH effect $\nu = 2/3$ shows a similar conductance, while the conductance at $\nu = 1/3$ at 60 mK is slightly higher. In all cases, the small and non-quantized junction conductance $G \ll e^2/h$ indicates that charge is strongly backscattered within the junction region.

In Fig. 9(a), integer $\nu$ are labelled with both the Landau index $m$ as well as spin $\sigma$. In diffusive 1D systems, one expects a stepwise increase in conductance with each additional mode, and in Fig. 9 such steps are observed to occur pairwise in $\nu; \nu = 1, 2 (m = 0); 3, 4 (m = 1);$ and $5, 6 (m = 2)$. The conductance thus behaves as though the Landau index $m$, not $\nu$, counts the modes at the junction. A suppression of spin splitting at this sharp QH junction could explain this result. A similar lack of spin-splitting at a sharp edge was already experimentally observed in tunneling experiments of sharp-edge systems [25] and deserves further scrutiny.

The length dependence $L$ of the conductance for sample B is shown in Fig. 9(b). At $\nu = 3$, 4, and 6, where the dependence could be measured, the short junction ($L = 0.45$ mm), black lines) conducts better than the long junction ($L = 4.2$ mm, grey lines), with conductance scaling approximately as $G \sim 1/L$. If backscattering is distributed uniformly along the length of the junction, 1D conductance can be written $G = (e^2/h) \lambda N/L$ where $\lambda N$ is the mean free path times the number of modes. These results suggest mean-free path $\lambda N = 7 \mu m (\nu = 3, 4)$ and $\lambda N = 27 \mu m (\nu = 6)$ at the temperatures shown, and provide evidence that the charge backscattering is distributed along the junction.

The junction conductance was also measured as a function of temperature $T$ and dc voltage bias $V_x$. Figure 10(a) shows the $T$ dependence of the conductance for $\nu = 1/3, 1, 2, 3,$ and 4. For each $\nu$, the same behaviour occurs across the entire minimum. With decreasing $T$, the conductivity along the corner junction either decreases ($\nu = 1, 2$), stays roughly constant ($\nu = 3, 4$), or increases ($\nu = 1/3$), illustrating what we label as strongly insulating, weakly insulating or metallic behaviour, respectively. In Fig. 10(b), the differential conductance $dI/dV$ of the corner QH junction is plotted for the same $\nu$ as a function of $V_x$. For the insulator, $dI/dV$ drops drastically with reduced bias ($\nu = 1, 2$), whereas $dI/dV$ increases for the metallic state, forming a cusp at zero bias ($\nu = 1/3$). Varying the temperature up to 170 mK while measuring this cusp reveals that most of the temperature dependence occurs at the small biases (not shown). The weakly insulating phase ($\nu = 3, 4$) shows the weakest bias dependence, with a mild dip at low bias indicating an insulator.

Possible explanations of these phases must be consistent with the experimentally measured sharp junction curvature of Fig. 7(a). We therefore base our discussion on the dispersisons of Fig. 7(b). The explanation for the insulator at $\nu = 1, 2$ is twofold. It can arise either from an anticrossing band insulator at the corner or from localization of 1D states. Gaps in the dispersion arise whenever the bands of Fig. 7(b) anticross, and if the Fermi level sits within such a gap, the junction would host a band insulator. At higher $\nu$ relevant for $\nu = 1, 2$, the anticrossing gaps from Eq. (6) increase, in contrast to planar barrier systems where the gaps vanish exponentially at high $\nu$. The increased gaps due to the strong coupling of the modes at the corner are likely to form a band insulator at high $\nu$. Alternately, the $\nu = 1, 2$ insulator could arise according to the scaling theory of localization for 1D systems, since all 1D systems are expected to become insulators in the presence of disorder [27] and repulsive interactions [28, 29]. The limited temperature range of the data in Fig. 10(a, b) is insufficient to identify which of these two mechanisms may be responsible, though we note that the conductance does drop faster than a power-law, consistent with both explanations. We also note that the $\nu = 1, 2$ temperature dependences perfectly overlap, suggesting a common mechanism.
The weakly insulating behaviour at $\nu = 3, 4$ may be related to weak localization. Examining the voltage dependence in Fig. 10, the zero-bias dip in $dI/dV$ suggests a crossover from a metal to a weakly insulating state below $V_s \sim 1 \text{ mV}$, which would represent an energy scale for the weak localization. A careful modeling of the multimode 1D conductance of Fig. 7(b) will be the first step towards identifying these energy scales in the model, and promises to be an interesting subject of future work.

Perhaps most intriguing is the metallic behavior at $\nu = 1/3$, with a junction conductance that increases as temperature is lowered. At such high fields $c_B > B$, the Hartree dispersions from Eq. (1) would have to be calculated self-consistently at finite $B$, and must include interactions to correctly account for the Laughlin ground state in the facets. Though such calculations are beyond the scope of this paper, qualitatively one expects a mixing of the accumulation wire magnetosubband dispersion for electrons [30] with the fractional QH edge dispersions for quasiparticles [31].

Looking at the $1/3$ voltage bias curves in Fig. 10, it is clear that the conductance is strongly temperature dependent at extremely low temperatures. The likeliest candidate for such low-energy scattering is electron–electron interactions. As discussed in Refs. [32] and [33], coupled fractional QH channels can result in such metallic behaviour, as long as electrons (not fractional quasiparticles) backscatter the charge between the counterpropagating $\nu = 1/3$ edges, creating an ‘antiwire’. The $T$-dependence is predicted to be metallic since low-temperature correlations at $\nu = 1/3$ suppress electron tunnelling and therefore backscattering. The conductance is predicted to behave as a power-law $G(T) \sim T^{-\alpha}$, and the data of Fig. 10 would fit an exponent $\alpha = -0.4$, corresponding to the Luttinger parameter $g = 1 - \alpha/2 = 1.2$ after Ref. [33]. We note that the same exponent occurs in the voltage dependence $d/dV \sim V^\alpha$ [dashed line, Fig. 10(a) and (b)]. It would appear that the accumulation wire effectively functions as a $\nu = 1$ ‘vacuum’ for counterpropagating fractional quasiparticles, constituting a $1/3 : 1 : 1/3$ junction where only electrons can tunnel and backscatter charge. We remark that the planar antwire geometry originally suggested in Refs. [32], [33] and implemented in Ref. [14] actually prohibits the desired strong coupling of fractional QH edges, since the intervening barrier exponentially suppresses tunnelling at high $B$ [34]. Only in the non-planar geometry introduced here can counterpropagating edge modes overlap sufficiently in real space that strong-backscattering in the high-$B$ limit may occur.

We note that the sharp confinement potential will play a decisive role in modeling the junction conductance. Experimentally, sharp edge potentials have been shown to eliminate the incompressible strips characteristic of soft QH edges [25]. Recent theory has been able to describe conduction in this sharp limit where these incompressible strips are expected to be absent [9].

7 Conclusion In conclusion we have provided an overview of quantum Hall edges, with special focus on recent experimental results in sharp edge structures. In the first $T$-junction experiment, we probed the QHE edge state dispersion relation $E_1^\nu(k)$ at a sharp cleaved edge. Associating conductance peaks with momentum-conserved tunneling into quantum Hall edge states, we deduced the real space position of the edge state orbit centers and deduce that incompressible strips as envisioned by Chklovskii cannot exist here. Subsequently, the corner-overgrowth technique was reviewed, and the resulting $L$-shaped quantum well was brought into the quantum Hall regime. Individual facets show well-developed quantum Hall effect, and at equal filling on both facets, conduction across the corner shows that the two facets are coupled and current
can scatter at this corner-junction. The conduction is characterized as a function of temperature, voltage, and filling factor, showing various metallic and insulating states.

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References

[29] Growth protocol for sample A is 05-23-02.4L3a, and sample B is 01-30-03.2L3a, and sample B is 01-30-03.2L3a.
[34] This simplified picture is valid as long as \( \theta_d \leq 90° - \theta_s \).