Measuring carrier density in parallel conduction layers of quantum Hall systems

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An experimental analysis for two parallel conducting layers determines the full resistivity tensor of the parallel layer, at magnetic fields where the other layer is in the quantum Hall regime. In heterostructures which exhibit parallel conduction in the modulation-doped layer, this analysis quantitatively determines the charge density in the doping layer and can be used to estimate the mobility. To illustrate one application, experimental data show magnetic freeze-out of parallel conduction in a modulation-doped heterojunction. As another example, the carrier density of a minimally populated second subband in a two-subband quantum well is determined. A simple formula is derived that can estimate the carrier density in a highly resistive parallel layer from a single Hall measurement of the total system. © 2005 American Institute of Physics.

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I. INTRODUCTION

Various high-mobility heterostructures which exhibit the quantum Hall effect contain parallel conducting layers whose individual conductance parameters need to be characterized. One such system of practical interest is a high-mobility two-dimensional electron system (2DES) with a parallel conducting modulation-dopant layer [Fig. 1(a)]. Other systems include quantum wells with a second occupied subband [Fig. 1(b)] and double-quantum well systems [Fig. 1(c)]. In all cases, the well-known signature of parallel conduction is that the minima in the longitudinal resistance do not go to zero in the quantum Hall limit. One standard method for separately characterizing the layer densities is the Shubnikov–de Haas (SdH) method, which requires that both systems show oscillations in the same magnetic-field range. These oscillations are then Fourier transformed against 1/B to give the densities of both systems. However, this method cannot be used if one of the systems has sufficiently low mobility that SdH oscillations are indiscernable, and this method cannot be used if one of the systems has sufficiently low mobility that SdH oscillations are indiscernable. Other methods have been introduced which can determine resistivities in nonquantized parallel channels, but to date an explicit treatment of the quantized case is needed.

II. PARALLEL CONDUCTION MODEL

We begin by modeling the resistivity tensor of the combined quantum Hall and parallel conducting layers. We define separate resistivity tensors for the quantum Hall and parallel conducting layers as

\[
\rho^Q = \begin{pmatrix} 0 & h/e^2 \\ -h/e^2 & 0 \end{pmatrix}, \quad \rho^p = \begin{pmatrix} \rho^p_{xx} & \rho^p_{xy} \\ -\rho^p_{xy} & \rho^p_{yy} \end{pmatrix},
\]

Assuming that the in-plane electric fields seen by the two layers are equal, the corresponding conductivities for each layer,

\[
\sigma^Q = \begin{pmatrix} 0 & -e^2/h \\ e^2/h & 0 \end{pmatrix}, \quad \sigma^p = \begin{pmatrix} \rho^p_{xx} & \rho^p_{xy} \\ -\rho^p_{xy} & \rho^p_{yy} \end{pmatrix},
\]

can be summed to yield the sheet conductivity of the combined system

\[
\sigma^{tot} = \sigma^Q + \sigma^p = \frac{1}{|\rho^p|} \begin{pmatrix} \rho^p_{xx} & \rho^p_{xy} - |\rho^p| e^2/h \\ -\rho^p_{xy} & \rho^p_{yy} + |\rho^p| e^2/h \end{pmatrix},
\]

where \(|\rho^p| = \det(\rho^p)| is the matrix determinant. Inverting \(\sigma^{tot}\) gives the total resistivity tensor, \(\rho^{tot} = (\sigma^{tot})^{-1}\) whose components are directly measured in experiment.

![FIG. 1. Conduction-band schematic for the three kinds of parallel conduction systems: (a) the dopant layer of a modulation-doped heterostructure, (b) a double-quantum well system, or (c) a quantum well with a second occupied subband.](image-url)
\[ \rho_{xx}^{\text{tot}} = \frac{|\rho|}{\rho_{xx}^2 + (|\rho|v/ev^2/h + \rho_{xy}^2)^2}, \]

\[ \rho_{xy}^{\text{tot}} = \frac{|\rho||v/ev^2/h + \rho_{xy}^2|}{\rho_{xx}^2 + (|\rho|v/ev^2/h + \rho_{xy}^2)^2}. \]

Just as in single-layer Hall measurements where expressions for the longitudinal and transverse resistance are solved for the two unknowns (density \( n \) and Hall mobility \( \mu_h \)), the above equations can be solved for the only two unknowns in this problem, namely, \( \rho_{xx}^\parallel \) and \( \rho_{xy}^\parallel \), the components of the resistivity tensor in the parallel layer. We will demonstrate this analysis for data measured in the general Van der Pauw geometry, although the same analysis can be adapted to Hall bar and Corbino geometries. All four-point resistances were measured in an Oxford \(^3\)He cryostat using standard lock-in techniques with a high input impedance preamplifier \( R_{in} \sim 100 \text{ G} \Omega \) to avoid measurement errors which might be mistaken for parallel conduction.\(^7\)

III. EXPERIMENTAL MEASUREMENTS

The first sample of interest is a heterojunction quantum well with parallel conduction in the modulation doping layer [Fig. 1(a)]. We start experimentally by measuring the longitudinal resistance of the sample at finite \( B \) field in two complementary geometries \( R_{xx}^B \) and \( R_{xx}^A \) [see Figs. 2(a) and 2(b)], and determine the total \( B \)-dependent sheet resistance \( \rho_{xx}^{\text{tot}}(B) \) according to the Van der Pauw equations,\(^8\)

\[ \rho_{xx}^{\text{tot}} = \frac{\pi}{\ln 2}(x) \frac{R_{xx}^A + R_{xx}^B}{2}, \]

\[ f(x) = 1 - \frac{\ln 2}{4} \left( \frac{x - 1}{x + 1} \right)^4 - \frac{(\ln 2)^2}{12} \left( \frac{x - 1}{x + 1} \right)^4. \]

Here \( x = R_{xx}^A / R_{xx}^B \) for \( R_{xx}^B > R_{xx}^A \), and \( f(x) \leq 1 \) is a weakly varying function of \( x \) of order unity. The Hall resistance \( \rho_{xy}^{\text{tot}} = R_{xy} \) can be measured directly [Fig. 2(c)].

We determine the resistivity tensor of the parallel layer \( \rho^\parallel \) in terms of the measured total resistivity tensor \( \rho^{\text{tot}} \) as follows, effectively inverting Eq. (4). First, the total resistivity tensor \( \rho^{\text{tot}} \) is inverted to \( \sigma^{\text{tot}} = (\rho^{\text{tot}})^{-1} \), and since \( \sigma^{\text{tot}} = \sigma^Q + \sigma^J \),

\[ \sigma_{xx}^{\text{tot}} = \frac{\rho_{xx}^{\text{tot}}}{|\rho^{\text{tot}}|} = 0 + \sigma_{xx}^\parallel, \quad \sigma_{xy}^{\text{tot}} = \frac{\rho_{xy}^{\text{tot}}}{|\rho^{\text{tot}}|} = \frac{v/ev^2/h}{h} + \sigma_{xy}^\parallel. \]

For a given filling factor index \( \nu \) of a quantum Hall minimum Eq. (6) can be solved for \( \sigma_{xx}^\parallel \) and \( \sigma_{xy}^\parallel \), the parallel conductivity tensor \( \sigma^\parallel \) can then be inverted to give the resistivity tensor of the parallel layer

\[ \rho_{xx}^\parallel = \frac{\sigma_{xx}^\parallel}{\sigma^Q} \left( \frac{\rho_{xx}^{\text{tot}}}{|\rho^{\text{tot}}|} \right)^2 + \frac{|\rho^{\text{tot}}|}{\rho_{xx}^{\text{tot}}} \left( \rho_{xx}^{\text{tot}} - |\rho^{\text{tot}}|v/ev^2/h \right)^2, \]

\[ \rho_{xy}^\parallel = \frac{\sigma_{xy}^\parallel}{\sigma^Q} \left( \frac{\rho_{xy}^{\text{tot}}}{|\rho^{\text{tot}}|} \right)^2 + \frac{|\rho^{\text{tot}}|}{\rho_{xy}^{\text{tot}}} \left( \rho_{xy}^{\text{tot}} - |\rho^{\text{tot}}|v/ev^2/h \right)^2, \]

which is nothing other than Eq. (4) solved for \( \rho_{xx}^\parallel \) and \( \rho_{xy}^\parallel \). The result is plotted in Fig. 4 for values of \( B \) where the Hall resistance is quantized for the high-mobility layer.

We can estimate the carrier density in the parallel layer with either of the following two methods. First, the slope of \( \rho_{xy} \) (both within each quantum Hall effect domain and across all filling factors) indicates a density of \( n^e = B/ep_{xy} = 0.30 \times 10^{11} \text{ cm}^{-2} \). Secondly, as a useful back-of-the-envelope shorthand, we note that in the limit of \( \rho_{xx}^{\text{tot}} < \rho_{xy}^{\text{tot}} \) as in Fig. 3, Eq. (8) can be simplified to

FIG. 2. The three measurements necessary to completely characterize the resistivity tensor at any magnetic field.

FIG. 3. Plot of measured \( \rho_{xx}^{\text{tot}} \) and \( \rho_{xy}^{\text{tot}} \) for a sample with parallel conduction in the modulation-doped layer. \( \rho_{xx}^{\text{tot}} \) is calculated from the Van der Pauw relation in Eq. (5) using \( R_{xx}^B \) and \( R_{xy} \) as measured in Fig. 2. Quantized Hall conductance values \( h/e^2 \) are indicated with lines and indexed by the filling factor \( \nu \).

FIG. 4. Plot of \( \rho_{xx}^{\text{tot}} \) and \( \rho_{xy}^{\text{tot}} \) for a sample with parallel conduction in the modulation-doped layer calculated from the data in Fig. 3 using Eqs. (7) and (8). The slope of \( \rho_{xx}^{\text{tot}} \) yields the carrier density in the parallel conductor \( n^e = eB/\rho_{xx}^{\text{tot}} \). A parabolic fit to \( \rho_{xx}^{\text{tot}} \) is included to determine a \( B = 0 \) intercept.
show the resistance quantum temperature and magnetic-field dependent. Inset: Plot of $\rho_{\parallel}$ for the parallel conducting layer. Note that the density of hopping carriers is independent of $B$ and temperature.

$$\rho_{\parallel} = \frac{B}{n e} = \frac{\rho_{\parallel}^{\text{tot}}}{1 - \rho_{\parallel, \text{ve}}^{\text{tot}}/h}. \tag{9}$$

Solving for $n$ gives

$$n = \frac{n}{n_e^0} \left( \frac{h/\nu e^2}{\rho_{\parallel, \text{ve}}^{\text{tot}}} - 1 \right), \tag{10}$$

where $n^0 = \nu e B(n)/h$ is the density of the QH system, and $B(n)$ is the magnetic-field value in the center of the $n$th plateau in $\rho_{\parallel}^{\text{tot}}$. In Fig. 3, the flat lines above each $\rho_{\parallel, \text{ve}}^{\text{tot}}$ plateau show the resistance quantum $h/\nu e^2$ demonstrating that the first term in parentheses in Eq. (10) is always greater than 1. The remarkable utility of Eq. (10) is that the parallel carrier density can be estimated from a single $\rho_{\parallel}^{\text{tot}}$ measurement. For example, in $\rho_{\parallel, \text{ve}}^{\text{tot}}$ from Fig. 3, the position of the $n=1$ plateau at $B(n=1)=8.07$ T determines $n^0=1.95 \times 10^{11}$ cm$^{-2}$ and the plateau resistance $\rho_{\parallel, \text{ve}}^{\text{tot}}=22.76$ kΩ gives $n=0.26 \times 10^{11}$ cm$^{-2}$—within 15% of the more exact graphical analysis of Eq. (8) and the resulting Fig. 4.

We can separately characterize the conductance of the parallel delta-doping layer as a result of this analysis. Looking at the two components of the resistivity tensor in Fig. 4, one observes first the sharp rise in $\rho_{\parallel, \text{ve}}$ at high $B$ fields resulting from so-called magnetic freeze-out in this highly disordered delta-doping layer, as expected in systems with hopping conduction. The total number of mobile carriers in the delta layer, however, stays fixed as confirmed by the constant slope of $\rho_{\parallel}$. We can also make a rough estimate of the mobility of this layer at 1.3 K, by extrapolating the parabolic fit shown in the figure to $B=0$. With an intercept resistivity of around $\rho_{\parallel, \text{ve}}=7$ kΩ and the known density, the $B=0$ mobility of the parallel conduction layer is about $\mu=3.0 \times 10^4$ cm$^2$/Vs.

The temperature dependence of the parallel layer can also be separately characterized. As shown in Fig. 5, we see that $\rho_{\parallel, \text{ve}}$ increases with decreasing temperature, while $\rho_{\parallel}$ remains unchanged, indicating a temperature-independent density. Both behaviors are consistent with hopping conduction as expected for the delta layer.

The same analysis can be applied to a two-subband quantum well sample [Fig. 1(b)]. In Fig. 6 we show the measured $\rho_{\parallel, \text{ve}}^{\text{tot}}$ and $\rho_{\parallel}^{\text{tot}}$ which are analyzed to give $\rho_{\parallel, \text{ve}}$ and $\rho_{\parallel}$ in the quantum Hall minima as plotted in Fig. 7. The measured parallel carrier density of $n^1=0.11 \times 10^{11}$ cm$^{-2}$ is in good agreement with self-consistent Poisson calculations which predict a second subband occupied to $n_{\text{calc}}=0.15 \times 10^{11}$ cm$^{-2}$. We note that the SdH signal from this low-density second subband was too weak to give an estimable density using the alternate Fourier transform method. Comparing the resistivity components in Fig. 7 with Fig. 4, one sees in $\rho_{\parallel}$ that once again the density of carriers is rather constant over the whole $B$-field range. The measured $\rho_{\parallel}$ is of order 50 Ω or less, implying a rather high mobility of over $10 \times 10^6$ cm$^2$/V s for this second subband. Although this value is not quantitatively conclusive, qualitatively it indicates that the low-density subband which would normally have a correspondingly low mobility, benefits from the Thomas–Fermi screening of the densely populated subband to screen most of the disorder.

As a final note, we remind the reader that measurement errors arise if the input impedance of the lock-in $R_{\text{in}}$ is too low, and these pathological signals look misleadingly like parallel conduction. With a low $R_{\text{in}}$, finite $R_{\text{xx}}$ minima of order $R_{\text{min}}=(h/\nu e^2)/R_{\text{xx}}$ are observed with the astonishing property that the false “parallel” signal disappears upon re-
versing the current contact polarity. Such subtleties in quantum Hall measurement phenomena are discussed in detail elsewhere.7

IV. CONCLUSION

In conclusion, we have demonstrated an analysis of parallel 2D systems that determines the resistivity tensor of one layer at magnetic fields where the other layer is in the quantum Hall regime. We demonstrate the validity of this analysis for determining the carrier density of parallel conducting modulation-doped layers. A temperature-dependent study of the hopping conduction in the dopant layer alone is demonstrated. The second case of a quantum well with two occupied subbands is also presented, and the density of the lightly populated upper subband is determined. This technique will be particularly useful for crystal growers in characterizing doping efficiency in modulation-doped structures, because a single Hall measurement of the total system can determine the excess dopant in the parallel layer.

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