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Influence of voltmeter impedance on quantum Hall measurements

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We report the influence of voltmeters on measurements of the longitudinal resistance R_{xx} in the quantum Hall-effect regime. We show that for input resistances typical of standard digital lock-in amplifiers, R_{xx} can show a nonzero minimum which might be mistaken for a parallel conduction in the doping layer. This residual impedance at the R_{xx} minima can be calculated with $Z^{\text{res}}=R_{xy}^2/R_{\text{in}}+j\omega CR_{xy}^2$, where R_{in} is the input resistance of the voltmeter, C is the measurement capacitance, and $R_{xy}=h/\nu e^2$ is the Hall resistance. In contrast to a real parallel conduction, the effect disappears when either the current source and ground contact are swapped or the polarity of the magnetic field is changed; examples with data are shown. We discuss how proper phasing of a lock-in amplifier is necessary to eliminate false residual minima which arise from stray capacitances. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1948530]

I. INTRODUCTION

The quantum Hall effect (QHE)[1,2] of a two-dimensional system (2DS) in a perpendicular magnetic field B is characterized by the quantization of the transverse resistance to $R_{xy}=h/\nu e^2$ ($\nu=1,2,\ldots$) coincident with minima in the longitudinal resistance R_{xx}. R_{xx} vanishes to $R_{xx}=0$ in high-mobility samples at sufficiently low temperatures, and a similar behavior occurs for the fractional quantum Hall effect (FQHE)[3,4] for odd denominator fractions ($\nu=1/3,2/5,2/3,\ldots$). A flat nonzero low-temperature minimum is normally attributed to a parallel conducting dopant layer,[5,6] and the QHE regime can be characterized to give the carrier density and mobility of the parallel layer itself.[7] In the following, we will show measurements where such a parallel conduction seems to be evident but, in reality, is due to a measurement error resulting from the finite input impedance of the measuring voltmeter.

II. CIRCUIT MODEL

The standard method to measure the quantum Hall effect is a four-point measurement with a Hall bar, as indicated in Fig. 1. A typical Hall bar has a current input I_i and a current output I_o. There are four voltage contacts V_1, V_2, V_3, and V_4 to measure the electrical potential at fixed points along the sample. For an ideally homogeneous and symmetric sample with $I_i=I_o=I$, R_{xy} is given by $R_{xy}=(V_1-V_3)/I=(V_2-V_4)/I$ and $R_{xx}=(V_1-V_2)/I=(V_3-V_4)/I$.

The analysis in this paper assumes that the sample is at a B-field such that the Hall resistance $R_{xy}=h/\nu e^2$ is quantized, and the longitudinal resistance $R_{xx}=1$ is zero. In the common QHE edge-state picture[8] the current flows only along a small strip at the edge of the sample, as indicated in Fig. 1 for electrons. Following the Landauer–Büttiker formalism for QHE edge channels,[9,10] the edge current flowing downstream from contact k is given by

$$I_k=\nu\frac{e}{h}\mu_k=\nu\frac{e^2}{h}V_k, \tag{1}$$

with ν being the filling factor, μ_k the chemical potential of the edge channel with index k, and V_k the corresponding voltage. In contrast to the conventional current flow, the direction of the edge channel flow equals the direction of the carrier movement and is determined by the sign of the charge and the polarity of the magnetic field. The following discussion assumes edge currents running clockwise, as in the case of electrons and a magnetic field pointing out of the Hall bar (see Fig. 1).

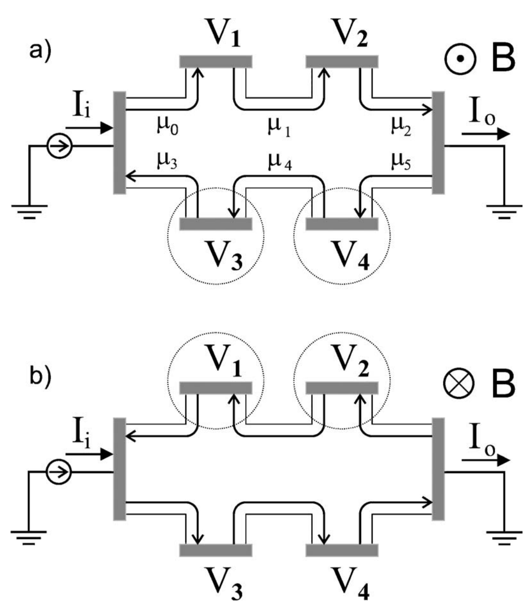

FIG. 1. Typical example of a Hall-bar geometry. I_i and I_o indicate the input and output currents. (a) For the indicated B field pointing out of the plane, the arrow lines represent electrons in the QHE regime traveling clockwise along the edge having a certain chemical potential μ_i ($i=1,\ldots,6$). (b) For the B field pointing into the plane, electrons travel counterclockwise. The circled contacts show proper $R_{xx}=0$ minima, whereas the voltage contacts opposite show residual minima Z_{xx}^{res} as a result of a finite voltmeter input impedance, as described in Eqs. (7a) and (7b). For hole edge channels one can simply flip the indicated B-field polarity.

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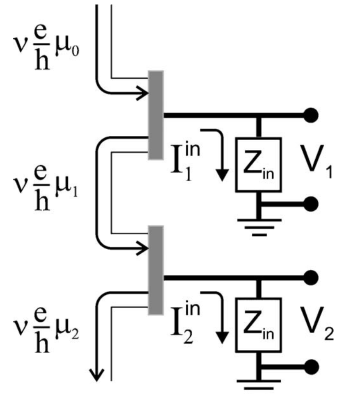

FIG. 2. Circuit diagram for a measurement of the longitudinal resitance. A measurement with input impedance Z_{in} measures the differential voltage $V_{xx}=V_1-V_2$.

The four-point longitudinal resistances can now be calculated. Measurements are normally performed with one of the current contacts set to ground and a constant dc or sinusoidal ac current sent through the sample. If a current is driven through a Hall bar in the QHE regime but the voltage contacts are not connected, then $\mu_0=\mu_1=\mu_2$ is the chemical potential of the current source and $\mu_3=\mu_4=\mu_5$ is the chemical potential of the ground. The same is applicable if the measurement devices connected to the voltage contacts V_i do not draw any current and the edge current flowing into a voltage contact equals the edge current flowing out. R_{xx} is then given by

$$R_{xx}^{1\text{-}2}=\frac{V_1-V_2}{I}=\frac{\mu_1-\mu_2}{eI}=0, \tag{2a}$$

$$R_{xx}^{3\text{-}4}=\frac{V_3-V_4}{I}=\frac{\mu_3-\mu_4}{eI}=0. \tag{2b}$$

Consider now that the voltage contacts 1 and 2 are connected to a voltmeter with a finite input resistance R_{in} and input capacitance C_{in} parallel to the stray capacitance in the lead C_s. The total capacitance is $C=C_{\mathrm{in}}+C_s$, and the total input impedance of the measurement apparatus as seen from the sample is $Z_{\mathrm{in}}=(R_{\mathrm{in}}^{-1}+j\omega C)^{-1}$. A current $I_n^{\mathrm{in}}=V_n/Z_{\mathrm{in}}=\mu_n/(eZ_{\mathrm{in}})$ then flows through the voltmeter to ground, where n is the lead index (see Fig. 2). Since the potential of a contact is defined by the chemical potential of the reduced outflowing edge current, the chemical potential drops slightly at every voltage contact. This effect gives a finite residual resistance $R_{xx}\neq 0$ in a QHE minimum. The measured voltages $V_{xx}^{1\text{-}2}$ and $V_{xx}^{3\text{-}4}$ are given by

$$V_{xx}^{1\text{-}2}=\frac{\mu_1-\mu_2}{e}=\frac{h}{\nu e^2}I_2^{\mathrm{in}}=\frac{h}{\nu e^2}\frac{1}{Z_{\mathrm{in}}}\frac{\mu_2}{e}, \tag{3a}$$

$$V_{xx}^{3\text{-}4}=\frac{h}{\nu e^2}\frac{1}{Z_{\mathrm{in}}}\frac{\mu_3}{e}, \tag{3b}$$

with $(\nu e/h)\mu_1=(\nu e/h)\mu_2+I_2^{\mathrm{in}}$ required by current conservation and $I_2^{\mathrm{in}}=V_2/Z_{\mathrm{in}}$.

Though these two expressions for V_{xx} look quite similar, the handedness of the Hall effect brings about two qualitatively different measurement results. In the QHE regime the voltage of the current–supply contact is given by $V_{xy}=R_{xy}I=h/\nu e^2 I$, with I being the driving current (dc or ac). By eliminating μ_1 in the following set of equations:

$$\mu_0=eI_1^{\mathrm{in}}R_{xy}+\mu_1, \tag{4a}$$

$$\mu_1=eI_2^{\mathrm{in}}R_{xy}+\mu_2, \tag{4b}$$

one deduces

$$\mu_2=\mu_0\left(\frac{R_{xy}}{Z_{\mathrm{in}}}+1\right)^{-2}, \tag{5a}$$

$$\mu_3=\mu_5\left(\frac{R_{xy}}{Z_{\mathrm{in}}}+1\right)^{-2}. \tag{5b}$$

In standard voltmeters $R_{xy}\ll R_{\mathrm{in}}$, since $R_{xy}\sim h/e^2=25\ \mathrm{k}\Omega$ and $R_{\mathrm{in}}\geq 10\ \mathrm{M}\Omega$. The above expressions are reduced to $\mu_2\approx\mu_0=V_{xy}e$ and $\mu_3=\mu_5=0$, accordingly. Assuming $I_o\sim I_i=I$,

$$V_{xx}^{1\text{-}2}=R_{xy}^2\frac{1}{Z_{\mathrm{in}}}I=\left(\frac{h}{\nu e^2}\right)^2\frac{1}{Z_{\mathrm{in}}}I=R_{xy}^2\left(\frac{1}{R_{\mathrm{in}}}+j\omega C\right)I, \tag{6a}$$

$$V_{xx}^{3\text{-}4}=0. \tag{6b}$$

The impedances measured at the respective R_{xx} minima are then given by

$$R_{xx}^{1\text{-}2}=Z^{\mathrm{res}}=R_{xy}^2/R_{\mathrm{in}}+jR_{xy}^2\omega C, \tag{7a}$$

$$R_{xx}^{3\text{-}4}=0, \tag{7b}$$

where $R_{xy}=h/\nu e^2$.

The results show that on the side of the Hall bar that is fed by the current source (dc or ac) a spurious residual resistance R_{xy}^2/R_{in} is measured, and on the other side fed by the ground contact there is none. This spurious effect can be eliminated by swapping the current–source contact with the ground contact by changing the polarity of the magnetic field or by using voltage contacts on the other side of the sample. It is worth reminding the reader that a parallel conducting layer would also lead to nonzero R_{xx} minima,[7] but such nonzero minima would be observed at *all* R_{xx} contact pairs, would not disappear either upon reversing the B field or swapping current contacts, and would not normally be quadratic in $1/\nu$.

III. EXPERIMENTAL MEASUREMENTS

A voltage measurement can either be done in dc or ac mode, the latter normally performed with lock-in amplifiers. Table I gives an overview of the input resistances R_{in} and capacitances C_{in} of commercially available voltmeters. In contrast to ac lock-in amplifiers, commercially available dc voltmeters usually have sufficiently high input impedances of $R>1\ \mathrm{G}\Omega$ to give a low residual signal $Z^{\mathrm{res}}<0.66\ \Omega/\nu^2$. The analog lock-in amplifiers have a residual signal of $\mathrm{Re}(Z^{\mathrm{res}})=6.6\ \Omega/\nu^2$ which can already be seen in measurements, especially in the FQHE regime. The digital lock-in amplifiers, on the other hand, have a residual signal of $\mathrm{Re}(Z^{\mathrm{res}})=66.6\ \Omega/\nu^2$ which can also be clearly seen in inte-

TABLE I. Input impedances of different commercially available voltmeters (data taken from the product specifications), along with the corresponding residual minima values Z^{res}.

Manufacturer	Model	Mode	R_{in}	C_{in}	$\mathrm{Re}(Z^{res})$ (Ω/ν^2)
Keithley	1801	dc	>1 GΩ	⋯	<0.66
Agilent	34420A	dc	>10 GΩ	<3.6 nF	<0.06
Signal recovery (EG&G)	7260	digital ac	10 MΩ	30 pF	66.6
Signal recovery (EG&G)	210	analog ac	100 MΩ	25 pF	6.66
Stanford research	810	digital ac	10 MΩ	25 pF	66.6
Stanford research	510	analog ac	100 MΩ	25 pF	6.66

ger QHE measurements. High-impedance preamplifiers are therefore recommended for use with lock-ins to eliminate this problem.

The measurement capacitance $C=C_{in}+C_s$ is usually dominated by stray capacitances and can also lead to residual minima in the real part of R_{xx} if the lock-in phasing is not properly adjusted. Sources of stray capacitance C_s include coaxial cables, twisted pair leads, and built-in line filters in dilution refrigerator systems to filter out radio frequencies. The most common source of phasing error is the line-rejection filter, included in standard lock-in amplifiers for frequencies of 50 and 100 Hz (or 60 and 120 Hz, depending on the line standard), which induces a noticeable phase shift at typical measurement frequencies (1–200 Hz). Internal line filters should therefore be used with care, and any induced phase shift must be rotated out. Since $\mathrm{Im}(Z^{res})\sim\omega$ scales linearly with frequency, frequency dependence in the R_{xx} minima is one clue for an improperly phased lock-in. Be warned that phasing errors give false residual signals even when using high- impedance preamplifiers since the effect is due entirely to the measurement capacitance.

The data in Fig. 3 show the real and imaginary parts of $R_{xx}^{1\text{-}2}$ (solid line) and $R_{xx}^{3\text{-}4}$ (dotted line) for a 2DS in an AlGaAs/GaAs heterostructure measured as in Fig. 1(a), with B coming out of the sample. The data were taken with an

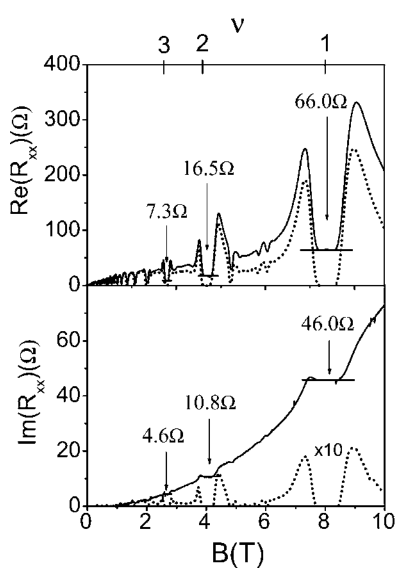

FIG. 3. Real part (top) and imaginary part (bottom) of R_{xx} corresponding to $R_{xx}^{1\text{-}2}$ (solid line) and $R_{xx}^{3\text{-}4}$ (dotted line), as measured in Fig. 1(a)).

EG&G 7260 lock-in amplifier in a van der Pauw geometry at an excitation frequency of $f=\omega/2\pi=17$ Hz. From the finite $R_{xx}^{1\text{-}2}$ resistance minima one can use Eq. (6a) to express the residual signal $Z^{res}=66\ \Omega/\nu^2+j46\ \Omega/\nu^2$ in terms of the input impedance of the lock-in amplifier and measurement capacitance, giving the result $Z_{in}=10.1\ \mathrm{M}\Omega+j\omega(650\ \mathrm{pF})$. While the real part is nearly exactly the same as the value given in Table I, the capacitance from the imaginary part is about a factor of $\times 20$ larger than the lock-in specifications, $C_{in}=30$ pF given in Table I. This is because the total capacitance $C=C_{in}+C_s=650$ pF includes a $C_s=620$ pF stray capacitance coming from the probe wiring.

IV. CONCLUSION

We conclude that the finite input impedance of voltmeters can lead to spurious signals in the R_{xx} minima of the QHE. Typical voltmeter input impedances are sufficiently low to induce a residual impedance of $Z^{res}=R_{xy}^2/R_{in}+j\omega CR_{xy}^2$ in the R_{xx} minima where $R_{xy}=h/\nu e^2$, and lead to the false assumption of a parallel channel. The following four tests will cause a finite R_{xx} minimum to drop to zero if the measurement impedance is indeed responsible for the residual R_{xx}.

1. Switch the polarity of the magnetic field.
2. Swap the current–source contact with the current–ground contact.
3. Use voltage contacts on the opposite side of the sample.
4. Properly phase the lock-in (real part frequency independent).

The ideal dc measurement setup for R_{xx} therefore has a high-input-impedance voltmeter, and the ideal ac setup has a high-impedance preamplifier with low input capacitance, while the stray capacitances are kept to a minimum, and the lock-in is properly phased with its line filters deactivated. When not working under these conditions, one should know the chirality of the edge states and measure R_{xx} on the side fed by the ground potential.

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