This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2007 proceedings.

Expected Error Based MMSE Detection Ordering for Iterative Detection-Decoding MIMO Systems

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***Abstract*— In multiple-input multiple-output (MIMO) systems, iterative receiver with turbo processing between detection and decoding can achieve near-capacity performance. In this paper, we introduce detection ordering into conventional soft interference cancellation minimum mean square error (SoIC-MMSE) detector which cancels the interference and detect signals in a 'parallel' manner. With soft information feedback, a novel expected error based (EEB) ordering algorithm is proposed and applied to Quadrature Amplitude Modulation (QAM) signaling. Simulation results show that with a little complexity increase in one Turbo iteration compared to the conventional SoIC-MMSE detection, our algorithm needs fewer iterations to achieve better performance.**

I. Introduction

Multiple-input multiple-output (MIMO) communication systems have received tremendous amount of attention due to its potentials in high wireless communication rate and high quality wireless multimedia services. In order to fully exploit the capacity and diversity advantages of MIMO channels, advanced signal processing techniques have to be applied at the transmitter and receiver. In particular, the "Turbo Principle" once used in traditional concatenated channel coding schemes (turbo-codes) [1] has applied to many iterative detection-decoding algorithms in MIMO systems [2], [3]. In such iterative schemes, soft information between the detector and the decoder exchanges so that the decoding performance measured by bit error rate (BER) can be significantly improved. Although there exists an optimal detection scheme – maximum *a posterior* probability (MAP) detection that can minimize the error probability, its complexity grows exponentially with the number of antennas and the constellation size. Thus, it is important to seek a detector with reasonable performance and manageable complexity. Such suboptimal detectors including soft interference cancellation minimum mean square error (SoIC-MMSE) detector [4], 'list' sphere decoder [5] and iterative tree search (ITS) detector [6] are proposed in the literature.

Conventional SoIC-MMSE detector makes use of extrinsic information provided by the soft-input soft-output (SISO) decoder to compute the statistical mean of the interfering signals. However, it processes in a 'parallel' manner, thus to result in performance degradation. In order to more effectively cancel the interference and better evaluate soft information for SISO decoder, the idea of successive interference cancellation with detection ordering (e.g. V-BLAST [7]) can be used for reference and extended to iterative receivers in MIMO systems.

In this paper, we propose a novel expected error based (EEB) ordering algorithm of successive inference cancellation for SoIC-MMSE detector. We use the expected error to scale the detection reliability of each sub-stream and the most reliable one is detected first in order to minimize error propagation. With a little computational increase in one Turbo iteration compared to the conventional SoIC-MMSE, simulation results reveal that for QAM modulation with Gray mapping, the proposed algorithm can achieve better performance with fewer Turbo iterations, and the performance advantage becomes stronger with the increase of modulation size.

The remainder of the paper is organized as follows. Section II describes the system model, and Section III presents a brief review of the conventional SoIC-MMSE detection. Our proposed EEB ordering detection algorithm is detailed in Section IV. In Section V, performance results based on computer simulation are provided. Conclusions are drawn in Section VI.

Throughout the paper, vectors and matrices are represented with bold face letters. The symbols $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^{-1}$ represent matrix transposition, Hermitian and inversion, respectively. All vectors are defined as column vectors with row vectors represented by transposition. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix and $diag$ represents a diagonal matrix.

II. System Description

We consider a system with the multiple-input multiple-output (MIMO) channel consisting of N_t transmit and N_r receive antennas. The modulation format is identical for all transmit antennas, and the number of bits per constellation point is denoted by M_c. First, the long information sequence $\mathbf{u}$ is encoded and bit-level interleaved. As part of the total coded sequence, we obtain an $N_t M_c \times 1$ dimensional binary vector $\mathbf{c} = [\mathbf{c}_1, \cdots, \mathbf{c}_{N_t}]^T$ with $\mathbf{c}_n = [c_{n,1}, \cdots, c_{n,M_c}]^T$. The sequence $\mathbf{c}$ is then serial-to-parallel converted and Gray mapped onto a transmit symbol vector $\mathbf{x}$, such that $c_{k,l}$ is the lth bit mapped onto the kth symbol. We assume the

This work is partially sponsored by Tsinghua-Qualcomm Joint Research Program and National Nature Science Foundation of China-No. 90204001.

binary digits $c_{k,l}$ to take independent values from $\{0,1\}$, so mapped symbols are equally likely chosen from a complex constellation set χ with cardinality $|\chi| = 2^{M_c}$.

The (N_t, N_r) MIMO channel is assumed to be rich-scattering and flat-fading. At any given time slot, the MIMO system is described by the well-known base-band model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is the $N_r \times N_t$ channel matrix whose elements are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, $\mathbf{x} = [x_1, \cdots, x_{N_t}]^T$ is the transmitted signal vector with unit variance for each component, $\mathbf{y} = [y_1, \cdots, y_{N_r}]^T$ is the received signal vector, and $\mathbf{n} = [n_1, \cdots, n_{N_r}]^T$ is assumed to be an i.i.d. complex Gaussian noise vector with each component having zero mean and variance $\sigma^2/2$ per dimension. The fading model we assume is a fast Rayleigh fading model in which the channel characteristics are changing for every transmit vector period. We assume that the receiver has perfect channel state information (CSI). In an iterative detection-decoding structure, the soft MIMO detector incorporates extrinsic information $L_A(\cdot)$ provided by the decoder, and the decoder incorporates soft information $L_E(\cdot)$ provided by the MIMO detector. A more detailed discussion of iterative MIMO detection and decoding falls outside the scope of this paper but can be found for example in [2].

III. SoIC-MMSE Detector: Review

The suboptimal SoIC-MMSE detector consists of a parallel interferences canceller followed by an MMSE filter as described in [3] and [4]. Below we review this algorithm.

As *a priori* information is available, SoIC-MMSE detector first forms symbol mean $\overline{x}_i, i = 1, 2, \cdots, N_t$, as

$$\overline{x}_i = \sum_{x \in \chi} x P(x_i = x) \tag{2}$$

where χ is the complex constellation set and $P(x_i = x)$ represents *a priori* symbol probability. Assuming bits within a symbol are independent and let x^l indicate the lth bit value of symbol x, then $P(x_i = x)$ can be computed as

$$P(x_i = x) = \prod_{l=1}^{M_c} \frac{e^{x^l L_A(c_{i,l})}}{1 + e^{L_A(c_{i,l})}} \tag{3}$$

For the kth transmit antenna, soft interference from other $N_t - 1$ antennas is 'parallel' cancelled to obtain

$$\widehat{\mathbf{y}}_k = \mathbf{h}_k x_k + \sum_{i=1, i \neq k}^{N_t} \mathbf{h}_i (x_i - \overline{x}_i) + \mathbf{n} \tag{4}$$

where $\mathbf{h}_i$ represents the ith column of $\mathbf{H}$. Then the MMSE filter $\mathbf{w}_k$ is chosen to minimize the mean square error (MSE) between the transmit symbol x_k and the filter output $\widehat{x}_k = \mathbf{w}_k^H \widehat{\mathbf{y}}_k$. It can be shown that the solution is given by

$$\mathbf{w}_k = \left[\sigma^2 \mathbf{I}_{N_r} + \mathbf{H}\mathbf{\Delta}_k \mathbf{H}^H\right]^{-1} \mathbf{h}_k \tag{5}$$

where the covariance matrix is

$$\mathbf{\Delta}_k = diag\left(\sigma_1^2, \cdots, \sigma_{k-1}^2, 1, \sigma_{k+1}^2, \cdots, \sigma_{N_t}^2\right) \tag{6}$$

and $\sigma_i^2, i = 1, 2, \cdots, N_t$ with $i \neq k$, is the variance of the ith symbol and can be computed as

$$\sigma_i^2 = \sum_{x \in \chi} |x - \overline{x}_i|^2 P(x_i = x) \tag{7}$$

Therefore the output of the MMSE filter to (4) is given by

$$\widehat{x}_k = \mathbf{w}_k^H \mathbf{h}_k x_k + \mathbf{w}_k^H \left[\sum_{i=1, i \neq k}^{N_t} \mathbf{h}_i (x_i - \overline{x}_i) + \mathbf{n}\right] \tag{8}$$

We approximate $\widehat{x}_k$ by the output of an equivalent AWGN channel with $\widehat{x}_k = \mu_k x_k + n_k$, where $\mu_k = \mathbf{w}_k^H \mathbf{h}_k$ and n_k is a complex Gaussian variable with zero mean and variance $\eta_k^2 = \mu_k - \mu_k^2$. Thus given the transmit symbol $x_k = x$, the likelihood function $P(\widehat{x}_k | x_k = x)$ is approximated by

$$P(\widehat{x}_k | x_k = x) \simeq \frac{1}{\pi \eta_k^2} e^{-\frac{1}{\eta_k^2} |\widehat{x}_k - \mu_k x|^2} \tag{9}$$

For the lth bit of the symbol x_k, by Max-log approximation [8], extrinsic information can be computed as

$$L_E(c_{k,l}) \simeq \min_{x \in \chi_l^0} \left\{ \frac{1}{\eta_k^2} |\widehat{x}_k - \mu_k x|^2 - \sum_{j=1}^{M_c} x^j L_A(c_{k,j}) \right\} - \min_{x \in \chi_l^1} \left\{ \frac{1}{\eta_k^2} |\widehat{x}_k - \mu_k x|^2 - \sum_{j=1}^{M_c} x^j L_A(c_{k,j}) \right\} - L_A(c_{k,l}) \tag{10}$$

where χ_l^0 is the subset of χ with cardinality $\left|\chi_l^0\right| = 2^{M_c - 1}$ for which the lth bit of each element is 0 (χ_l^1 is similarly defined).

IV. Proposed SoIC-MMSE Ordering

In this section, we will derive the Expected Error Based (EEB) ordering detection algorithm and its application to QAM modulation, we will also analyze its complexity and make comparison with the conventional SoIC-MMSE detector.

A. EEB Ordering Detection Algorithm

As the conventional SoIC-MMSE detector, symbol mean is first computed by (2) and then all cancelled from transmit vector $\mathbf{x}$ to obtain

$$\widehat{\mathbf{y}} = \sum_{i=1}^{N_t} \mathbf{h}_i (x_i - \overline{x}_i) + \mathbf{n} = \sum_{i=1}^{N_t} \mathbf{h}_i \Delta x_i + \mathbf{n} \tag{11}$$

where $\Delta x_i = x_i - \overline{x}_i$ denotes the residuum of symbol x_i. We make linear least mean square estimation for residual symbol Δx_i rather than transmit symbol x_i. For the kth transmit antenna, we choose MMSE filter $\mathbf{v}_k$ to minimize $E\left[\left|\mathbf{v}_k^H \widehat{\mathbf{y}} - \Delta x_k\right|^2\right]$, which can be computed as

$$\mathbf{v}_k = \sigma_k^2 \left[\sigma^2 \mathbf{I}_{N_r} + \mathbf{H}\mathbf{\Delta}\mathbf{H}^H\right]^{-1} \mathbf{h}_k \tag{12}$$

where the covariance matrix is

$$\mathbf{\Delta} = diag\left(\sigma_1^2, \cdots, \sigma_{k-1}^2, \sigma_k^2, \sigma_{k+1}^2, \cdots, \sigma_{N_t}^2\right) \tag{13}$$

with $\sigma_i^2, i = 1, 2, \cdots, N_t$ can be computed by (7). Denote $\mathbf{V}_k = \mathbf{v}_k/\sigma_k^2$, then the output of the MMSE filter to (11) can be written by

$$\mathbf{V}_k^H \widehat{\mathbf{y}} = \mathbf{V}_k^H \mathbf{h}_k \left(x_k - \overline{x}_k \right) + \mathbf{V}_k^H \left(\sum_{i=1, i \neq k}^{N_t} \mathbf{h}_i \Delta x_i + \mathbf{n} \right) \quad (14)$$

Equivalently,

$$\begin{aligned} \widehat{x}_k &\triangleq \mathbf{V}_k^H \widehat{\mathbf{y}} + \mathbf{V}_k^H \mathbf{h}_k \overline{x}_k \\ &= \mathbf{V}_k^H \mathbf{h}_k x_k + \mathbf{V}_k^H \left(\sum_{i=1, i \neq k}^{N_t} \mathbf{h}_i \Delta x_i + \mathbf{n} \right) \end{aligned} \quad (15)$$

As the conventional SoIC-MMSE detector, we approximate $\widehat{x}_k$ by the output of an equivalent AWGN channel with $\widehat{x}_k = \mu_k x_k + n_k$, where

$$\mu_k = \mathbf{V}_k^H \mathbf{h}_k \quad (16)$$

and n_k is a complex Gaussian variable with zero mean and variance η_k^2 given by

$$\eta_k^2 = \mu_k - \sigma_k^2 \mu_k^2 \quad (17)$$

Thus the likelihood function $P\left(\widehat{x}_k \left| x_k = x \right.\right)$ and extrinsic information can be computed as in (9) and (10).

Without loss of generality, we think about the detection of the kth symbol. Let

$$x_k^{\mathrm{MAP}} = \arg\max_{\tilde{x} \in \chi} P\left(x_k = \tilde{x} \left| \widehat{x}_k \right.\right) \quad (18)$$

be the maximum *a posteriori* (MAP) decision for the kth symbol. Given the transmit symbol equal to x and the *a priori* symbol probability computed by (3), let

$$\begin{aligned} \Delta\left(\tilde{x}, x\right) &\triangleq \left[\left| \widehat{x}_k - \mu_k \tilde{x} \right|^2 - \eta_k^2 \ln P\left(x_k = \tilde{x}\right) \right] \\ &\quad - \left[\left| \widehat{x}_k - \mu_k x \right|^2 - \eta_k^2 \ln P\left(x_k = x\right) \right] \\ &= \mu_k^2 \left| x - \tilde{x} \right|^2 + 2\mu_k \mathrm{Re}\left[\left(x - \tilde{x}\right) n_k^* \right] \\ &\quad - \eta_k^2 \ln \frac{P\left(x_k = \tilde{x}\right)}{P\left(x_k = x\right)} \end{aligned} \quad (19)$$

denote the metric difference between $\tilde{x}$ and x. Then by some manipulation, (18) can be rewritten as

$$x_k^{\mathrm{MAP}} = \arg\min_{\tilde{x} \in \chi} \Delta\left(\tilde{x}, x\right) \quad (20)$$

Therefore, the expected symbol error of MAP detection is

$$E_k = \sum_{x \in \chi} P\left(x_k = x\right) P\left(\Delta\left(x_k^{\mathrm{MAP}}, x \right) < 0 \right) \quad (21)$$

We make a assumption that x_k^{MAP} is either x or the nearest neighbor of x, then (21) can be approximated by

$$E_k \simeq \sum_{x \in \chi} P\left(x_k = x\right) P\left(\Delta\left(\hat{x}, x \right) < 0 \right) \quad (22)$$

where $\hat{x} = \arg \min_{\tilde{x} \in \chi, \left| \tilde{x} - x \right| = d_m} \Delta\left(\tilde{x}, x\right)$ with d_m represents the minimum Euclidian distance of symbols in χ.

TABLE I

PROPOSED EEB ORDERING DETECTION ALGORITHM

Initialization:
for $i = 1, 2, \cdots, N_t$
1. Find *a priori* symbol probability $P(x_i = x)$ as in (3).
2. Find symbol mean $\overline{x}_i$ in (2) and variance σ_i^2 in (7).
end
Recursion:
for $m = 1, 2, \cdots, N_t$
3. Perform soft interference cancellation as in (11).
4. Compute $\left[\sigma^2 \mathbf{I}_{N_r} + \mathbf{H} \mathbf{\Delta} \mathbf{H}^H\right]^{-1}$ with $\mathbf{\Delta}$ shown in (13).
for $k = 1, 2, \cdots, N_t$
5. Obtain $V_k = v_k/\sigma_k^2$ with v_k shown in (12).
6. Form Gaussian approximated sub-stream $\widehat{x}_k = \mu_k x_k + n_k$ with $\widehat{x}_k$ in (15), μ_k in (16) and variance of n_k in (17).
7. Calculate the expected error according to (22).
end
8. Find the sub-stream with the minimum expected error, denoted as the k_mth.
9. Compute extrinsic information of all bits mapped to the k_mth symbol by (10) and find the MAP estimate $x_{k_m}^{\mathrm{MAP}}$ in (18).
10.Cancel the interference caused by k_mth symbol by renewing $\overline{x}_{k_m}$ with $x_{k_m}^{\mathrm{MAP}}$ and setting the k_mth diagonal element of $\mathbf{\Delta}$ to zero.
end

TABLE II

THE METRIC DIFFERENCE BETWEEN x AND ITS NEAREST NEIGHBOR $\tilde{x}$

Metric Difference	Positions of x and $\tilde{x}$
$\Delta_{x,R}^l = \mu_k^2 d_m^2 + 2\mu_k d_m R - \eta_k^2 \ln \frac{P(x_k = \tilde{x})}{P(x_k = x)}$	$\tilde{x}$ is on the left of x
$\Delta_{x,R}^r = \mu_k^2 d_m^2 - 2\mu_k d_m R - \eta_k^2 \ln \frac{P(x_k = \tilde{x})}{P(x_k = x)}$	$\tilde{x}$ is on the right of x
$\Delta_{x,I}^d = \mu_k^2 d_m^2 + 2\mu_k d_m I - \eta_k^2 \ln \frac{P(x_k = \tilde{x})}{P(x_k = x)}$	$\tilde{x}$ is beneath x
$\Delta_{x,I}^u = \mu_k^2 d_m^2 - 2\mu_k d_m I - \eta_k^2 \ln \frac{P(x_k = \tilde{x})}{P(x_k = x)}$	$\tilde{x}$ is above x

After computing expected errors according to (22) of all N_t transmit sub-streams, we pick out one assumed to be the k_1th with the minimum expected error. We use (10) to compute extrinsic information of all bits mapped to the k_1th symbol, and in this process we can also find the MAP estimate $x_{k_1}^{\mathrm{MAP}}$. Then, the interference cause by x_{k_1} is subtracted as in (11) by renewing $\overline{x}_{k_1}$ with $x_{k_1}^{\mathrm{MAP}}$ and setting the k_1th diagonal element of $\mathbf{\Delta}$ in (13) to zero, leading to a new system with only $N_t - 1$ transmit sub-streams. This procedure is repeated for the reduced systems until extrinsic information of all bits is calculated. We summarize above mentioned steps in Table I.

B. Application to QAM Modulation

In this subsection, assuming QAM modulation with Gray mapping, we intend to simplify the process of calculating the expected error in (22) so that the computational complexity of the proposed algorithm can be greatly reduced.

For any $x \in \chi$, we consider the metric difference between x and its nearest neighbors. For the kth sub-stream, according to (19), we can obtain Table II, where $R = \mathrm{Re}(n_k)$ and $I = \mathrm{Im}(n_k)$ are statistically independent real Gaussian variables with zero mean and variance $\eta_k^2/2$. Set

$$\Delta_{x,R} \triangleq \min\left(\Delta_{x,R}^l, \Delta_{x,R}^r \right), \Delta_{x,I} \triangleq \min\left(\Delta_{x,I}^d, \Delta_{x,I}^u \right) \quad (23)$$

and since they are statistically independent, we can obtain

$$P(\Delta(\hat{x}, x) < 0) = 1 - P\left(\Delta_{x,R} \geq 0\right) P\left(\Delta_{x,I} \geq 0\right) \quad (24)$$

TABLE III
ELEMENTS USED FOR EXPECTED ERROR CALCULATION

	Image set B	$P\left(\Delta_{j,R} \geq 0\right), j = 1, 2, \cdots, 2^{M_c/2}$
QPSK	$\{1, 0\}$	$f(-\Lambda_1), f(\Lambda_1)$
16QAM	$\{10, 11, 01, 00\}$	$f(\Lambda_2), g(-\Lambda_1, -\Lambda_2), g(\Lambda_1, -\Lambda_2), f(\Lambda_2)$
64QAM	$\{100, 101, 111, 110, 010, 011, 001, 000\}$	$f(\Lambda_3), g(\Lambda_2, -\Lambda_3), g(-\Lambda_2, -\Lambda_3), g(-\Lambda_1, \Lambda_3), g(\Lambda_1, \Lambda_3), g(-\Lambda_2, -\Lambda_3), g(\Lambda_2, -\Lambda_3), f(\Lambda_3)$

Assuming that for any QAM symbol x mapped by M_c bits, its column index and row index of the constellation can determine the first and the other $M_c/2$ bits, which are denoted by two bijective mappings $\omega_R, \omega_I : A = \{1, 2, \cdots, 2^{M_c/2}\} \rightarrow \{0, 1\}^{M_c/2}$ with $\omega_R(i) = \omega_I(2^{M_c/2} + 1 - i)$, respectively. For $\forall i, j \in A$, we denote symbol x which lies in the ith row and jth column of χ as $x_{i,j}$. Due to the Gray mapping, from Table II and (23) we can obtain that $\Delta_{x_{i,j},R}$ is independent of i and $\Delta_{x_{i,j},I}$ is independent of j. Define

$$\Delta_{j,R} \triangleq \Delta_{x_{i,j},R}, \Delta_{i,I} \triangleq \Delta_{x_{i,j},I} \tag{25}$$

With (24), (25) and the assumption that bits within a symbol are independent of each other, (22) can be reformulated as

$$\begin{aligned} E_k &\simeq 1 - \sum_{i \in A} \sum_{j \in A} P\left(x_k = x_{i,j}\right) P\left(\Delta_{j,R} \geq 0\right) P\left(\Delta_{i,I} \geq 0\right) \\ &= 1 - \sum_{i \in A} \left[\prod_{l=1}^{\frac{M_c}{2}} P\left(x_k^{l + \frac{M_c}{2}} = \omega_I^l(i) \right) \right] P\left(\Delta_{i,I} \geq 0\right) \\ &\quad \times \sum_{j \in A} \left[\prod_{l=1}^{\frac{M_c}{2}} P\left(x_k^l = \omega_R^l(j) \right) \right] P\left(\Delta_{j,R} \geq 0\right) \end{aligned} \tag{26}$$

where $\omega_R^l(j)$ denotes the lth bit of the sequence $\omega_R(j)$ ($\omega_I^l(i)$ is similarly defined).

We consider three different QAM modulation: QPSK, 16QAM and 64QAM. Without loss of generality, we compute the third term in (26) (the second term can be similarly calculated). Assuming that the multiplications of *a priori* probability can be obtained in the process of Step 1 in Table I, only $P\left(\Delta_{j,R} \geq 0\right)$ should be calculated. As detailed in Table III, the second column shows the ordered image set B of mapping ω_R, which satisfies for $\forall j \in A, \omega_R(j)$ equals to jth element of B. And the third column gives the value of $P\left(\Delta_{j,R} \geq 0\right)$, in which

$$f(x) = \frac{1}{2} \mathrm{erfc} \left(\frac{\eta_k}{2\mu_k d_m} x - \frac{\mu_k d_m}{2\eta_k} \right) \tag{27}$$

$$g(x, y) = \max\left(0, f(x) + f(y) - 1\right) \tag{28}$$

and Λ_i denotes the *a priori* information of the ith bit mapped to the kth symbol.

C. Complexity Analysis

Although compared to the conventional SoIC-MMSE detection, one matrix inversion should be calculated in each successive interference cancellation step of our proposed algorithm, it is easy to point out that the total number of matrix inversions per turbo iteration is the same for both schemes. The main difference will lie in the computation of expected error which consists of step 6 and 7 in Table I.

It is assumed that there is an equal number of transmit and receive antennas $N_t = N_r = N$ and QAM modulation with Gray mapping as shown in the subsection above is used. We adopt an 'operand-counting' approach specified in [8] where the total number of operations performed on elements fetched from memory to produce a given output is accrued. It can be computed that $4N$ flops are needed to setup one Gaussian approximated sub-stream in step 6. Additionally for one sub-stream, since table lookups $\frac{1}{2}\mathrm{erfc}(\cdot)$ is omitted as it does not involve additional operand fetches, only $f(x), x = \pm\Lambda_1, \cdots, \pm\Lambda_{M_c/2}$ and at most $2^{M_c/2}/2$ additional additions in function g of Table III should be calculated. Therefore the cost can be computed as $4 + 2M_c + 2^{M_c/2}/2$ flops, where 4 refers to the cost of the part without x in function f which is needed to be calculated only once for one sub-stream. Due to some equal items in $P\left(\Delta_{j,R} \geq 0\right)$, the cost of the third term in (26) is $2\left[(2^{M_c/2} - 2)/2 + 2\right]$ flops. Then the overall cost of (26) can be computed as $2(6 + 2M_c + \frac{3}{2} \cdot 2^{M_c/2})$ flops. Therefore in one Turbo iteration, the cost increase of step 6 and 7 is approximately given by

$$\begin{aligned} &4N \cdot (\sum_{k=1}^{N} k - N) + 2(6 + 2M_c + \frac{3}{2} \cdot 2^{M_c/2}) \cdot \sum_{k=2}^{N} k \\ &= 2N^2(N-1) + (6 + 2M_c + \frac{3}{2} \cdot 2^{M_c/2})(N^2 + N - 2) \end{aligned} \tag{29}$$

Compared to the cost of SoIC-MMSE given by $7N^4/3 + 3N^3 + 7N^2 + 4NM_c \cdot 2^{M_c} + 9N \cdot 2^{M_c}$ flops [8], it can be concluded that ratio of overall complexity increase of our algorithm will become less with the increase of N and M_c. Especially, when $N = 4$ and $M_c = 6$, the increase percentage is only 6.8%.

V. NUMERICAL RESULTS

We now access the performance of our proposed EEB ordering algorithm for SoIC-MMSE detector. We consider an iterative detection-decoding MIMO system using a rate-$1/2$ 4 state convolutional code with octal generators $(7, 5)$. We assumed an equal number of transmit and receive antennas $N_t = N_r = 4$ and the MIMO channel had i.i.d. Gaussian matrix entries with zero mean and unit variance. To simulate fast fading, the channel was independently generated for each time slot. We incorporated EEB ordering into SoIC-MMSE from the second Turbo iteration to form a novel detector. We considered our proposed detector and the conventional SoIC-MMSE detector with different QAM modulation using Gray mapping for comparison.

Fig. 1 to Fig. 3 shows the average bit-error rate (BER) of this $(4, 4)$ MIMO system versus the average E_b/N_0 per receive antenna for QPSK, 16QAM and 64QAM, respectively. For each modulation format, the number of detection-decoding iterations is chosen to satisfy that the performance gain by

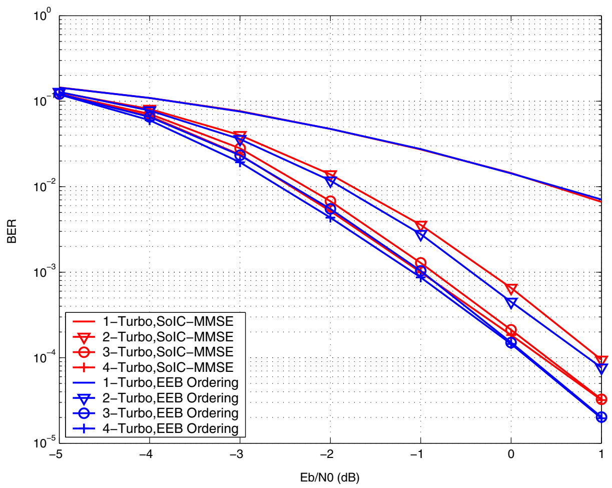

Fig. 1. Performance comparison between conventional SoIC-MMSE detector and the proposed EEB ordering detector for QPSK modulation

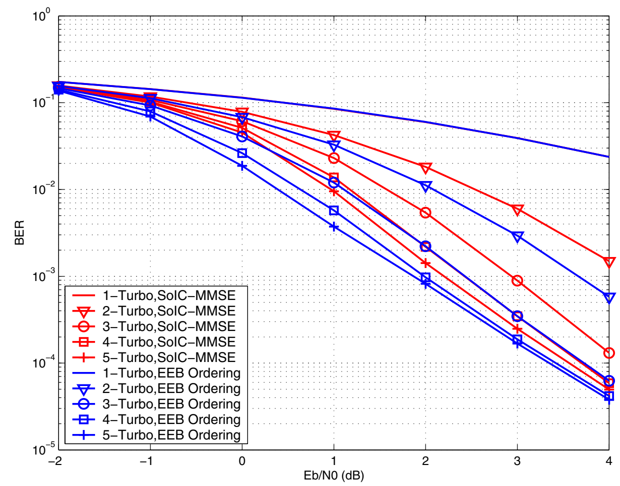

Fig. 2. Performance comparison between conventional SoIC-MMSE detector and the proposed EEB ordering detector for 16QAM modulation

doing one extra Turbo iteration is diminishing and convergence is nearly achieved. The following conclusions can be drawn from these results:

- For all different modulation formats, our proposed detector outperforms the conventional SoIC-MMSE detector in every Turbo iteration except for the first one.
- For QPSK, 16QAM and 64QAM respectively, 1,1 and 2 fewer Turbo iterations are needed for our proposed detector to achieve the convergence performance of the conventional SoIC-MMSE detector.
- The convergence BER is lower for our proposed detector than the conventional SoIC-MMSE detector. And the performance advantage is stronger for large modulation size. At 0.1% BER, we observe that our proposed detector outperforms the conventional one by nearly 0.15dB for QPSK modulation. And the performance gain increases to 0.25dB and 0.4dB for 16QAM and 64QAM, respectively.

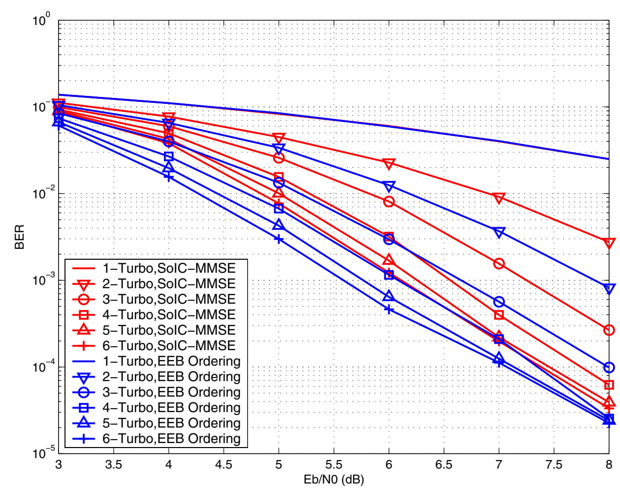

Fig. 3. Performance comparison between conventional SoIC-MMSE detector and the proposed EEB ordering detector for 64QAM modulation

VI. Conclusion

In this paper, we have presented a novel detection ordering algorithm for iterative detection-decoding MIMO system. The main idea is to detect and cancel sub-streams in order of the magnitude of the expected error. When applied to QAM modulation with Gray mapping, complexity analysis demonstrates that the ratio of overall complexity increase of our proposed algorithm compared to the conventional SoIC-MMSE detection will become much less with the increase of antenna numbers and modulation size. Moreover, simulation results in $(4, 4)$ MIMO system show that our algorithm outperforms the conventional SoIC-MMSE, and the performance advantage becomes stronger with the increase of modulation size.

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