Wireless Peer-to-Peer Mutual Broadcast via Sparse Recovery

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***Abstract*—This paper studies a problem frequently seen in wireless networks: Every node wishes to broadcast information to nodes within a single hop, which are referred to as its peers. We call this problem *mutual broadcast*. A novel solution is proposed, which exploits the multiaccess nature of the wireless medium and addresses the half-duplex constraint at the fundamental level. The defining feature of the scheme is to let all nodes send their messages at the same time, where each node broadcasts a codeword (selected from its unique codebook) consisting of on-slots and off-slots, where it transmits only during its on-slots, and listens to its peers through its own off-slots. Decoding can be viewed as a problem of *sparse support recovery* based on linear measurements. In case each message consists of a small number of bits, an iterative message-passing algorithm based on belief propagation is developed, the performance of which is characterized using a state evolution formula in the limit where each node has a large number of peers. Numerical results demonstrate that, to achieve the same reliability for mutual broadcast, the proposed scheme achieves three to five times the rate of ALOHA and carrier-sensing multiple-access (CSMA) in typical scenarios.**

I. Introduction

Consider a frequent situation in wireless peer-to-peer networks, where every node wishes to broadcast messages to all nodes within its one-hop neighborhood, called its *peers*. We refer to this problem as *mutual broadcast*. Such broadcast traffic can be dominant in many applications, such as messaging or video conferencing of multiple parties in a spontaneous social network, or on a site of disaster relief or battlefield. Wireless mutual broadcast is also critical to efficient network resource allocation, where messages are exchanged between nodes about their demands and local states, such as queue length, channel quality, code and modulation format, and request for certain resources and services.

A major challenge in wireless networks is the half-duplex constraint, namely affordable radio cannot receive useful signal at the same time over the same frequency band within which it is transmitting. This is largely due to the limited dynamic range of the radio frequency circuits, which is likely to remain a physical restriction in the foreseeable future. An important consequence of the half-duplex constraint in a typical implementation of wireless networks is that, if two peers transmit their packets at the same time, they do not hear each other. To achieve reliable mutual broadcast using a usual packet-based scheme, nodes have to repeat their packets a number of times interleaved with random delays, so that peers can hear each other after enough retransmissions. This leads to the ubiquitous random channel access solution.

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A closer examination of the half-duplex constraint, however, reveals that a node does not need to transmit an entire packet before listening to the channel. An alternative solution is conceivable: Let a frame (typically of a few thousand symbols) be divided into some number of slots, where each node transmits over a subset of the slots and assumes silence over the remaining slots, then the node can receive useful signals over those nontransmission slots. If nodes activate different sets of on-slots, then nodes can all transmit information during a frame and receive useful signals within the same frame, and decode messages from peers as long as sufficiently strong error-control codes are applied. In fact, reliable mutual broadcast can be achieved using a single frame interval.

The on-off signaling described in above is called *rapid on-off-division duplex (RODD)*, which is originally proposed in [1]. Not only is the signaling applicable to the mutual broadcast problem, it can also be the basis of a clean-slate design of the physical and medium access control (MAC) layers of wireless peer-to-peer networks. Importantly, RODD achieves virtual full-duplex communication using half-duplex radios. RODD-based schemes have advantages over state-of-the-art designs of MAC protocols, which either apply ALOHA-type random access or use a mixture of random access and scheduling/reservation. This is in part because RODD eliminates retransmissions due to collisions.

In this paper, we focus on a special use of RODD signaling and a special case of mutual broadcast, where each node has a small number of bits to send to its peers. The goal here is to provide a practical algorithm for encoding and decoding the short messages to achieve reliable and efficient mutual broadcast. Decoding is in fact a problem of support recovery based on linear measurements, since the received signal is basically a noisy superposition of peers' codewords selected from their respective codebooks. There are many algorithms developed in the compressed sensing (or *sparse recovery*) literature to solve the problem, the complexity of which is often polynomial in the size of the codebook (see, e.g., [2]–[6]). In this paper, an iterative message-passing algorithm based on belief propagation (BP) with linear complexity is developed. Its performance is characterized using a state evolution formula in the large-system limit, where the number of peers each node has tends to infinity. Numerical results show that the proposed RODD scheme significantly outperforms

ALOHA and CSMA-type random-access schemes in terms of data rate. Moreover, as far as the scenarios studied numerically, a fair comparison shows that the message-passing algorithm outperforms two popular sparse recovery algorithms, namely, compressive sampling matching pursuit (CoSaMP) [2] and approximate message passing (AMP) [3].

The remainder of the paper is organized as follows. Section II describes the proposed coding scheme. The message-passing decoding algorithm is developed and analyzed in Section III. In Section IV, competing random-access schemes are studied. Numerical comparisons are presented in Section V. Section VI concludes the paper.

II. Encoding for Mutual Broadcast

Suppose all transmissions use the same single carrier frequency. Let time be slotted and all nodes be perfectly synchronized.[1] Each node k is assigned a unique codebook of 2^l on-off signatures (codewords) of length M_s, denoted by $\{\boldsymbol{S}_k(1), \dots, \boldsymbol{S}_k(2^l)\}$. For simplicity, let each element of each signature be generated randomly and independently, which is 0 with probability $1-q$ and 1 and -1 with probability $q/2$ each.

Node k broadcasts its l-bit message (or information index) $w_k \in \{1, \dots, 2^l\}$ by transmitting the codeword $\boldsymbol{S}_k(w_k)$. For simplicity, let us assume that the physical link between every pair of neighboring nodes is an additive white Gaussian noise channel of the same signal-to-noise ratio (SNR), denoted by γ. Let the set of peers of node k be denoted by $\mathcal{N}_k$, the population of which is its cardinality $|\mathcal{N}_k|$. The received signal of node k, if it could listen over the entire frame, is then described by

$$\widetilde{\boldsymbol{Y}}_k = \sqrt{\gamma} \sum_{j \in \mathcal{N}_k} \boldsymbol{S}_j(w_j) + \boldsymbol{W}_k \tag{1}$$

where $\boldsymbol{W}_k$ is Gaussian noise consisting of independent identically distributed (i.i.d.) entries of zero mean and unit variance. For simplicity, transmissions from non-neighbors, if any, are accounted for as part of the additive Gaussian noise.

Without loss of generality, we focus on the neighborhood of node 0 and omit the subscript k in (1). Suppose $|\mathcal{N}_0| = K$ and the neighbors of node 0 are indexed by $1, \dots, K$. The total number of signatures of all neighbors is $N = 2^l K$. Due to the half-duplex constraint, however, node 0 can only listen during its off-slots, the number of which has binomial distribution, denoted by $M \sim \mathcal{B}(M_s, 1-q)$, whose expected value is $\mathsf{E}\{M\} = M_s(1-q)$. Let the matrix $\boldsymbol{S} \in \mathbb{R}^{M \times N}$ consist of columns of the signatures from all neighbors of node 0, observable during the M off-slots of node 0, and then normalized by $\sqrt{M_s(1-q)q}$ so that the expected value of the l_2 norm of each column in $\boldsymbol{S}$ is equal to 1. Based on (1), the M-vector observed through all off-slots of node 0 can be expressed as

$$\boldsymbol{Y} = \sqrt{\gamma_s} \boldsymbol{S} \boldsymbol{X} + \boldsymbol{W} \tag{2}$$

[1]See [1] for a discussion of synchronization issues. In [7], cyclic codes are proposed to resolve the user delays in a multiaccess channel.

where $\gamma_s = \gamma M_s(1-q)q$, and $\boldsymbol{X}$ is a binary N-vector for indicating which K signatures are selected to form the sum in (1). Precisely, $X_{(j-1)2^l+i} = 1_{\{w_j = i\}}$ for $1 \le j \le K$ and $1 \le i \le 2^l$. Note that the sparsity of $\boldsymbol{X}$ is exactly 2^{-l}, which can be very small. The average system load is defined as $\beta = N/\big(M_s(1-q)\big)$.

In general, the decoding problem each node k faces is to identify, out of a total of $2^l|\mathcal{N}_k|$ signatures from all its neighbors, which $|\mathcal{N}_k|$ signatures were selected. This requires every node to know the codebooks of all neighbors. One solution is to let the codebook of each node be generated using a pseudo-random number generator using its network interface address (NIA) as the seed, so that it suffices to acquire all neighbors' NIAs. This, in turn, is a neighbor discovery problem, which has been studied in [8]–[10]. The discovery scheme proposed in [9], [10] uses similar on-off signalling and also solves a compressed sensing problem.

III. Sparse Recovery (Decoding) via Message Passing

The problem of recovering the support of the sparse input $\boldsymbol{X}$ based on the observation $\boldsymbol{Y}$ has been intensively studied in the compressed sensing literature. In this section, we develop an iterative message-passing algorithm based on belief propagation, and characterize its performance in a certain limit. The computational complexity of the algorithm is in the order of $\mathcal{O}(MNq)$, which is of the same order as the complexity of CoSaMP and AMP.

A. The Message-Passing Algorithm

First we construct a Forney-style bipartite factor graph to represent the model (2), which is rewritten as

$$y_\mu = \sqrt{\gamma_s} \sum_{k=1}^{N} s_{\mu k} x_k + w_\mu \tag{3}$$

where $\mu = 1, \dots, M$ and $k = 1, \dots, N$ index the measurements and the "input symbols," respectively. For simplicity, we ignore the dependence of the symbols $\{X_k\}$ for now, which shall be addressed toward the end of this section. Each X_k then corresponds to a symbol node and each Y_μ corresponds to a measurement node. For every (μ, k), symbol node k and measurement node μ are connected by an edge if $s_{\mu k} \neq 0$.

For convenience, let $\partial\mu$ (resp. ∂k) denote the subset of symbol nodes (resp. measurement nodes) connected directly to measurement node μ (resp. symbol node k), called its neighborhood. Also, let $\partial\mu \backslash k$ denote the neighborhood of measurement node μ excluding symbol node k and let $\partial k \backslash \mu$ be similarly defined.

The message-passing algorithm, which decodes the information indexes $w_1, \dots, w_K$, is described as Algorithm 1. In fact, Algorithm 1 can be derived from the original iterative BP algorithm by certain Gaussian approximation. The computational complexity of the original BP algorithm is exponential in $|\partial\mu| = \mathcal{O}(qN)$, which would be too high for the problem at hand. However, since $qN \gg 1$, the computation carried out at each measurement node admits a good approximation by using the central limit theorem. A similar technique has been used in the CDMA detection problem, for fully-connected bipartite graph in [11]–[13], and for a graph with large node degrees in [14]. The derivation is omitted here due to space limitations.

Algorithm 1 Message-Passing Decoding Algorithm

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Input: S, Y, γ_s, β, q.
Initialization:
Λ ← −log(2^l − 1), L ← Σ_μ |∂μ|, L_2 ← Σ_μ |∂μ|^2
R ← 2γ_s^{−1/2} Y − S · 1
m̂^0_{μk} ← 0 for all μ, k
Main iterations:
for t = 1 to T − 1 do
    for all μ, k with s_{μk} ≠ 0 do
        m^t_{kμ} ← tanh( Λ/2 + Σ_{ν∈∂k\μ} tanh^{−1} m̂^{t−1}_{νk} )
    end for
    Q^t ← (1/L_2) Σ_μ |∂μ| Σ_{j∈∂μ} (m^t_{jμ})^2
    A^t ← [ 4/γ_s + (β L_2)/(N q L) (1 − Q^t) ]^{−1}
    for all μ, k with s_{μk} ≠ 0 do
        m̂^t_{μk} ← tanh( A^t s_{μk} (r_μ − Σ_{j∈∂μ\k} s_{μj} m^t_{jμ}) )
    end for
end for
m_k ← tanh( Λ/2 + Σ_{ν∈∂k} tanh^{−1} m̂^{T−1}_{νk} ) for all k
Output: ŵ_j = arg max_{i=1,...,2^l} m_{(j−1)2^l+i}, j = 1, . . . , K
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We now revisit the assumption that $\boldsymbol{X}$ has independent elements. In fact, $\boldsymbol{X}$ consists of K sub-vectors of length 2^l, where the entries of each sub-vector are all zero except for one position corresponding to the transmitted message. Algorithm 1 outputs the position of the largest element of each of the K sub-vectors of $[m_1, \dots, m_N]$. In fact, the factor graph can be modified by including K additional nodes, each of which puts a constraint on one sub-vector. Slight improvement over Algorithm 1 can be obtained by carrying out message passing on the modified graph.

B. Performance Characterization of Algorithm 1

The iterative decoding dynamics of Algorithm 1 can be quantified using state evolution in the (large-system) limit, where $K, M \to \infty$ with the average system load β and all other parameters kept constant. Recently, in the compressed sensing literature, a rigorous foundation to state evolution for sensing matrices with i.i.d Gaussian entries is provided in [15]. But it cannot be directly applied here due to the on-off nature of the elements of $\boldsymbol{S}$.

For simplicity, we declare decoding success if node 0 can correctly recover all the messages its peers transmitted; otherwise it is regarded as a decoding error. The probability of decoding error averaged over all realizations of all possible messages, signatures and noise is denoted by P^e_s. The main result of the large-system analysis is summarized in the following proposition:

Proposition 1: In the large-system limit, with $T-1$ iterations of message-passing decoding described in Algorithm 1, the decoding error probability P^e_s is given by

$$\mathsf{P}^e_s = 1 - \left(\int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-\sqrt{E})^2} \left[1 - Q(x+\sqrt{E})\right]^{2^l-1} \frac{dx}{\sqrt{2\pi}} \right)^K \tag{4}$$

where $Q(x) = \int_x^\infty \exp(-z^2/2)\, dz/\sqrt{2\pi}$ and $E = \eta^T \gamma_s/4$ is obtained using the following recursive equation with $\eta^0 = 0$:

$$\eta^{t+1} = \frac{1}{1 + \frac{1}{4}\gamma_s \beta \mathcal{E}\left(\frac{1}{4}\gamma_s \eta^t\right)} \tag{5}$$

where we let $r = \tanh(\Lambda/2)$ and

$$\mathcal{E}(E) = 1 - \sum_{x=\pm 1} \int \frac{1+xr}{2\sqrt{2\pi}} \tanh\left(z\sqrt{E} + E + \frac{x\Lambda}{2}\right) e^{\frac{-z^2}{2}} dz.$$

The proof of Proposition 1 makes use of the idea of *density evolution* originally developed for analyzing graphical error-control codes. A similar technique has also been used in [11], [13], [14] to analyze CDMA systems. Proposition 1 is basically a single-letter characterization of the performance in the large-system limit (cf. [16]). That is, from the view point of an individual neighbor, the mutual broadcast system with message-passing decoding is asymptotically equivalent to sending a simplex code through a scalar Gaussian channel with some degradation in the SNR, where the degradation factor is determined from the iterative formula depending on the number of iterations.

IV. Random-Access Schemes

In this section we describe two random-access schemes, namely slotted ALOHA and CSMA, and provide lower bounds on the minimum number of symbol transmissions required for achieving a given error probability. The lower bounds shall be compared to the performance of the RODD scheme.

Let each frame encode L bits including an l-bit message and a few additional bits which identify the sender. A message is assumed to be always decoded correctly if no collision occurs. Each broadcast period consists of a number of frames to allow for retransmissions. Without loss of generality, consider node 0 and let K denote the number of its neighbors. An error event is defined as the situation where node 0 does not correctly receive all K messages from its neighbors until the end of the broadcast period. We denote the total number of symbol transmissions of ALOHA and CSMA by M_a and M_c, respectively.

A. Slotted ALOHA

In slotted ALOHA, each node chooses independently with the same probability p to transmit in every frame interval. The error probability is lower bounded by the probability that one particular peer cannot successfully transmitting its frame without collision by the end of n frame intervals:

$$\mathsf{P}^e \geq \left[1 - p(1-p)^K\right]^n \geq \left[1 - \frac{K^K}{(K+1)^{K+1}}\right]^n \tag{6}$$

where the second inequality becomes equality if and only if $p = 1/(K+1)$. Over the Gaussian channel with SNR

γ, in order to send L bits reliably through the channel, the number of symbols in a frame must exceed $2L/\log_2(1+\gamma)$. Therefore, in order to achieve a given error probability P^e, M_a must satisfy

$$M_a \geq \frac{2L}{\log_2(1+\gamma)} \cdot \frac{\log \mathsf{P}^e}{\log\left(1 - K^K(K+1)^{-K-1}\right)}. \tag{7}$$

B. CSMA

As an improvement over ALOHA, CSMA lets nodes use a brief contention period to negotiate a schedule in such a way that nodes in a small neighborhood do not transmit data simultaneously. Consider the following generic scheme: Each node senses the channel continuously. If the channel is busy, the node remains silent and disables its timer; as soon as the channel becomes available, the node starts its timer with a random offset, and waits till the timer expires to transmit its frame. Clearly, the node whose timer expires first in its neighborhood captures the channel and transmits its frame.

By using the Matérn hard core model [17], the performance of CSMA can be characterized in the following result.

Proposition 2: If each node has $\overline{K}$ neighbors on average, then

$$\mathsf{E}(M_c) \geq \frac{2L \log \frac{1}{\mathsf{P}^e}}{\log_2(1+\gamma)}\left(K - \frac{3\sqrt{3}}{4\pi}\left(K - 1 - \overline{K}\right)\right). \tag{8}$$

This lower bound underestimates the number of slots needed by CSMA since the hidden terminal problem is neglected and the contention overhead is also ignored.

V. Numerical Results

In order for a fair comparison, we assume the same power constraint for both scheme based on sparse recovery and random-access schemes, i.e., the average transmitted power in each active slot (in which the node transmits energy) is the same. Recall that, in the proposed scheme, where the frame length is M_s, the SNR in the model (2) is $\gamma_s = \gamma M_s q(1-q)$.

We compare the number of symbol transmissions required by the three schemes, denoted by M_s, M_a and M_c, respectively, for node 0 to achieve the same error probability. We consider the special case where node 0 has $K = \overline{K}$ neighbors, which is the average over all nodes. We always choose $q = 1/(K+1)$ in the sparse-recovery-based scheme.

First consider a node with $K = 4$ peers, each with a message of $l = 10$ bits. In random-access schemes, at least 2 additional bits are needed to identify a sender, so $L = l + 2 = 12$. In Fig. 1, the error performance of random-access schemes computed from (7) and (8) are compared with that of the proposed mutual broadcast scheme with three different sparse recovery algorithms (message passing, CoSaMP, and AMP). The sparse-recovery-based scheme with message-passing decoding requires much fewer number of symbol transmissions than ALOHA and CSMA to achieve the same error probability. The simulation results show that the message-passing decoding algorithm noticeably outperforms CoSaMP and AMP at both SNRs, $\gamma = 0$ dB and $\gamma = 10$ dB. Here, we have utilized in AMP the prior information that each

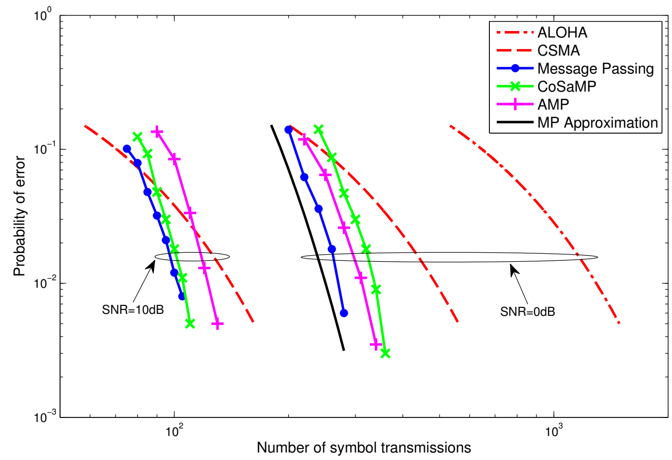


Fig. 1. Performance comparison with 4 neighbors and 10 bits per node.

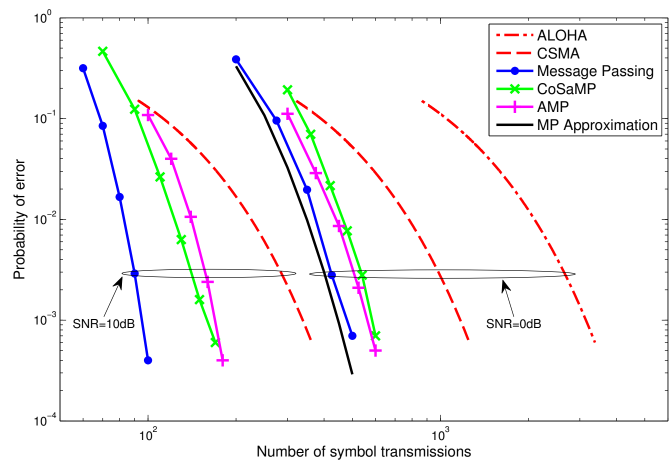
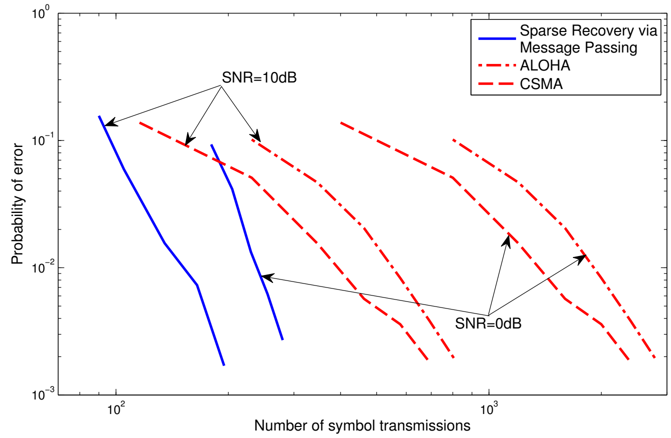


Fig. 2. Performance comparison with 9 neighbors and 5 bits per node.

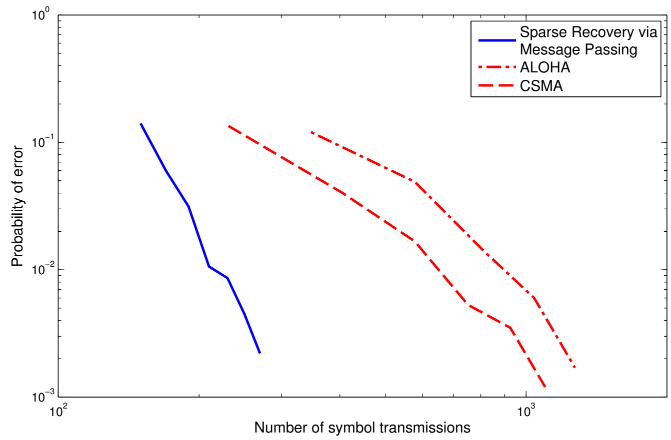
entry in input $\boldsymbol{X}$ is i.i.d. which is 1 with probability 2^{-l} and 0 otherwise. At $\gamma = 0$ dB, we also make a comparison between the simulation result and the large system approximation from (4) for the message-passing algorithm. The approximation is seen to provide a good characterization of the performance of the message-passing algorithm. Note that the performance of ALOHA is much worse than that of CSMA at $\gamma = 10$ dB and is omitted to avoid clutter.

Fig. 2 repeats the simulation with the number of neighbors changed to $K = 9$ and the message length changed to $l = 5$. In this case, random-access schemes require each node to transmit at least 4 additional bits so that node 0 can identify its sender. The relative performance of the different schemes remains similar as in the previous case depicted in Fig. 1.

In the remainder of this section, we consider some system-level experiments. Suppose there are 1000 nodes in a wireless network, and they are uniformly distributed in a 100×100 square. Assume that each node has a sensing radius of R, then each node has an average number of $\overline{K} = \pi R^2/10$ neighbors by considering periodic boundary condition. We always choose $q = \frac{1}{\overline{K}+1}$ in the sparse-recovery-based scheme. The metric



(a) $\overline{K} = 4$



(b) $\overline{K} = 9$

Fig. 3. System-level performance comparison with 10 bits per node. (a) $\overline{K} = 4$, (b) $\overline{K} = 9$

for performance comparison between different schemes is the probability for one node to miss a given neighbor, averaged over all pairs of neighbors in the network.

First we consider one realization of the network where each node has $\overline{K} = 4$ neighbors and each node has a message of $l = 10$ bits. Suppose the identification overhead in random-access schemes is ignored and L is set to 10. We still consider $\gamma = 0$ dB and 10 dB. As shown in Fig. 3(a), in the same system setting, the proposed sparse-recovery-based scheme needs much less symbol transmissions than both ALOHA- and CSMA-based random-access schemes. Similar results can be observed in Fig. 3(b) when simulating one realization of the network where $\overline{K} = 9$ and $l = 10$ at 10 dB SNR. To achieve the error rate of 0.01, the proposed scheme requires 200 symbol transmissions, whereas CSMA and ALOHA require about 700 and 1,000 symbol transmissions, respectively.

VI. Concluding Remarks

The proposed scheme departs from the usual networking solution where a highly-reliable, capacity-approaching, point-to-point physical-layer code is paired with a rather unreliable MAC layer. By treating the physical and MAC layers as a whole, the proposed scheme achieves better overall reliability at much higher efficiency.

The proposed scheme can serve as a highly desirable sub-layer of any network protocol stack to provide the important function of simultaneous peer-to-peer mutual broadcast. This sub-layer provides the missing link in many advanced resource allocation schemes, where it is often *assumed* that nodes are provided the state and/or demand of their peers.

We have assumed that the channel between every pair of nodes is Gaussian. Using a more realistic model by incorporating fading and path loss and addressing the near-far problem are future research directions.

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