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STABILITY OF FINITE-USER SLOTTED ALOHA UNDER PARTIAL INTERFERENCE IN WIRELESS MESH NETWORKS

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ABSTRACT

We study the stability of finite-user infinite-buffer slotted ALOHA with partial interference. For the case of two users, there is a *gradual transition* from the collision channel to the orthogonal channel when the link separation increases. The stability region can be either convex or nonconvex, depending on the link separation and the transmission probability vector. A partial characterization on the boundary of the stability region in *closed form* for the case of general number of users is also given. We hope this work can provide insight in designing traffic engineering algorithms in wireless mesh networks with practical random access protocols like 802.11.

I. INTRODUCTION

The relationship between interference and capacity has been a main focus in the study of *wireless mesh networks*. In [1], a new interference model, *partial interference*, was proposed, stating that even if the signal-to-noise ratio (SNR) of a transmission does not reach the required threshold, it is still possible for the transmission to be successful probabilistically. It was shown that the gain in terms of *capacity across unit cut* by considering partial interference in scheduling in a regular wireless network can be as high as 67%. Therefore it is beneficial to consider the performance gain by partial interference in emerging *traffic engineering algorithms* in wireless mesh networks.

Existing traffic engineering algorithms [2–4] presume the existence of a centralized scheduler, which can coordinate interfering links to access the wireless channel at different times. However, practical wireless networks pre-dominantly use distributed *random access* protocols. Therefore, as a first attempt to exploit the gain from partial interference in traffic engineering in wireless mesh networks with random access protocols, [1] studied the stability (admissible) region of an 802.11 network with two links and infinite buffer. The result showed that the stability region is convex when partial interference exists. However, the result was computed numerically and was not analytically tractable. Therefore, we consider a simpler random access protocol, *i.e.*, slotted ALOHA, to obtain insight on exploiting partial interference in 802.11 networks.

In this paper, we make the following contributions:

- By considering partial interference, in Section IV. we extend the model in [5] to derive the exact stability region of slotted ALOHA with two users.
- In Section V. we show that as the link separation increases, the stability region obtained in Section IV. expands *gradually* under partial interference. Due to the similarities in the results with those in [1], the results here can be applied to 802.11 networks. The stability region can be either convex or nonconvex, depending on the link separation and the transmission probability vector.
- In Section VI. we give in *closed form* a partial characterization on the boundary of the stability region under partial interference with general number of users.

For the rest of the paper, in Section II. we present related works. In Section III. the model of slotted ALOHA is described. In Section VII. we conclude the paper and discuss future works.

II. RELATED WORKS

The study of the stability region of M-user, $M < \infty$, infinite-buffer slotted ALOHA was initiated by [6] decades before, and is still an ongoing research. [6] obtained the boundary of the stability region when $M = 2$ with the *collision channel* model. [7] and [8] used *stochastic dominance* and derived the same result as in [6] for the case of $M = 2$. Unlike these works, we consider partial interference [1] in slotted ALOHA networks.

For general M, [6] obtained separate sufficient and necessary conditions for stability. [7] and [8] derived tighter bounds on the stability region by using stochastic dominance in different ways. [9] established the boundary of the stability region for general M, but it involves stationary joint queue statistics, which still do not have closed form to date. [10] introduced *instability rank* and used it to improve the bounds on the stability region. Contrasting to these works, we provide part of the boundary of the stability region in closed form.

With the advances in multi-user detection, researchers also studied this problem with the *multipacket reception* (MPR) model. [11] studied this problem in the infinite-user, single-buffer and symmetric MPR case. In [5], the authors removed the symmetric assumption and considered the problem with finite users and infinite buffer. They obtained the boundary for the asymmetric MPR case with two users, and also the inner bound on the stability region for general M. The MPR model is similar to partial interference that in both models simultaneous transmissions are allowed; but in MPR one receiver can receive multiple packets, while in partial interference two or more receivers in proximity can receive packets from the corresponding transmitters simultaneously.

III. THE FINITE-USER SLOTTED ALOHA MODEL

A. Assumptions

Let $\mathcal{M} = \{n\}_{n=1}^{M}$ be the set of links in the slotted ALOHA system. The following assumptions apply to all links $n \in \mathcal{M}$. Let

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T_n and R_n be the transmitter and the receiver of link n respectively. T_n has an infinite buffer. The packet arrival process at T_n is Bernoulli with mean λ_n and is independent of the arrivals at other transmitters. T_n attempts a *virtual transmission* with probability p_n, *i.e.*, if its buffer is nonempty, T_n attempts an *actual transmission* with probability p_n; otherwise, T_n always remains silent. Also define $\overline{p}_n = 1 - p_n$.

In the system, time is slotted and each slot is just enough for transmission of one packet. Packets are assumed to have equal lengths. We assume transmission results are independent in each slot. For $n \in \mathcal{A} \subseteq \mathcal{M}$, let $\mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}}$ be the probability that the transmission on link n is successful when $\{T_{n'}\}_{n' \in \mathcal{A}}$ is the set of active transmitters. $\mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}}$ depends on the SNR at the receiver and the modulation scheme used. We also assume the transmitters know immediately the transmission results, so that the transmitters remove successfully transmitted packets and retain only those unsuccessful ones.

B. Stability of Slotted ALOHA

We let $Q_n(t), t \in \mathbb{N}$ be the queue length in T_n at the beginning of slot t, and use a M-dimensional Markov chain $\mathbf{Q}^{\mathcal{M}}(t) = \big(Q_n(t)\big)_{n \in \mathcal{M}}$ to represent the queue lengths in all transmitters. We denote by $A_n(t)$ the number of packets arrived at T_n in slot t, and $D_n(t)$ the number of packets successfully transmitted in slot t by T_n when $Q_n(t) > 0$. Then $Q_n(t+1) = [Q_n(t) - D_n(t)]^+ + A_n(t)$, where $[z]^+ = \max\{0, z\}$ is used to account for the case that there is no packet transmitted when $Q_n(t) = 0$. We use the definition of stability in [5, 9, 10].

Definition 1 *A M-dimensional stochastic process $\mathbf{Q}^{\mathcal{M}}(t)$ is stable if for $\mathbf{x} \in \mathbb{N}^M$ the following holds:*

$$\lim_{t \to \infty} \Pr\Big\{\mathbf{Q}^{\mathcal{M}}(t) < \mathbf{x}\Big\} = F(\mathbf{x}) \text{ and } \lim_{\mathbf{x} \to \infty} F(\mathbf{x}) = 1.$$

If the following weaker condition holds instead,

$$\lim_{\mathbf{x} \to \infty} \liminf_{t \to \infty} \Pr\Big\{\mathbf{Q}^{\mathcal{M}}(t) < \mathbf{x}\Big\} = 1,$$

then the process is substable. The process is unstable if it is neither stable nor substable.

The stability problem of slotted ALOHA we consider in this paper is to determine whether the slotted ALOHA system with the set of links $\mathcal{M}$ is stable given the parameters $\{\lambda_n\}_{n \in \mathcal{M}}$ and $\{p_n\}_{n \in \mathcal{M}}$. We use the result from [12]: On the assumption that the arrival and the service processes of a queue are stationary, the queue is stable if the average arrival rate is less than the average service rate, and the queue is unstable if the average arrival rate is larger than the average service rate. We also define the slotted ALOHA system to be stable when all queues in the system are stable.

IV. Stability Region of 2-User Slotted ALOHA under Partial Interference

We extend the model in [5] to obtain the following results. For $n \in \mathcal{M}$, let P_n and N_n be the transmission power used by T_n and the background noise power at R_n respectively. Assume the signal propagation follows the path loss model $pl(d) = Cd^{-\alpha}$, where d is the propagation distance, α is the path loss exponent and C is a constant. We let $\gamma^{\mathcal{M}}_{n,\mathcal{A}}$ be the SNR attained at R_n when $\{T_{n'}\}_{n' \in \mathcal{A}}$ is the set of active transmitters. Assume a packet consists of L bits. Let $e(\gamma)$ be the bit error rate when the SNR is γ. In particular, if differential binary phase shift keying (DBPSK) is used in the physical layer, $e(\gamma) = \dfrac{1}{2}\exp(-\gamma)$ [13]. Under binary interference, we let the SNR threshold γ_0 be the case that the packet error rate is ϵ, *i.e.*, $1 - \left[1 - \dfrac{1}{2}\exp(-\gamma_0)\right]^L = \epsilon$. Consider $M = 2$. When only T_1 is active, the SNR attained at R_1 is $\gamma^{\mathcal{M}}_{1,\{1\}} = \dfrac{P_1 C d^{-\alpha}_{T_1,R_1}}{N_1}$, and

$$\mathsf{q}^{\mathcal{M}}_{1,\{1\}} = \begin{cases} 1, & \gamma^{\mathcal{M}}_{1,\{1\}} \geq \gamma_0 \\ 0, & \gamma^{\mathcal{M}}_{1,\{1\}} < \gamma_0 \end{cases}, \tag{1}$$

where $d_{X,Y}$ is the distance between X and Y. When both T_1 and T_2 are active, then $\gamma^{\mathcal{M}}_{1,\{1,2\}} = \dfrac{P_1 C d^{-\alpha}_{T_1,R_1}}{P_2 C d^{-\alpha}_{T_2,R_1} + N_1}$ is the SNR attained at R_1, and

$$\mathsf{q}^{\mathcal{M}}_{1,\{1,2\}} = \begin{cases} 1, & \gamma^{\mathcal{M}}_{1,\{1,2\}} \geq \gamma_0 \\ 0, & \gamma^{\mathcal{M}}_{1,\{1,2\}} < \gamma_0 \end{cases}. \tag{2}$$

If we consider partial interference instead, we can calculate $\mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}}$ as follows. When only T_1 is active,

$$\mathsf{q}^{\mathcal{M}}_{1,\{1\}} = \Big[1 - e\Big(\gamma^{\mathcal{M}}_{1,\{1\}}\Big)\Big]^L. \tag{3}$$

When both T_1 and T_2 are active,

$$\mathsf{q}^{\mathcal{M}}_{1,\{1,2\}} = \Big[1 - e\Big(\gamma^{\mathcal{M}}_{1,\{1,2\}}\Big)\Big]^L. \tag{4}$$

Similarly, we can derive expressions for $\mathsf{q}^{\mathcal{M}}_{2,\{2\}}$ and $\mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}$ under binary and partial interference.

To evaluate the boundary of the stability region for the 2-user slotted ALOHA system, we use stochastic dominance as introduced in [7]. We use $\mathsf{S}_{\mathcal{P}}$ to represent a *dominant system* of the original system S, with $\mathcal{P}$ being the *persistent* set. The transmitters of the links in this set transmit dummy packets when they decide to transmit but do not have packets queued in their buffer. The remaining transmitters behave identically as those in S. We first consider the dominant system $\mathsf{S}_{\{1\}}$. In this dominant system, the successful transmission probability of link 2 is $p_2\overline{p}_1\mathsf{q}^{\mathcal{M}}_{2,\{2\}} + p_2 p_1 \mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}$. For link 1, the queue in T_2 is empty with probability $1 - \dfrac{\lambda_2}{p_2\overline{p}_1\mathsf{q}^{\mathcal{M}}_{2,\{2\}} + p_2 p_1 \mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}}$, in this case the successful transmission probability is $p_1\mathsf{q}^{\mathcal{M}}_{1,\{1\}}$; otherwise, the successful transmission probability is $p_1\overline{p}_2\mathsf{q}^{\mathcal{M}}_{1,\{1\}} + p_1 p_2 \mathsf{q}^{\mathcal{M}}_{1,\{1,2\}}$. Hence, the average successful transmission probability of link 1 is

$$p_1\mathsf{q}^{\mathcal{M}}_{1,\{1\}}\left(1 - \frac{\lambda_2}{p_2\overline{p}_1\mathsf{q}^{\mathcal{M}}_{2,\{2\}} + p_2 p_1 \mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}}\right) + \left(p_1\overline{p}_2\mathsf{q}^{\mathcal{M}}_{1,\{1\}} + p_1 p_2 \mathsf{q}^{\mathcal{M}}_{1,\{1,2\}}\right)\frac{\lambda_2}{p_2\overline{p}_1\mathsf{q}^{\mathcal{M}}_{2,\{2\}} + p_2 p_1 \mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}}$$

Table 1: Parameters used for the analytical results.

P_1, P_2	24.5 dBm	N_1, N_2	-88 dBm
G_T, G_R	1	h_T, h_R	1.5 m
ϵ	0.001	d_{T_1,R_1}, d_{T_2,R_2}	450 m

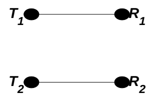


Figure 1: A sample topology.

With the following notations,

$$\begin{aligned} \lambda_1' &= p_1\overline{p}_2\mathsf{q}^{\mathcal{M}}_{1,\{1\}} + p_1p_2\mathsf{q}^{\mathcal{M}}_{1,\{1,2\}}, \\ \lambda_2' &= p_2\overline{p}_1\mathsf{q}^{\mathcal{M}}_{2,\{2\}} + p_2p_1\mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}, \\ \mathsf{Q}^{\mathcal{M}}_{1,\{1\},\{2\}} &= \mathsf{q}^{\mathcal{M}}_{1,\{1\}} - \mathsf{q}^{\mathcal{M}}_{1,\{1,2\}}, \\ \mathsf{Q}^{\mathcal{M}}_{2,\{2\},\{1\}} &= \mathsf{q}^{\mathcal{M}}_{2,\{2\}} - \mathsf{q}^{\mathcal{M}}_{2,\{1,2\}}, \end{aligned}$$

the stability region of $\mathsf{S}_{\{1\}}$ is

$$\lambda_1 < p_1\mathsf{q}^{\mathcal{M}}_{1,\{1\}} - \frac{\lambda_2 p_2 p_1 \mathsf{Q}^{\mathcal{M}}_{1,\{1\},\{2\}}}{\lambda_2'} \text{ and } \lambda_2 < \lambda_2', \quad (5)$$

and by symmetry, the stability region of $\mathsf{S}_{\{2\}}$ is

$$\lambda_2 < p_2\mathsf{q}^{\mathcal{M}}_{2,\{2\}} - \frac{\lambda_1 p_1 p_2 \mathsf{Q}^{\mathcal{M}}_{2,\{2\},\{1\}}}{\lambda_1'} \text{ and } \lambda_1 < \lambda_1'. \quad (6)$$

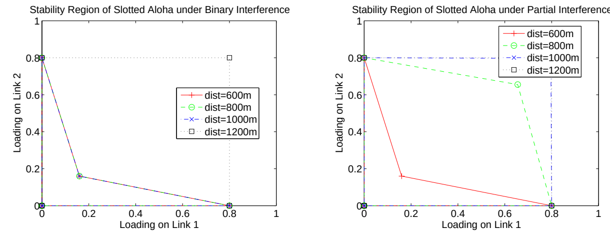
The union of these two regions constitutes the inner bound on the stability region of the original system S.

The reason for the union of these two regions to be the outer bound on the stability region follows from the indistinguishability argument [5, 7]. Consider the dominant system $\mathsf{S}_{\{1\}}$. With a particular initial condition on the length of the queues, if the queue in T_1 is unstable, it is equivalent to the case that the queue in T_1 never empties with nonzero probability. Then $\mathsf{S}_{\{1\}}$ and S will be indistinguishable, in the sense that the packets transmitted from T_1 in $\mathsf{S}_{\{1\}}$ are always real packets, and S is also unstable. Therefore, the union of the regions defined by (5) and (6) is the *exact* stability region for $M = 2$.

V. Some Illustrations

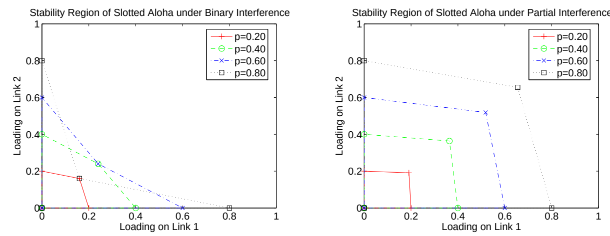
In this section, we depict the stability region derived in Section IV. by considering the parallel-link topology in Fig. 1. We use the two-ray ground reflection model $pl(d) = \frac{G_T G_R h_T^2 h_R^2}{d^4}$ to represent the path loss, where G_T and G_R are the gain of transmitter and receiver antenna respectively, and h_T and h_R are the height of transmitter and receiver antenna respectively. The values of various parameters are shown in Table 1.

We first assume $p_1 = p_2 = 0.8$ and vary the link separation, *i.e.*, the perpendicular distance between the links, to obtain the results under binary interference in Fig. 2(a). The stability region has only two possible shapes. For the separations of 600, 800 and 1000 meters, the SNR attained at either receiver



(a) binary interference. (b) partial interference.

Figure 2: Stability region for $M = 2$ with transmission probabilities 0.8 under binary and partial interference.



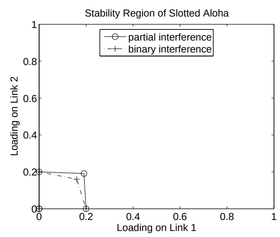
(a) binary interference. (b) partial interference.

Figure 3: Stability region for $M = 2$ with link separation 800 meters under binary and partial interference.

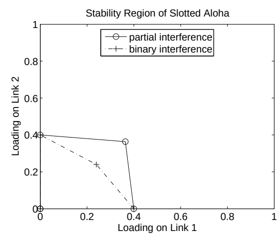
when both transmitters are active is smaller than the threshold. Therefore the underlying channel follows the collision channel model and the stability region is nonconvex. When the separation is 1200 meters, the links are separated far enough so that transmissions on both links are independent. The channel can be regarded as the orthogonal channel and the stability region is convex. Therefore, the threshold in binary interference determines when to *switch* between the collision channel and the orthogonal channel.

Fig. 2(b) shows the corresponding results under partial interference. When the link separation is small, the amount of interference is so large that partial interference degenerates to the collision channel. As the link separation increases, the stability region expands gradually and changes from nonconvex to convex. At another extreme, when the links are sufficiently far apart, partial interference is identical to the orthogonal channel. Therefore, partial interference can be viewed as a generalization of binary interference that it *interpolates* the transition from the collision channel to the orthogonal channel. Notice that our results here are similar to the case considered in [1], therefore our results should be applicable to networks with practical random access protocols like 802.11.

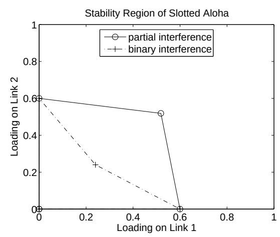
Next, we assume the links are separated by 800 meters. We let both links transmit with probability p, and illustrate the effect of p on the convexity of the stability region under binary interference in Fig. 3(a). When p is small, *i.e.*, 0.2 and 0.4, the links are too conservative in attempting transmissions. It leads to better channel utilization by adding one more link to the system, and the stability region is convex. On the other hand, when p is large, *i.e.*, 0.6 and 0.8, the links are too aggressive. When one more link is added to the system, it increases



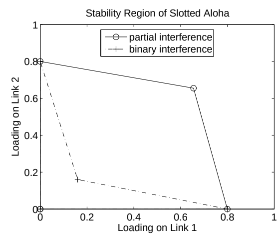
(a) $p_1 = p_2 = 0.2$.



(b) $p_1 = p_2 = 0.4$.



(c) $p_1 = p_2 = 0.6$.



(d) $p_1 = p_2 = 0.8$.

Figure 4: Stability region for $M = 2$ under binary and partial interference with various transmission probabilities.

contention and hence reduces the loading supported by each link drastically. As a result, the stability region is nonconvex. The convexity of the stability region can therefore be regarded as a measure of the contention level in a network.

Fig. 3(b) illustrates the stability region when partial interference is considered instead, under the same settings. Although the SNR attained at a receiver when both transmitters are active is smaller than the threshold, the SNR is large enough to support a sustainable throughput probabilistically. Therefore, it is possible to receive more packets opportunistically by exploiting partial interference, thereby increasing the loading supported by each link and allowing the stability region to be convex. If we compare the stability region under binary and partial interference in identical settings, as shown in Figs. 4(a)-4(d), the stability region under partial interference is always larger than that under binary interference. This implies that by considering partial interference, more combinations of flows on the links can be admitted, and the capacity of a wireless network can be potentially increased.

VI. Generalization to the M-link Case

In this section, we give in closed form a partial characterization on the boundary of the stability region of M-user slotted ALOHA under partial interference. First, for $n \in \mathcal{A} \subseteq \mathcal{M}$, $\gamma_{n,\mathcal{A}}^{\mathcal{M}} = \dfrac{P_n C d_{T_n,R_n}^{-\alpha}}{\sum_{n' \in \mathcal{A}\setminus\{n\}} P_{n'} C d_{T_{n'},R_n}^{-\alpha} + N_n}$. Therefore, under binary interference,

$$\mathsf{q}_{n,\mathcal{A}}^{\mathcal{M}} = \begin{cases} 1, & \gamma_{n,\mathcal{A}}^{\mathcal{M}} \geq \gamma_0 \\ 0, & \gamma_{n,\mathcal{A}}^{\mathcal{M}} < \gamma_0 \end{cases}, \tag{7}$$

while under partial interference,

$$\mathsf{q}_{n,\mathcal{A}}^{\mathcal{M}} = \left[1 - e\left(\gamma_{n,\mathcal{A}}^{\mathcal{M}}\right)\right]^L. \tag{8}$$

For each $\mathcal{M}' \subseteq \mathcal{M}$, let $\mathsf{p}^{\mathcal{M}}(\mathcal{M}') = \left(\mathsf{p}_n^{\mathcal{M}}(\mathcal{M}')\right)_{n\in\mathcal{M}}$ be a M-dimensional 0-1 vector such that

$$\mathsf{p}_n^{\mathcal{M}}(\mathcal{M}') = \begin{cases} 1, & n \in \mathcal{M}' \\ 0, & n \notin \mathcal{M}' \end{cases},$$

where $\mathcal{M}'$ is a set of persistent links and all other links are empty. Define $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')} = \left(\Pi_n^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')}\right)_{n\in\mathcal{M}}$, where

$$\Pi_n^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')} = \sum_{\mathcal{A}:\, n\in\mathcal{A}\subseteq\mathcal{M}'} \prod_{n'\in\mathcal{A}} p_{n'} \prod_{n''\in\mathcal{M}'\setminus\mathcal{A}} \overline{p}_{n''} \mathsf{q}_{n,\mathcal{A}}^{\mathcal{M}}, \tag{9}$$

to be a *corner point* corresponding to the case that $\mathcal{M}'$ is the set of persistent links. Notice RHS of (9) is zero when $n \notin \mathcal{M}'$. Then we obtain the following Theorem.

Theorem 1 *All corner points lie on the boundary of the stability region.*

By using stochastic dominance and the indistinguishability argument, we obtain the following Theorem.

Theorem 2 *Let $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P})}, \Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P}\cup\mathcal{D})}$ be two corner points such that $\mathcal{D} = \{\bar{n}\} \subseteq \mathcal{M}\setminus\mathcal{P}$. Then the line segment joining these two points lies on the boundary of the stability region. This line segment represents the case that $\mathcal{P}$ is the set of persistent links while $\bar{n}$ is the only non-empty non-persistent link in the system.*

When $|\mathcal{P}| = 0$, it is trivial that the line segment between $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P})}$ and $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P}\cup\mathcal{D})}$ lies on the boundary because it is part of the positive $\lambda_{\bar{n}}$-axis. Assume $|\mathcal{P}| > 0$. Then by Theorem 2,

$$\begin{aligned} &\lambda_{\bar{n}} < \lambda'_{\bar{n}}, \bar{n} \in \mathcal{D}, \\ &\lambda_n < \sum_{\mathcal{A}:\, n\in\mathcal{A}\subseteq\mathcal{P}} \prod_{n'\in\mathcal{A}} p_{n'} \prod_{n''\in\mathcal{P}\setminus\mathcal{A}} \overline{p}_{n''} \mathsf{q}_{n,\mathcal{A}}^{\mathcal{M}} \\ &\qquad - \frac{\lambda_{\bar{n}}}{\lambda'_{\bar{n}}} p_{\bar{n}} \sum_{\mathcal{A}:\, n\in\mathcal{A}\subseteq\mathcal{P}} \prod_{n'\in\mathcal{A}} p_{n'} \prod_{n''\in\mathcal{P}\setminus\mathcal{A}} \overline{p}_{n''} \mathsf{Q}_{n,\mathcal{A},\mathcal{D}}^{\mathcal{M}}, n \in \mathcal{P}, \\ &\lambda_{\tilde{n}} = 0, \tilde{n} \in \mathcal{M}\setminus(\mathcal{P}\cup\mathcal{D}) \end{aligned} \tag{10}$$

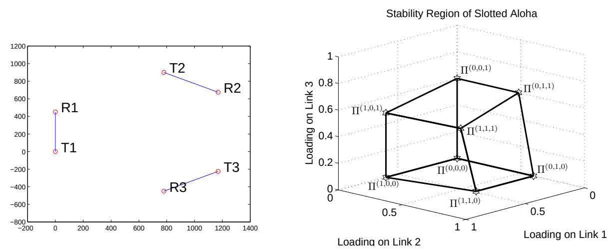
with

$$\begin{aligned} \lambda'_{\bar{n}} &= \sum_{\mathcal{A}:\, \bar{n}\in\mathcal{A}\subseteq(\mathcal{P}\cup\mathcal{D})} \prod_{n'\in\mathcal{A}} p_{n'} \prod_{n''\in(\mathcal{P}\cup\mathcal{D})\setminus\mathcal{A}} \overline{p}_{n''} \mathsf{q}_{\bar{n},\mathcal{A}}^{\mathcal{M}}, \\ \mathsf{Q}_{n,\mathcal{A},\mathcal{D}}^{\mathcal{M}} &= \mathsf{q}_{n,\mathcal{A}}^{\mathcal{M}} - \mathsf{q}_{n,\mathcal{A}\cup\mathcal{D}}^{\mathcal{M}}, \mathcal{A}\colon n \in \mathcal{A} \subseteq \mathcal{P}. \end{aligned}$$

lies on the boundary of the stability region. The proofs of these Theorems can be found in Appendices A and B respectively.

Theorems 1 and 2 cover all cases with zero or one non-empty non-persistent link in the system respectively. However, if there are at least two non-empty non-persistent links, the stationary joint queue statistics must be involved in calculating the boundary. Hence, the results here are the best we can obtain without the stationary joint queue statistics.

We illustrate the results of these Theorems by considering $M = 3$ with the ring topology in Fig. 5(a). The distance between a receiver and the nearest interfering transmitter is 900 meters. Each link transmits with probability 0.6. Other parameters are the same as in Table 1. From Theorem 1, each



(a) A sample topology. (b) Part of the boundary.

Figure 5: Stability region with $M = 3$.

of the eight 3-dimensional 0-1 vector corresponds to a corner point shown in Fig. 5(b), and their coordinates can be obtained from (9). By Theorem 2, the solid lines in Fig. 5(b) are part of the boundary of the stability region. As another example, for $M = 2$, notice that (5) and (6) are special cases of (10). As a direct consequence of our Theorems 1 and 2, the stability region of slotted ALOHA with two users under partial interference is piecewise linear.

VII. Conclusion

In this paper, we have derived the exact stability region of slotted ALOHA with two users under partial interference. We showed that there is a gradual transition from the collision channel to the orthogonal channel when the link separation increases. We demonstrated that the stability region can be either convex or nonconvex, depending on the link separation and the transmission probability vector. For the case of M users, we obtained a partial characterization on the boundary of the stability region in closed form.

In view of this work, we may investigate whether it is possible for the stability region with general M to be convex as in the case of $M = 2$. Also, we may derive convex and piecewise linear bounds on the stability region, to reduce the traffic engineering problem into convex or linear programming.

A Proof of Theorem 1

When $\mathcal{M}' = \emptyset$, (9) becomes $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')} = \mathbf{0}$, which is obviously on the boundary. If $\mathcal{M}' \neq \emptyset$, each link $n \in \mathcal{M}'$ operates as *M/M/1*. If at a certain instant, only the links in $\mathcal{A} \subseteq \mathcal{M}'$ are active, which occurs with probability $\prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{M}' \setminus \mathcal{A}} \overline{p}_{n''}$, the probability of successful transmission of link n is $\mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}}$. Therefore, by unconditioning on $\mathcal{A}$ while noticing $\mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}} = 0$ if $n \notin \mathcal{A}$, the successful transmission probability of link n is

$$\sum_{\mathcal{A}:\, n \in \mathcal{A} \subseteq \mathcal{M}'} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{M}' \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}}.$$

Therefore $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')}$ lies on the boundary.

B Proof of Theorem 2

We prove (10) is part of the boundary of the stability region. For any $\tilde{n} \notin \mathcal{P} \cup \mathcal{D}$, $T_{\tilde{n}}$ has no packet, hence $\lambda_{\tilde{n}} = 0$. Therefore we consider the dominant system $\mathsf{S}_{\mathcal{P}}$, assuming the system contains only the links in $\mathcal{P} \cup \mathcal{D}$. For the sufficiency part, the queue in $T_{\bar{n}}$ in $\mathsf{S}_{\mathcal{P}}$ is stable if

$$\begin{aligned} \lambda_{\bar{n}} &< p_{\bar{n}} \sum_{\mathcal{A} \subseteq \mathcal{P}} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{P} \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{\bar{n},\mathcal{A} \cup \mathcal{D}} \\ &= \sum_{\mathcal{A}:\, \bar{n} \in \mathcal{A} \subseteq (\mathcal{P} \cup \mathcal{D})} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in (\mathcal{P} \cup \mathcal{D}) \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{\bar{n},\mathcal{A}} = \lambda'_{\bar{n}} \end{aligned}$$

For any $n \in \mathcal{P}$, the queue in T_n in $\mathsf{S}_{\mathcal{P}}$ is stable if

$$\begin{aligned} \lambda_n &< \left(1 - \frac{\lambda_{\bar{n}}}{\lambda'_{\bar{n}}}\right) \sum_{\mathcal{A}:\, n \in \mathcal{A} \subseteq \mathcal{P}} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{P} \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}} \\ &\quad + \frac{\lambda_{\bar{n}}}{\lambda'_{\bar{n}}} \sum_{\mathcal{A}:\, n \in \mathcal{A} \subseteq (\mathcal{P} \cup \mathcal{D})} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in (\mathcal{P} \cup \mathcal{D}) \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}} \\ &= \sum_{\mathcal{A}:\, n \in \mathcal{A} \subseteq \mathcal{P}} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{P} \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{q}^{\mathcal{M}}_{n,\mathcal{A}} \\ &\quad - \frac{\lambda_{\bar{n}}}{\lambda'_{\bar{n}}} p_{\bar{n}} \sum_{\mathcal{A}:\, n \in \mathcal{A} \subseteq \mathcal{P}} \prod_{n' \in \mathcal{A}} p_{n'} \prod_{n'' \in \mathcal{P} \setminus \mathcal{A}} \overline{p}_{n''} \mathsf{Q}^{\mathcal{M}}_{n,\mathcal{A},\mathcal{D}} \end{aligned}$$

The necessity follows directly from the indistinguishability argument. We observe that λ_n varies linearly with $\lambda_{\bar{n}}$ on the boundary, $\forall n \notin \mathcal{D}$. It is trivial that $\lambda_{\bar{n}} = 0$ and $\lambda_{\bar{n}} = \lambda'_{\bar{n}}$ correspond to $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P})}$ and $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{P} \cup \mathcal{D})}$ respectively.

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