FRASA: Feedback Retransmission Approximation for the Stability Region of Finite-User Slotted ALOHA

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ABSTRACT

We propose FRASA, *Feedback Retransmission Approximation for Slotted ALOHA*, to study the stability region of finite-user slotted ALOHA under collision channel. With FRASA, we derive in *closed form* the boundary of the stability region for *any* number of users in the system, which is shown to be accurate via simulations. We use convex hulls and supporting hyperplanes to construct convex and piecewise linear outer and inner bounds on the stability region of FRASA respectively to facilitate network optimization. We hope the analytical findings with FRASA can provide more insights on the characterization of the capacity region of other types of wireless random access networks, and enable traffic engineering with linear constraints in the design of wireless mesh networks.

1. INTRODUCTION / MOTIVATION

The study of the stability region of slotted ALOHA has attracted many researchers [1–6]. Despite the simplicity of slotted ALOHA, this problem is extremely difficult when M, the number of users in the system, exceeds two, even on the collision channel assumption. Under this assumption, successful transmissions occur if and only if there is one active transmitter, because of the interference among the stations. The inherent difficulty in the analysis is due to the effect of queueing in each transmitter. More specifically, the probability of successful transmission depends on the number of active transmitters, which in turn depends on whether the queues in the transmitters are empty or not. However, it is still an open problem to obtain the stationary joint queue statistics in closed form.

Instead of finding the exact stability region, previous researchers have attempted to bound the stability region [5,6]. However, they did not require the bounds to be *convex* or *piecewise linear*, which are important in traffic engineering. Requiring such properties reduces the traffic engineering problem into convex or linear programming, which are relatively more tractable. Therefore, we are motivated to derive convex and piecewise linear bounds on the stability region. We hope this work can serve as a basis and can be extended to consider multi-hop networks and interference models other than collision channel.

In this paper, we propose FRASA, *Feedback Retransmission Approximation for Slotted ALOHA*, as a surrogate to approximate finite-user slotted ALOHA. We obtain in *closed form* the boundary of the stability region of FRASA under collision channel for *any* number of users. This approximation gives the exact stability region when there are two users, and is shown to be accurate in other cases via simulations. We also provide convex and piecewise linear bounds on the stability region of FRASA by using convex hulls and supporting hyperplanes.

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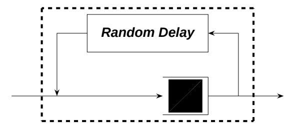


Figure 1: FRASA - Feedback Retransmission Approximation for Slotted ALOHA.

2. THE FRASA MODEL

In FRASA, the buffer in each transmitter can hold one packet only. Whenever there is a packet in the buffer, if the transmitter decides not to transmit the packet, or the transmitter cannot successfully transmit the packet due to collision, the packet will be removed and put back in the buffer again after a random delay which is geometrically distributed. Therefore, the *aggregate arrival* of packets to the buffer, which is defined as the sum of the new arrivals and the retransmissions, is assumed to be Bernoulli or memoryless. Similar approximation was introduced by [7]. FRASA is shown in Fig. 1.

Assume there are M links in the network, and the set of links is denoted by $\mathcal{M} = \{n\}_{n=1}^{M}$. Let $\mathbf{p} = (p_n)_{n \in \mathcal{M}}$ be the transmission probability vector. Let $\overline{p}_n = 1 - p_n, \forall n \in \mathcal{M}$. We first consider a *reduced FRASA system* $\mathsf{S}_{\hat{n}}$, in which we let link $\hat{n}$ have infinite backlog and for $n \neq \hat{n}$, link n has a fixed aggregate arrival rate $\chi_n \in [0, 1]$. Therefore, link $\hat{n}$ is active with probability $p_{\hat{n}}$, while for $n \neq \hat{n}$, link n is active with probability $\chi_n p_n$. Hence, $\overline{\boldsymbol{\lambda}} = \left(\overline{\lambda}_n\right)_{n \in \mathcal{M}}$ is the

Table 1: Comparison for λ_M for $M=3$ and $p_1=p_2=p_3=0.5$ (upper and lower bounds from [5]).

λ_1	λ_2	Upper	FRASA(Exact Sim.)	Lower
0	0	0.5	0.5(0.4996)	0.5
0	0.12	0.38	0.38(0.3831)	0.38
0.06	0.06	0.38	0.3703(0.3652)	0.3405
0.12	0.123	0.257	0.1704(0.1585)	0.1402
0.12	0.13	0.25	0.13(0.1315)	0.13

successful transmission probability vector, where

$$\overline{\lambda}_n = \begin{cases} \chi_n p_n (1-p_{\hat{n}}) \prod_{n' \in \mathcal{M} \setminus \{n,\hat{n}\}} (1-\chi_{n'} p_{n'}), & n \neq \hat{n} \\ p_{\hat{n}} \prod_{n' \in \mathcal{M} \setminus \{\hat{n}\}} (1-\chi_{n'} p_{n'}), & n = \hat{n} \end{cases},$$

with $\overline{\lambda}_{\hat{n}} > 0$. Then we obtain the following result.

THEOREM 1. *From FRASA, $\overline{\mathcal{R}} = \bigcup_{\hat{n} \in \mathcal{M}} \overline{\mathcal{R}}_{\hat{n}}$ is the stability region, where $\overline{\mathcal{R}}_{\hat{n}}$ is represented by:*

$$\frac{\lambda_{\hat{n}}(1-p_{\hat{n}})}{p_{\hat{n}}} \geq \frac{\lambda_n(1-p_n)}{p_n} \geq 0, \forall n \in \mathcal{M} \setminus \{\hat{n}\}, \quad (1)$$

$$\prod_{n' \in \mathcal{M}} [\lambda_{\hat{n}}(1-p_{\hat{n}}) + \lambda_{n'} p_{\hat{n}}] < p_{\hat{n}} [\lambda_{\hat{n}}(1-p_{\hat{n}})]^{M-1}. \quad (2)$$

REMARK 1. *With some algebraic manipulations, it can be shown that the stability region of FRASA in Theorem 1 coincides exactly with that of slotted ALOHA when $M=2$ given by [1–3]. For $M>2$, our simulation results, e.g., Table 1, show that FRASA provides an excellent approximation.*

3. CONVEX BOUNDS ON THE STABILITY REGION OF FRASA

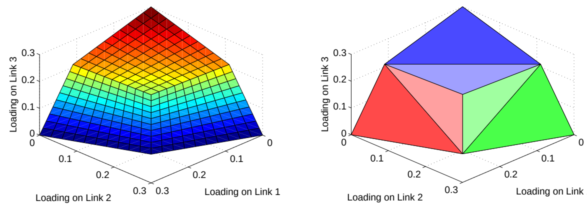
Though Theorem 1 gives the closed-form solution for the stability region of FRASA, it can be proved that such region cannot be convex and piecewise linear for $M>2$. Therefore, we develop bounds on the stability region of FRASA that are convex and piecewise linear to facilitate the application of FRASA to routing optimization of wireless random access networks.

First we use *corner points* of the stability region of FRASA to construct outer bounds. For each $\mathcal{M}' \subseteq \mathcal{M}$, we obtain a corner point $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')} = \left(\Pi_n^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')}\right)_{n \in \mathcal{M}}$, where

$$\Pi_n^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')} = \begin{cases} p_n \prod_{n' \in \mathcal{M}' \setminus \{n\}} \overline{p}_{n'}, & n \in \mathcal{M}' \\ 0, & n \in \mathcal{M} \setminus \mathcal{M}' \end{cases}.$$

We observe that for all $\hat{n} \in \mathcal{M}$, the boundary of the stability region of $\overline{\mathsf{S}}_{\hat{n}}$, *i.e.*, equality form of (2), is contained in the convex hull $\mathsf{H}_{\hat{n}}$ generated by the corner points $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}' \cup \{\hat{n}\})}$ for all $\mathcal{M}' \subseteq \mathcal{M} \setminus \{\hat{n}\}$. This helps us obtain the following two outer bounds on the stability region of FRASA.

THEOREM 2. (Bound of Convex-Hull Union). *$\mathcal{H}_{\hat{n}}$, the convex hull generated by $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}' \cup \{\hat{n}\})}$ for all $\mathcal{M}' \subseteq \mathcal{M} \setminus \{\hat{n}\}$ together with $\mathbf{0}$, i.e., the origin, is a piecewise linear outer bound on $\overline{\mathcal{R}}_{\hat{n}}$. Therefore, the union of $\mathcal{H}_{\hat{n}}$ for all $\hat{n} \in \mathcal{M}$, i.e., $\overline{\mathcal{H}} = \bigcup_{\hat{n} \in \mathcal{M}} \mathcal{H}_{\hat{n}}$, is a piecewise linear outer bound on the stability region of FRASA.*



(a) (b)

Figure 2: Illustrations of (a) the stability region of FRASA (Theorem 1) and (b) the corresponding convex and piecewise linear outer bound (Theorem 3) with $M=3$ and $p_1=p_2=p_3=0.3$.

THEOREM 3. (Convex Hull Bound). *$\mathcal{H}$, the convex hull generated by $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M}')}$ for all $\mathcal{M}' \subseteq \mathcal{M}$, is a convex and piecewise linear outer bound on the stability region of FRASA.*

REMARK 2. *While both bounds are piecewise linear, only the convex hull bound is guaranteed to be convex. In general, the bound of convex-hull union is tighter than the convex hull bound. However, it can be proved that the two bounds are identical if and only if $\sum_{n \in \mathcal{M}} p_n \leq 1$.*

Next, we give a convex and piecewise linear inner bound on the stability region of FRASA by using *supporting hyperplanes*. This inner bound is derived based on the observation about the convex hull $\mathsf{H}_{\hat{n}}$ for all $\hat{n} \in \mathcal{M}$ stated previously.

THEOREM 4. (Supporting Hyperplane Bound). *For each $\hat{n} \in \mathcal{M}$, we construct a supporting hyperplane $\mathsf{P}_{\hat{n}}$ which supports the convex hull $\mathsf{H}_{\hat{n}}$ at $\Pi^{\mathsf{p}^{\mathcal{M}}(\mathcal{M})}$ such that*

1. *it lies below $\mathsf{H}_{\hat{n}}$; and*
2. *it has positive intercepts on all coordinate axes.*

We let $\mathcal{S}_{\hat{n}}$ be the closed half space below $\mathsf{P}_{\hat{n}}$. Then the intersection of all these half spaces in the positive orthant, i.e., $\mathcal{S} = \bigcap_{\hat{n} \in \mathcal{M}} \mathcal{S}_{\hat{n}} \cap \{\boldsymbol{\lambda} \colon \lambda_n \geq 0, \forall n \in \mathcal{M}\}$, is a convex and piecewise linear inner bound on the stability region of FRASA.

For more information, refer to `http://mobitec.ie.cuhk.edu.hk/doc/frasa.pdf`.

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