$n$-way Mutual Exclusion

EECS 3/495 “Rust”

Spring 2017
Filter algorithm for $n$ threads

template <int N>
class Filter_lock : public Lock_base
{
    int level_[N] = {};  // class field
    int waiting_[N];

    bool exists_competition(int level)
    {
        for (auto k : thread::all_ids())
            if (k != i() && level_[k] >= level)
                return true;

        return false;
    }
};
template <int N>
class Filter_lock : public Lock_base
{
;
public:
    virtual void lock() override
    {
        for (int level = 1; level < N; ++level) {
            level_[i()] = level;
            waiting_[level] = i();
            while (exists_competition(level) &&
                   waiting_[level] == i()) {}
        }
    }

    virtual void unlock() override
    {
        level_[i()] = 0;
    }
}
Filter lock properties

- Mutual exclusion
  - By induction, one thread gets stuck in each level...
- Deadlock freedom
  - Like Peterson—one thread can wait per level
- Starvation freedom
  - Like Peterson—every thread advances if any does
Filter lock properties

- Mutual exclusion
  - By induction, one thread gets stuck in each level...
- Deadlock freedom
  - Like Peterson—only one thread can wait per level
- Starvation freedom
  - Like Peterson—every thread advances if any does
- Fairness?
  - No—threads can overtake others
Bounded waiting

Idea: If thread A “starts before” B, then A enters CS before B.
Bounded waiting

Idea: If thread A “starts before” B, then A enters CS before B. But what is “starts before”?
Bounded waiting

Divide \textit{lock()} operation into two intervals:

- Doorway (\(D_A\)), finite steps
- Waiting (\(W_A\)), possibly unbounded
Bounded waiting

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- Doorway \((D_A)\), finite steps
- Waiting \((W_A)\), possibly unbounded

\( r \)-Bounded Waiting Guarantee: If \( D^k_A \to D^j_B \), then \( CS^k_A \to CS^{j+r}_B \).
Bounded waiting

Divide lock() operation into two intervals:

- Doorway \((D_A)\), finite steps
- Waiting \((W_A)\), possibly unbounded

\(r\)-Bounded Waiting Guarantee: If \(D^k_A \rightarrow D^j_B\), then \(CS^k_A \rightarrow CS^{j+r}_B\).

“If A enters the doorway for the \(k\)th time before B enters it for the \(j\)th time, then A’s \(k\)th critical section happens before B’s \((j + r)\)th critical section.”
$r$-Bounded waiting

- Peterson’s Algorithm (for 2) has $r = 0$ (first-come-first-served)
- Filter algorithm (for $n$) has $r = \infty$
$r$-Bounded waiting

- Peterson’s Algorithm (for 2) has $r = 0$ (first-come-first-served)
- Filter algorithm (for $n$) has $r = \infty$
- Bakery algorithm (for $n$) has $r = 0$ (first-come-first-served)
Helper class: lexicographically-ordered pairs

template<typename A, typename B>
struct LP
{
    A x;
    B y;
};

template<typename A, typename B>
bool operator>=(const LP<A, B> & p, const LP<A, B> & q)
{
    return p.x > q.x || (p.x == q.x && p.y > q.y);
}
Bakery algorithm for \( n \) threads

template <int N>
class Bakery_lock : public Lock_base
{
    // Initialize flags and labels
    bool flag_[N] = {false};
    int label_[N] = {0};
    int max_label_ = 0;

    // Check if someone is ahead
    bool someone_is_ahead()
    {
        for (auto k : thread::all_ids())
            if (flag_[k] && LP{label_[i()], i()} > LP{label_[k], k})
                return true;

        return false;
    }

    // Additional code
    ...
}
template <int N>
class Bakery_lock : public Lock_base
{
  
public:
  
    virtual void lock() override
    {
        flag_[i()] = true;
        label_[i()] = ++max_label_; 
        while (someone_is_ahead()) {} 
    }

    virtual void unlock() override 
    { flag_[i()] = false; } 
}
Bakery Y2\textsuperscript{32}K bug

Does overflow matter?

<table>
<thead>
<tr>
<th>Bits</th>
<th>Does it?</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>quite</td>
</tr>
<tr>
<td>32</td>
<td>maybe</td>
</tr>
<tr>
<td>64</td>
<td>no</td>
</tr>
</tbody>
</table>
Bakery lock properties

- Mutual exclusion
  - Between any two (label[k], k) pairs, one will defer to the other…
- Deadlock freedom
  - Must be some least (label[k], k) pair
- Starvation freedom
  - No thread takes the same number twice
- First-come-first-served (0-bounded waiting)
  - First through the door has lower label, goes first
Bakery lock properties

- **Mutual exclusion**
  - Between any two \((\text{label}[k], k)\) pairs, one will defer to the other…

- **Deadlock freedom**
  - Must be some least \((\text{label}[k], k)\) pair

- **Starvation freedom**
  - No thread takes the same number twice

- **First-come-first-served (0-bounded waiting)**
  - First through the door has lower label, goes first

- **Practical?**
  - Have to readh \(n\) variables to lock
“Registers” (shared memory locations)

Flavors:

- Multi-reader/single-writer (flag[])
- Multi-reader/multi-writer (waiting)
- (Not that interesting: SRMW and SRSW)
Theorem

At least $n$ MRSW (multi-reader/single-writer) registers are needed to solve deadlock-free mutual exclusion.
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Proof sketch. For \( n = 2 \), one register is insufficient because neither thread necessarily sees the other’s write. Then by induction, the record of the first thread to enter always gets obliterated by the rest.
Summary

For deadlock-free mutual exclusion of $n$ threads:

- Best known algorithm uses $2n$ MRSW registers
- Lower bound for MRMW is $n$
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- Best known algorithm uses $2n$ MRSW registers
- Lower bound for MRMW is $n$

$O(n)$ reads is too inefficient—we need something better, and we’ll get it from the hardware
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