A (min-)priority queue provides these operations:

- **insert**: adds an element
- **remove_min**: removes the smallest element
Some implementation complexities

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>remove_min</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>unsorted list</td>
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<td>$\mathcal{O}(\log n)$</td>
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Introducing the binary heap

A *binary heap* is complete binary tree that is *heap-ordered*. A tree is heap-ordered if every element is *less than or equal* to its children.
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A tree is heap-ordered if every element is less than or equal to its children.

Which of these is a binary heap?:

```
      2
     / \
    5   97
   / \   / \
  40  7  99  5
```

```
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     / \
    5   97
   / \   / \
  40  7  99  5
```
Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant
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Binary heap removal

1. Replace the root with the last element of the heap
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The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
                  2
                /   \
               5     6
             /     /  \
           40    7   8
          /   \
        45    12  14
```

a binary heap is stored in level-order in an array:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
2 5 6 40 7 8 90 45 60 12 14 75
```
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```
Finding parents and children

Because the structure is *implicit*, we can’t just follow pointers.

Suppose $i$ is the index of a node:

- How can we find its parent (if any)?
- How can we find its children (if any)?
Next time: another graph algorithm and another data structure to go with it