The Binary Heap
Implementing a priority queue

A (min-)priority queue provides these operations:

- `insert`: adds an element
- `remove_min`: removes the smallest element
Some implementation complexities

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>remove_min</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>unsorted list</td>
<td>$O(1)$</td>
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<td>binary heap</td>
<td>$\mathcal{O}(\log n)$</td>
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Introducing the binary heap

A *binary heap* is complete binary tree that is *heap-ordered*

A tree is heap-ordered if every element is *less than or equal* to its children
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Which of these is a binary heap?:
Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant
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Binary heap removal

1. Replace the root with the last element of the heap
2. Sink down to restore invariant
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The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
2
5
  40
  45
  60
  7
  12
  14
  75
  90
```

a binary heap is stored in level-order in an array:

```
[2, 5, 6, 40, 7, 8, 90, 45, 60, 12, 14, 75, 16, 17, 18, 19, 20, 21, 22, 23]
```
The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

![Binary Heap Diagram]

a binary heap is stored in level-order in an array:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
2 5 6 40 7 8 90 45 60 12 14 75 4
```
The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
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  5
 / \
40 7
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 / \\ \
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```

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![Binary Heap Diagram]

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2 5 4 40 7 6 90 45 60 12 14 75 8
```
Finding parents and children

Because the structure is *implicit*, we can’t just follow pointers.

Suppose $i$ is the index of a node:

- How can we find its parent (if any)?
- How can we find its children (if any)?
Next time: another graph algorithm and another data structure to go with it