Implementing a priority queue

A (min-)priority queue provides these operations:

- **insert**: adds an element
- **remove_min**: removes the smallest element
Some implementation complexities

<table>
<thead>
<tr>
<th>List Type</th>
<th>Insert</th>
<th>Remove_min</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list</td>
<td>$\mathcal{O}(n)$</td>
<td>$\mathcal{O}(1)$</td>
</tr>
<tr>
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Introducing the binary heap

A *binary heap* is complete binary tree that is *heap-ordered*

A tree is heap-ordered if every element is *less than or equal* to its children
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Which of these is a binary heap?:
Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant
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Binary heap removal

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The super cool thing about binary heaps

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```
  2
   
  5   6
   
 40  7
   
45  60
```

a binary heap is stored in level-order in an array:

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
2 5 6 40 7 8 90 45 60 12 14 75 4
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Finding parents and children

Because the structure is *implicit*, we can’t just follow pointers.

Suppose $i$ is the index of a node:

- How can we find its parent (if any)?
- How can we find its children (if any)?
Next time: another graph algorithm and another data structure to go with it