The Binary Heap

EECS 214, Fall 2017
Implementing a priority queue

A (min-)priority queue provides these operations:

- insert: adds an element
- removeMin: removes the smallest element
Some implementation complexities

<table>
<thead>
<tr>
<th></th>
<th>insert</th>
<th>removeMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>sorted list</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>unsorted list</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
</tbody>
</table>

- Binary heap: $O(\log n)$
Some implementation complexities

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Introducing the binary heap

A *binary heap* is complete binary tree that is *heap-ordered*. A tree is heap-ordered if every element is *less than or equal* to its children.
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A tree is heap-ordered if every element is less than or equal to its children.

Which of these is a binary heap?:

```
2
/ \  \
5 97
/ \  \
40 7 99
```

```
2
/ \  \
5 97
/ \  \
40 7 99
```

```
5
/ \  \
2 97
/ \  \
40 7 99
```
Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant
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Diagram:
```
        5
       / \
      40   7
     / \ / \ / \
    45 60 12 14
```
Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant
Binary heap remove-min

1. Replace the root with the last element of the heap
2. Sink down to restore invariant
Binary heap remove-min

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The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
      2
     / \
    5   6
   / \
  40  7
 / \
45 60
```

a binary heap is stored in level-order in an array:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
2 5 6 40 7 8 90 45 60 12 14 75
```
The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
2
/\  
5 6
/\  
40 7
/\  
45 12 14
/\  
8
/\  
75 4
```

A binary heap is stored in level-order in an array:

```
2 5 6 40 7 8 90 45 60 12 14 75 4
```
The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
      2
     / \
    5   6
   / \ / \ \
  40 7 4 90 9
 / \ / \ / \ \
45 60 12 14 75 8
```

A binary heap is stored in level-order in an array:

```
[2, 5, 6, 40, 7, 4, 90, 45, 60, 12, 14, 75, 8, ...]
```
The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:

```
2
/  \
5   4
/  \
40  7
/  \
45  60 12 14
```

a binary heap is stored in level-order in an array:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
2 5 4 40 7 6 90 45 60 12 14 75 8
```
Finding parents and children

Because the structure is *implicit*, we can’t just follow pointers

Suppose $i$ is the index of a node:

- How can we find its parent (if any)?
- How can we find its children (if any)?
Next time: another graph algorithm and another data structure to go with it