Economics of Public Wi-Fi
Monetization and Advertising

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Public Wi-Fi is everywhere
Background

Venues: largest public Wi-Fi providers

- **Top 3:** Retailers > Cafes & Restaurants > Hotels

<table>
<thead>
<tr>
<th>Venues</th>
<th>Retails</th>
<th>Cafes &amp; Restaurants</th>
<th>Hotels</th>
<th>Others</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wi-Fi hotspots</td>
<td>5,763,907</td>
<td>4,259,351</td>
<td>397,905</td>
<td>1,808,234</td>
<td>12,229,397</td>
</tr>
</tbody>
</table>

Predicted Wi-Fi Ownership by Venue Type, 2018 ©WBA

- **Reasons to provide Wi-Fi**
  - Enhance customers’ experiences
  - Provide location-based services (e.g., navigation, billing, social interaction)

- **Question:** It is costly to deploy and operate the public Wi-Fi networks. How do venues generate revenue?
First Approach: Ad Sponsored Wi-Fi Access

**Users** watches an ad (e.g., 30sec) and then connect Wi-Fi for a certain period (e.g., 30min)

play sponsored video (advertisement) and connect
First Approach: Ad Sponsored Wi-Fi Access

- **Advertising platform** (e.g., SOCIFI) organizes a two-sided market between **venues** and **advertisers**
- **Example:**
  - Starbucks (**venue**) displays Apple’s (**advertiser**) ad to **users** in Wi-Fi;
    - User watches the ad, and uses Wi-Fi for 30min for free
  - Apple pays Starbucks based on the ad display times;
  - Starbucks shares 30% revenue with SOCIFI (**advertising platform**)
First Approach: Ad Sponsored Wi-Fi Access

Illustration of ad sponsored Wi-Fi access

one advertising platform

one venue

ad sponsored Wi-Fi access

% × $

$ 

one

multiple users

multiple advertisers
Second Approach: Premium Wi-Fi Access

Users directly pay the venue to use Wi-Fi

Look for Wi-Fi without ads?
pay $ based on connection time
Understand each decision maker’s **optimal** behavior

- **Advertising platform**: What is the ad revenue sharing proportion?
- **Venue**:
  - How much to charge **advertisers** for displaying ads?
  - How much to charge **users** for premium access?
- **Each advertiser**: How many ads to display at the **venue**?
- **Each user**: Which Wi-Fi access type to choose?
Ad revenue sharing ratio $\delta \in [0, 1]$: the fraction of the ad revenue the venue needs to transfer to advertising platform.
System Model: Venue

Two decision variables:

- **Wi-Fi price** $p_f$ (premium access): if a **user** chooses premium access, **venue** charges the **user** $p_f$ per session;

- **Advertising price** $p_a$ (ad sponsored access): if a **user** chooses ad sponsored access, **venue** charges the corresponding **advertisers** $p_a$ per displayed ad.

For example,

1 session = 30 min

this **user** demands 1h (=2 sessions)
System Model: Users

- Consider $N$ users, and each user’s type $\theta \sim U[0, \theta_{\text{max}}]$ describes its valuation for Wi-Fi access.

- A user’s access choice $d \in \{0, 1\}$:
  - $d = 0$ denotes the ad sponsored access;
  - $d = 1$ denotes the premium access.

- A type-$\theta$ user’s payoff in one session is:

$$
\Pi_{\text{user}}(\theta, d) = \begin{cases} 
\theta (1 - \beta), & \text{if } d = 0 \text{ (ad sponsored access)}, \\
\theta - p_f, & \text{if } d = 1 \text{ (premium access)},
\end{cases}
$$

where $\beta < 1$ captures the inconvenience of watching ads.

- The number of sessions that a user demands within the considered time period (e.g., one week) follows the Poisson distribution with parameter $\lambda > 0$. Parameter $\lambda$ describes users’ visiting frequency at the venue.
System Model: Advertisers

- Consider $M$ advertisers, and each advertiser’s type $\sigma \sim U[0, \sigma_{\text{max}}]$ describes its popularity (the popularity decreases with $\sigma$).
- An advertiser’s strategy $m \geq 0$: number of ads to display at the venue
- A type-$\sigma$ advertiser’s payoff

$$\Pi_{\text{advertiser}}(\sigma, m) = as(\sigma)N\phi_a(p_f) \left(1 - e^{-\frac{m}{N\phi_a(p_f)}}\right) - p_a m$$

- $a$: unit profit of showing the ad to a targeted user
- $s(\sigma)$: popularity of the advertiser (decrease with $\sigma$)
- $N\phi_a(p_f)$: number of users choosing the ad sponsored access
- $\left(1 - e^{-\frac{m}{N\phi_a(p_f)}}\right)$: probability for a user to see the advertiser’s ad (obtained via computation), and is concavely increasing in $m$
- $p_a$: advertising price (set by the venue)
Three-Stage Stackelberg Game

Solution: backward induction (Stage III $\rightarrow$ Stage II $\rightarrow$ Stage I)

<table>
<thead>
<tr>
<th>Stage I</th>
<th>Advertising platform specifies the sharing policy $\delta$.</th>
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<td>Stage II</td>
<td>Venue specifies Wi-Fi price $p_f$ and advertising price $p_a$.</td>
</tr>
<tr>
<td>Stage III</td>
<td>Each User with type $\theta$ makes access choice $d$; Each Advertiser with type $\sigma$ purchases $m$ ad spaces.</td>
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</table>
Stage III: Users’ Optimal Access Choices

- **Users' threshold policy:**
  - If $\theta < \theta_T(p_f)$, use ad sponsored access;
  - If $\theta \geq \theta_T(p_f)$, use premium access.

- Threshold $\theta_T(p_f)$ is non-decreasing in $p_f$
Stage III: Advertisers’ Optimal Advertising

Advertisers’ threshold policy:
If $0 \leq \sigma \leq \sigma_T(p_a)$, advertise, and number of ads decreases with $\sigma$;
If $\sigma_T(p_a) < \sigma \leq \sigma_{\text{max}}$, do not advertise

$m^*(\sigma, p_f, p_a)$: number of ads to display

linearly decrease with $\sigma$

do not advertise

$\sigma$: advertiser type
(small $\sigma$ = large popularity)
Stage III: Advertisers’ Optimal Advertising

If Wi-Fi price $p_f$ increases (more users choose the sponsored access),
- threshold $\sigma_T(p_a)$ does not change: the number of advertiser types that need to advertise does not change
- slope increases: the advertisers who originally advertise should display more ads

$m^*(\sigma, p_f, p_a)$: number of ads to display

if increase Wi-Fi price $p_f$
Stage II: Venue’s Optimal Advertising Price

Venue’s optimal advertising price $p_a^*$ (limit case $M \to \infty$ and $\sigma_{\text{max}} \to \infty$)

1. $p_a^*$ is independent of the advertising platform’s sharing ratio $\delta$;
2. $p_a^*$ decreases with $\lambda$ for small $\lambda$ region (reason: limited ad spaces);
3. $p_a^*$ is independent of $\lambda$ for large $\lambda$ region (reason: enough ad spaces).

$p_a^*$: venue’s advertising price

$\lambda$: users’ visiting frequency

convexly decrease with $\lambda$

independent of $\lambda$
Stage II: Venue’s Optimal Wi-Fi Price

Define indicator $\Omega \triangleq \frac{\lambda \beta \theta_{\text{max}}}{a g(\lambda, \gamma, \eta)}$

- Parameters’ meanings
  - $\lambda$: users’ visiting frequency
  - $\beta$: users’ payoff reduction due to watching ads
  - $\theta_{\text{max}}$: users’ maximum valuation on Wi-Fi access
  - $a$: unit profit for an advertiser of showing the ad to a targeted user
  - $\gamma$: the venue’s advertising concentration level
  - $\eta$: the expected number of advertisers that a user likes

- Intuition: a large $\Omega$ implies that the venue can earn more revenue by providing the premium access comparing to the ad sponsored access.
Stage II: Venue’s Optimal Wi-Fi Price

- **Indicator** $\Omega$: a large $\Omega$ implies that the venue can earn more revenue by providing premium access comparing to ad sponsored access.
- Wi-Fi price $p_f^*$ is non-increasing in $\Omega$.
Stage I: Advertising Platform’s Optimal Sharing Policy

- **Indicator $\Omega$:** a large $\Omega$ implies that the venue can earn more revenue by providing premium access comparing to ad sponsored access.
- **Sharing ratio $\delta^*$:**
  - first decreases with $\Omega$: attract venue to provide ad sponsored access;
  - second increases with $\Omega$: directly extract more ad revenue from venue.

\[ \delta^*: \text{advertising platform's sharing policy} \]

![Diagram showing the relationship between $\delta^*$ and $\Omega$]

more beneficial for venue to provide premium access
Conclusion and Future Work

- Public Wi-Fi monetization problem
  - Five threshold strategies for decision makers

- Future work
  - QoS differentiation (e.g., premium access with QoS guarantee)
  - Influence of Wi-Fi capacity
THANK YOU

Network Communications and Economics Lab
http://ncel.ie.cuhk.edu.hk
AD’s optimal advertising strategy

A type-$\sigma$ AD’s optimal advertising strategy is

$$m^*(\sigma, p_f, p_a) = \begin{cases} N\varphi_a(p_f)\left(\ln\left(\frac{a\gamma}{p_a}\right) - \gamma\sigma\right), & \text{if } 0 \leq \sigma \leq \sigma_T(p_a), \\ 0, & \text{if } \sigma_T(p_a) < \sigma \leq \sigma_{\text{max}}, \end{cases}$$

where $\sigma_T(p_a) \triangleq \min \left\{ \frac{1}{\gamma} \ln \left(\frac{a\gamma}{p_a}\right), \sigma_{\text{max}} \right\}$ is the threshold AD type.

(1) $\sigma_T(p_a)$ and $m^*(\sigma, p_f, p_a)$ decrease with $p_a$;
(2) $m^*(\sigma, p_f, p_a)$ decreases with type $\sigma$. 
Stage III: Advertisers’ Optimal Advertising

Threshold $\sigma_T(p_a)$ is non-increasing in advertising price $p_a$.