Homework 3

Instructions:

- This homework is due in class on **Monday, November 20th**.
- Each student may submit one homework assignment during the quarter up to five days late at no penalty. I will not accept additional late assignments except under extreme circumstances.
- You are welcome to work in small groups, but you must write up and submit your own assignment. You must also list the name of everyone you work with.
- The primary resources for this class are the lectures, textbook, lecture notes, and course Piazza page. Try to solve each question only using primary resources. If you use any other resource, you must cite it and explain why you needed it.

Problems:

Exercises 4.11, 7.1, and 7.10 in the textbook, and the problem below.

1. Read the introduction to Chapter 11 in the textbook which gives “Mixing Things” as an application of convex hulls, and which gives another equivalent definition of the convex hull of a set of points as the set of all convex combinations of the points.

Now consider the following generalization of the setting in the book. You are given \( n \) points \( p_1, \ldots, p_n \in \mathbb{R}^d \) (corresponding to \( n \) mixtures), a point \( q \in \mathbb{R}^d \) (corresponding to the the desired output mixture), and costs \( c_1, \ldots, c_n \in \mathbb{R}^+ \) (where \( c_i \) corresponds to the cost of a unit of mixture \( p_i \)).

Your goal is to find a convex combination \( \sum_{i=1}^n \lambda_i p_i \) of the \( p_i \)’s which is equal to \( q \) and which is of minimum cost.

E.g., in the book’s example, \( p_1 := (0.1, 0.35), p_2 := (0.16, 0.2), p_3 := (0.07, 0.15); q := (0.13, 0.22); \) and \( \lambda_1 := 1/5, \lambda_2 := 3/5, \lambda_3 := 1/5. \) (The book’s example just looks for a feasible solution, and does not consider its cost.)

Formulate this problem as a linear program.