EECS 395/495: Algorithmic Mechanism Design	Homework 3
Assigned: 11/17/11	**Due: 12/1/11**

1. Given a valuation profile $\mathbf{v}$ in sorted order, i.e., $v_1 \geq v_2 \geq \cdots \geq v_n$, and any (single-dimensional) downward-closed permutation environment, show that the envy-free revenue for $\mathbf{v}^{(2)} = (v_2, v_2, \ldots, v_n)$ and $\mathbf{v}_{-1} = (v_2, v_3, \ldots, v_n, 0)$ are within a factor of two of each other.

2. Consider the following single-agent prior-free pricing game. There is a value $v \in [1, h]$. If you offer a price $p \leq v$ you get p otherwise you get zero.
   (a) Design a randomized pricing strategy to minimize the ratio of the value to the revenue.
   (b) Prove that your randomized pricing strategy is optimal. Hint: use the lower-bounding technique for digital-goods auctions from class.
   (c) Discuss the connection between your above results and the claim from class that it is impossible for a digital-goods auction to approximate the envy-free benchmark $\text{EFO}(\mathbf{v}) = \max_i iv_{(i)}$.

3. Consider the design of prior-free incentive-compatible mechanisms with revenue that approximates the (optimal) social-surplus benchmark, i.e., $\text{OPT}(\mathbf{v})$, when all values are known to be in a bounded interval $[1, h]$.
   (a) For single-dimensional downward-closed environments, give a $\Theta(\log h)$-approximation mechanism. (Extra credit if your mechanism is revenue monotone, i.e., for any valuation profiles $\mathbf{v}$ and $\mathbf{v}'$ with $v_i \geq v'_i$ for all i, your expected revenue for $\mathbf{v}$ is at least that for $\mathbf{v}'$.)
   (b) For general (multi-dimensional) combinatorial auctions, i.e., m items, each agent i has a value $v_i(S') \in [1, h]$ for each subset $S' \subseteq S = \{1, \ldots, m\}$ of the m items, give a prior-free $\Theta(\log h)$-approximation mechanism.

4. Consider the design of prior-independent mechanisms for (multi-dimensional) unit-demand agents. Suppose there are n agents and n houses and agent i's value for house j is drawn independently from a regular distribution F_j. (I.e., the agents are i.i.d., but the houses are distinct.) Give a prior-independent mechanism that approximates the Bayesian optimal mechanism. What is your mechanism's approximation factor?