A Unified Framework for Statistical Timing Analysis with Coupling and Multiple Input Switching

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Abstract

As technology scales to smaller dimensions, increasing process variations, coupling induced delay variations and multiple input switching effects make timing verification extremely challenging. In this paper, we establish a theoretical framework for statistical timing analysis with coupling and multiple input switching. We prove the convergence of our proposed iterative approach and discuss implementation issues under the assumption of a Gaussian distribution for the parameters of variation. A statistical timer based on our proposed approach is developed and experimental results are presented for the ISCAS benchmarks. We juxtapose our timer with a single pass, non iterative statistical timer that does not consider the mutual dependence of coupling with timing and another statistical timer that handles coupling deterministically. Monte Carlo simulations reveal a distinct gain (up to 24%) in accuracy by our approach in comparison to the others mentioned.

1 Introduction

Variability in modern deep sub-micron VLSI circuits has not scaled down in proportion to the scaling down of their feature sizes. Manufacturing process variations (e.g. V_T, L_e), environmental variations (e.g. V_{dd}, Temperature), and device fatigue phenomenon contribute to uncertainties. These sources of variation are called parameters and the range in which they can collectively vary is called the the parameter space. Uncertainty due to parametric variations deeply impacts the timing characteristics of a circuit and makes timing verification extremely difficult. This necessitates the consideration of the parametric variations in timing analysis for accurate timing estimation. Statistical timing analysis has thus emerged in an attempt to capture the statistical behavior of circuit delays under parametric variations.

With the progress of deep sub-micron technologies, shrinking geometries have led to a reduction in the self-capacitance of wires. Meanwhile, coupling-capacitances have increased as wires have a larger aspect ratio and are brought closer together. For present day processes, the coupling-capacitance of a net can be as high as the sum of its area capacitance and fringing capacitance. Trends indicate that the role of coupling-capacitances will be even more dominant in the future as feature-sizes shrink [1]. Simultaneous switching on coupled nets in a circuit affects the delay on each net, and cause delay variations in the circuit. The delay variations are often significant (up to 40% stage delay error [2]) and should not be ignored. In addition, the mutual dependence of coupling effects and timing make timing analysis a chicken-and-egg problem. Iterative approaches are consequently employed in approaches to timing analysis with coupling. A situation of coupling is initially assumed (often the worst case situation) and the computed timing information is used to modify the coupling situation. This procedure is repeated until convergence.

The gate delay for multiple input CMOS gates often depends on the number of inputs switching at the same time. For example, turning on two transistors in parallel is faster than using only one as the path resistance is lower. The assumption of only single input switchings for timing analysis and consequently ignoring the effect of multiple input switchings (MIS) can result in significant errors (up to 20% stage delay error [3]). Although the probability of several multiple-input switchings adding up along a path in a circuit is small, delay variations due to the temporal proximity of switching inputs should be considered in accurate timing analysis.

The increasing significance of the above factors on timing accuracy motivates considering them in a unified timing analysis framework. Approaches to statistical timing analysis have gained wide acceptance recently [4–8]. Statistical timers treat delays as correlated random variables and propagate their distributions through the circuit. These approaches, however, do not discuss coupling induced delay variations or effects due to MIS. On the other hand, the majority of prevalent timing analysis approaches that consider coupling treat delays as deterministic values and do not discuss the impact of variability [2,9–14]. A statistical timing analysis algorithm that accounts for correlations and accommodates dominant interconnect coupling is proposed by Le *et al.* in [15]. While considering coupling in their proposed approach, they estimate switching window overlaps deterministically, based on worst case values. Our experiments conclude that such an approach is pessimistic and over estimates the switching windows at the outputs. A gate delay model that accounts for the effects of temporal proximity and input transitions is proposed in [16]. Another statistical gate delay model that considers MIS is proposed in [3]. Dartu *et al.* present results on the significance of the mentioned effects on timing accuracy and recommend increased focus on consideration of these effects in timing analysis for modern circuits [17].

In this paper, we establish a theoretical framework for statistical timing analysis with coupling and MIS. We prove the convergence of our proposed iterative approach and discuss implementation issues under the assumption of a Gaussian distribution for the parameters of variation. We develop a statistical timer that considers coupling based on our proposed approach and present results obtained on the ISCAS benchmarks [18]. We additionally develop the following two timers for comparison.

- A single pass statistical timer based on [8], which does not consider the mutual dependence of coupling and timing.
- An iterative statistical timer that considers coupling deterministically, based on [15].

Monte Carlo simulations reveal a distinct gain in accuracy (up to 24%) of our approach in comparison to the others.

The rest of the paper is organized as the following. Section 2 describes statistical timing analysis with coupling as a fixpoint computation problem on a lattice. Section 3 presents the generalized computation of coupling induced delay pushouts as random variables. We discuss the implications of the assumption of a Gaussian distribution for the parameters of variation and simplify the computation of the delay pushout under the given assumption in Section 4. We show that the effects of MIS are amicable to our framework in Section 5 and present experimental results and comparisons to other approaches in Section 6. Conclusions and future work are described in Section 7.

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2 Statistical timing as fixpoints

We consider a combinational circuit and select a set of interconnect wires where timing information needs to be computed. Symbolically, we use a variable x_i to denote the timing information on a wire i and X to represent the vector $(x_1, x_2, \ldots, x_n)$, that is the timing information of the whole circuit. Depending on the actual delay model, the timing information on a wire i depends only on a subset of other wires $i_1, i_2, \ldots, i_k$. These wires may include all inputs of the gate fanning out to i and the wires coupled to i. Mathematically,

$$x_i \;=\; t_i(x_{i_1}, x_{i_2}, \ldots, x_{i_k}),$$

where the function t_i is called the local timing transformation for i. Putting all local transformations together, we get a transformation T for the entire circuit, which can be written as

$$X = T(X) \tag{1}$$

A solution of timing analysis must be an X satisfying (1). Such a solution is also called a fixpoint of T. Though this formulation is amicable to timing analysis considering variability, coupling and MIS, we first consider timing analysis with variability and coupling. Coupling causes mutual dependence of the timing information on multiple wires, and thereby creates cycles during timing analysis. For a complex transformation T, an iterative method is perhaps the only possible way to find its fixpoint. First an initial solution X_0 is guessed, then new solutions are iteratively computed from previous solutions until we find a fixpoint, that is $X_n = T^n(X_0)$ such that $T(X_n) = X_n$.

2.1 Statistical timing with coupling

Considering functional information in timing analysis involves enumerating all input vectors and is consequently very expensive. As a result, static timing analysis does not use functional information of a circuit. Such a treatment makes timing information on each wire to be a set of possible signal switchings, instead of a single signal switching. The set is represented by a switching window and a set of slew rates such that any signal switching falling in the window and having a slew in the range is in the set. The switching window x_i on a wire i denotes an interval $[x_i^l, x_i^u]$, such that any actual signal switching x_i^* lies in the interval. Thus,

$$\{x_i^* : x_i^l \;\leq\; x_i^* \;\leq\; x_i^u\}$$

denotes the set of all possible signal switchings. The switching window representation is a worst case representation as it only gives bounds (best and worst case) on the elements of the set. It does not provide any information on the distribution of signal switchings within the window.

Statistical timing analysis considers the uncertainty due to the parameters of variation. Assumptions on the distribution of the parameters of variation and on the delay model as a function of these parameters provide a known distribution of signal switchings on every wire in the circuit.

Statistical timing with coupling involves uncertainties from both variability and ignorance of functional information. Although both uncertainties prohibit the solution to timing analysis on a wire being a single signal switching, we do not treat them in the same way. This is because we have distribution information on variability, but not on the functionality of the circuit. Consequently, we cannot obtain the solution to statistical timing analysis with coupling as a known distribution of signal switchings on each wire of the circuit.

The solution to statistical timing analysis on any wire i is a distribution obtained from a statistical sampling of the timing information on i at various corners in the parameter space. For any corner in the parameter space, that is, a given assignment of parametric values, the timing information on i is in the form of a switching window. Consequently, the solution to statistical timing analysis with coupling is a **distribution of switching windows** on each wire of the circuit. This view of the solution is obtained by first considering the uncertainty from ignorance of functional information and then the uncertainty from variability.

Each window in the above view of the solution is denoted by a best and a worst case value. Consequently, the distribution of the windows contains the distributions of these best and worst case values. The two distributions thus obtained, are represented as the distributions of two correlated random variables respectively. The window formed using these random variables as the best and worst case value respectively, contains all possible signal switching distributions in the solution. This transformation of the original view of the solution gives an alternate view of the solution to statistical timing analysis with coupling as a **window of signal switching distributions**.

The latter view of the solution is also obtained by considering the uncertainty from variability first. Temporarily assuming that functional information and input configuration of each gate is known, statistical timing analysis on a wire i computes a distribution of a single signal switching. However, functional information and input configuration is not considered in reality. The timing information on i is therefore a set of signal switching distributions, which is represented by a window of signal switching distributions.

2.2 Statistical switching windows

Each view of the solution to statistical timing analysis contains the exact same information. We consider the solution as a window of signal switching distributions, since it allows us to leverage the theoretical foundations of timing analysis with coupling as a fixpoint computation developed in [14]. We cannot use traditional switching windows in our formulation as they represent a set of deterministic quantities, while our set consists of signal switchings as random variables with known distributions.

We introduce a *statistical switching window* as a representation for a set of random variables. For any random variable x_i^* in the set, we consider providing a lower and an upper bound on the probability that x_i^* is not less (or more) than a given real value c. The statistical switching window extends the bounds over the entire range of c, in the form of two distributions of **correlated** random variables x_i^l and x_i^u respectively. We call the two random variables the bounding Random Variables (or *bounding RVs*) of the statistical switching window x_i. Mathematically, the statistical switching window x_i is defined as the following.

$$x_i = [x_i^l, x_i^u] \triangleq \tag{2}$$
$$\{x_i^* \mid \forall c \in \Re : \; Pr(x_i^l \leq c) \geq Pr(x_i^* \leq c) \geq Pr(x_i^u \leq c)\},$$

where $Pr(k)$ denotes the probability of an event k. We observe that the cumulative distribution functions (CDFs) of x_i^l and x_i^u provide a statistical interval for the CDF of the uncertain actual timing variable x_i^*. This implies that two random variables with known CDFs can represent a statistical switching window and is illustrated graphically in Figure 1(a). In the rest of this paper, we consider the solution to statistical timing analysis with coupling on a wire i as a statistical switching window x_i of signal switching distributions.

2.3 Structure of fixpoints

We consider the family of all sets of signal switching distributions on a wire. The inclusion relation ($\subseteq$) forms a partial order on the family as it satisfies the properties of reflexivity, transitivity, and antisymmetry.

The inclusion relation between two statistical switching windows x_i and x_j is formally defined as the following and is graphically illustrated in Figure 1(b).

$$\begin{aligned} x_i &\subseteq x_j \\ &\triangleq \forall c \in \Re : \ \left(Pr(x_i^l \le c) \le Pr(x_j^l \le c)\right) \ \wedge \\ &\qquad \left(Pr(x_i^u \le c) \ge Pr(x_j^u \le c)\right) \end{aligned} \tag{3}$$

The sets of signal switching distributions on a wire are actually subsets of the whole set that consists of all possible signal switching distributions. According to lattice theory [19], a partially ordered set forms a *complete lattice* if any subset has a least upper bound and a greatest lower bound of its members. The family of all subsets of a given set with inclusion relation forms a complete lattice. The partial order on each wire can be extended point-wise to get a partial order on vectors. We say that timing information vectors $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_n)$ satisfy $X \subseteq Y$ if and only if $x_i \subseteq y_i$ for all $1 \le i \le n$. It can be shown that the vectors with such a partial order also form a complete lattice.

We now consider the transformation T on a vector of sets of signal switching distributions. We say that T is a *monotonic* transformation when, for any two vectors X and Y, if $X \subseteq Y$, then $T(X) \subseteq T(Y)$. The monotonicity of T is proved based on the monotonicity of its member transformations $t_1, t_2, \dots, t_n$. If a transformation t_i is not monotonic, it implies less possible switching distributions are produced under the condition of the same or more possible switching distributions on the fanin and coupling wires. This contradicts with the causality of physical effects. The existence of a fixpoint of the transformation T is now guaranteed by Knaster-Tarski Theorem [19].

2.4 Optimal fixpoint

It is known that if fixpoints of a monotonic transformation are found by the iterative method starting from the bottom element of the lattice, it must be the least fixpoint [14]. It is also known that the fixpoints of a monotonic transformation form a complete lattice. The least fixpoint is therefore the intersection of all the fixpoints. In terms of statistical switching windows, a fixpoint with the smallest window will result from an initial assumption that the probability that any two windows overlap with each other is zero. An initial solution that satisfies this condition will subsequently yield the optimal fixpoint, that is the vector of tightest switching windows on the real solution or the one with minimal uncertainty.

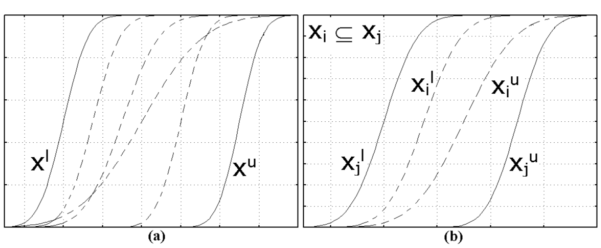

Figure 1: (a) A statistical switching window for a set of distributions (b) Inclusion relation between two statistical switching windows

3 Coupling induced delay pushouts as random variables

3.1 Overlap between statistical switching windows

The overlap between two statistical switching windows is not a deterministic quantity. For two statistical switching windows x_i and x_j, we denote the overlap between the windows by a random variable O_{ij}, which is defined as the following.

$$O_{ij} \ \triangleq \ min(x_i^u, x_j^u) \ - \ max(x_i^l, x_j^l) \tag{4}$$

We consider a probabilistic view of the overlap O_{ij}. If we denote the CDF of O_{ij} by $\Psi_{ij}(t)$, the probability of an overlap between x_i and x_j is given by

$$Pr(O_{ij} > 0) \ = \ 1 - \Psi_{ij}(0) \tag{5}$$

3.2 Coupling induced delay pushout computation

Simultaneous switching on a pair of coupled nets cause delay variations on the nets. The amount of additional delay (positive or negative) induced is called the *coupling induced delay pushout* and is a function of the overlap between the switching windows on the coupled nets. Since the overlap between two statistical switching windows is a random variable, the coupling induced delay pushout is consequently a random variable, even if the delay pushouts for a deterministic overlap are constants. The delay pushout widens the switching window on a wire, that is, if x_i and x_i' denote the switching windows on a wire i when effects due to coupling are not considered and are considered respectively, we have the following relation.

$$x_i \ \subseteq \ x_i' \tag{6}$$

Without any loss of generality, we consider the computation of the worst case delay on a wire i when coupling effects are considered. The worst case delay on i is given by the sum of the delay on the wire when coupling effects are not considered and the coupling induced delay pushout due to each wire it couples with.

As an example, we illustrate the computation of a coupling induced delay pushout D as a random variable based on a simple coupling model, which is given by the following.

$$D \triangleq \begin{cases} D^O & \text{overlap} \\ D^N & \text{no overlap} \end{cases} \tag{7}$$

where D^O and D^N are the values assigned to D, depending on whether the statistical switching windows between the coupled wires overlap or not respectively. We denote the overlap as a random variable O.

Using principal component analysis, we express all random variables as a weighted sum of independent and orthonormal random variables ξ_i, $i = 1, 2, \dots, n$, such that $E[\xi_i \xi_j] = 0$ if $i \ne j$, and 1 otherwise. The delay pushouts D^O and D^N are random variables and are expressed as the following.

$$\begin{aligned} D^O &= d_0^O + \sum_{i=1}^{n} d_i^O \xi_i \qquad (8) \\ D^N &= d_0^N + \sum_{i=1}^{n} d_i^N \xi_i \\ \Rightarrow D &= d_0 + \sum_{i=1}^{n} d_i \xi_i \triangleq \begin{cases} d_0^O + \sum_{i=0}^{n} d_i^O \xi_i & \text{overlap} \\ d_0^N + \sum_{i=0}^{n} d_i^N \xi_i & \text{no overlap} \end{cases} \end{aligned}$$

We define a new set of random variables $\mathbf{d_i}$s, $i = 0, 1, \dots, n$

as the following.

$$\mathbf{d_i} \triangleq \begin{cases} d_i^O & \text{overlap} \\ d_i^N & \text{no overlap} \end{cases} \quad (9)$$

We therefore have the following.

$$d_0 + \sum_{i=1}^{n} d_i \xi_i \;=\; \mathbf{d_0} + \sum_{i=1}^{n} \mathbf{d_i} \xi_i \quad (10)$$

We multiply (10) by a new independent and orthogonal random variable ξ_{n+1} and take the expected value ($E[.]$) on both sides. We consider the orthogonality property of ξ_i, and that the overlap is a function of ξ_i, $i = 1, \ldots, n$ but independent of ξ_{n+1}. Using conditional expectation, we obtain the following results.

$$\begin{aligned} d_0 &= \frac{1}{E[\xi_{n+1}]}\big(E[\mathbf{d_0}\xi_{n+1}] + \textstyle\sum_{i=1}^{n} E[\mathbf{d_i}\xi_i\xi_{n+1}]\big) \\ E[\mathbf{d_0}\xi_{n+1}] &= d_0^O E[\xi_{n+1}|\text{ overlap}]Pr(\text{overlap}) \quad (11) \\ &\quad + d_0^N E[\xi_{n+1}|\text{ no overlap}]Pr(\text{no overlap}) \\ &= d_0^O E[\xi_{n+1}|\; O > 0]Pr(O > 0) \\ &\quad + d_0^N E[\xi_{n+1}|\; O \le 0]Pr(O \le 0) \\ &= \big\{d_0^O Pr(O > 0) \;+\; d_0^N Pr(O \le 0)\big\}E[\xi_{n+1}] \\ \Rightarrow\; d_0 &= d_0^O Pr(O > 0) \;+\; d_0^N Pr(O \le 0) \\ &\quad + \frac{1}{E[\xi_{n+1}]}\textstyle\sum_{i=1}^{n} E[\mathbf{d_i}\xi_i\xi_{n+1}] \quad (12) \\ E[\mathbf{d_i}\xi_i\xi_{n+1}] &= d_i^O E[\xi_i\xi_{n+1}|\; O > 0]Pr(O > 0) \\ &\quad + d_i^N E[\xi_i\xi_{n+1}|\; O \le 0]Pr(O \le 0) \quad (13) \end{aligned}$$

We multiply (10) again by ξ_k for $k = 1, 2, \ldots, n$ and take the expected value on both sides to obtain the following.

$$\begin{aligned} & d_k \quad (14) \\ &= \frac{1}{E[\xi_k^2]}\big(E[\mathbf{d_0}\xi_k] + \textstyle\sum_{i=1}^{n} E[\mathbf{d_i}\xi_i\xi_k] - d_0 E[\xi_k]\big) \\ & E[\mathbf{d_i}\xi_i\xi_k] \quad (15) \\ &= d_i^O E[\xi_i\xi_k|\; O > 0]Pr(O > 0) \\ &\quad + d_i^N E[\xi_i\xi_k|\; O \le 0]Pr(O \le 0) \\ & E[\mathbf{d_0}\xi_k] \quad (16) \\ &= d_0^O E[\xi_k|\; O > 0]Pr(O > 0) \\ &\quad + d_0^N E[\xi_k|\; O \le 0]Pr(O \le 0) \end{aligned}$$

The above illustration of the computation of the delay pushout is extensible to an arbitrary coupling model and does not imply that the proposed approach is limited to a given coupling model. The model can be trivially extended to having M delay pushouts for given ranges of an overlap value and expressed as the following.

$$D \triangleq \begin{cases} D^N & \text{no overlap} \\ D_i^O & \text{overlap} \in (O_i, O_{i+1}],\ \forall i = 1, 2, \ldots, M-1 \\ D_M^O & \text{overlap} > O_M \end{cases}$$

Under such a model, the above equations are modified to obtain the desired coefficients d_is. The conditional expectations are now evaluated for each of the possible overlap intervals. Even for a continuous coupling model (for example, the delay pushout as a exponential function of overlap), a similar approach is used to compute the desired coefficients. The conditional expectation computations in this case involves the PDF of the overlap instead of probability that the overlap lies in some interval. We do not discuss details of such models as our work is model independent and guarantees convergence under the monotonicity properties of the timing transformation.

4 Practical considerations under a Gaussian assumption

4.1 Gaussian statistical switching windows

The parameters of variation in statistical timing are often assumed to be random variables having a Gaussian distribution. Gate delays are expressed as a linear weighted sum of these parameters and thereby have a Gaussian distribution too. We consider the following definition.

$$\Phi(y) \;\triangleq\; \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} exp(-\frac{x^2}{2})\; dx \quad (17)$$

$\Phi(y)$ denotes the CDF of a unit normal Gaussian. The CDF of an arbitrary Gaussian with mean μ and variance σ^2 can be shown to be $\Phi(\frac{y-\mu}{\sigma})$.

The assumption of a Gaussian distribution for the bounding RVs when used in describing statistical switching windows prohibits the generic use of the $\subseteq$ relation between the windows. We discuss the cause of this problem and propose a remedy in this section.

Definition 1 *We say that a random variable X strictly dominates another random variable Y if and only if the CDF of X never lies to the left of the CDF of Y, that is*

$$\forall c \in \Re : Pr(X \le c) \le Pr(Y \le c)$$

This dominance relationship between two random variables is denoted as the upper bound on a CDF in [5].

Theorem 1 *The CDFs of two arbitrary Gaussians with non-equal variances always intersect at exactly one point.*

Proof: We consider two arbitrary Gaussians having means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively. The point(s) of intersection of the CDFs is(are) found by equating the CDFs and solving for c. We thus solve for c in

$$\Phi(\frac{c-\mu_1}{\sigma_1}) \;=\; \Phi(\frac{c-\mu_2}{\sigma_2}) \quad (18)$$

It is known that $\Phi(y)$ is strictly increasing with y, that is $\Phi(x) > \Phi(y)$ if and only if $x > y$. The above equation is therefore equivalent to solving the following.

$$\frac{c-\mu_1}{\sigma_1} \;=\; \frac{c-\mu_2}{\sigma_2} \quad (19)$$

(19) is a linear equation in c and has a single root at $c^* = \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}$, given $\sigma_1 \neq \sigma_2$. Since both the CDFs are monotonically non-decreasing, c^* denotes the single point of intersection of the CDFs. ∎

Corollary 1.1 *For two arbitrary Gaussians with equal variance, the CDF of the Gaussian with a larger mean strictly dominates the CDF of the other.*

Theorem 2 *An arbitrary Gaussian can never strictly dominate another Gaussian, given that their variances are non-identical.*

Proof: For two arbitrary Gaussians having means μ_1, μ_2 and variances σ_1^2, σ_2^2 respectively, we assume without any loss of generality that $\sigma_2 > \sigma_1$. Based on Theorem 1 and given the monotonicity of the CDFs, the following results are immediate.

$$\Phi(\frac{c-\mu_1}{\sigma_1}) \geq \Phi(\frac{c-\mu_2}{\sigma_2}) \quad \forall c \in [\frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}, \infty) \quad (20)$$

$$\Phi(\frac{c-\mu_1}{\sigma_1}) \leq \Phi(\frac{c-\mu_2}{\sigma_2}) \quad \forall c \in (-\infty, \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}] \quad (21)$$

Thus, neither of the Gaussians strictly dominate the other. □

Corollary 2.1 *If the ratio of the means of two arbitrary Gaussians is equal to the ratio of their variances, the CDF of the Gaussian with the smaller mean (or variance) strictly dominates the other in the interval $c \in [0, \infty)$.*

For two statistical switching windows x_i and y_i, $x_i \subseteq y_i$ implies that x_i^l strictly dominates y_i^l and y_i^u strictly dominates x_i^u.

Definition 2 *A statistical switching window x_i is said to be a Gaussian switching window if and only if both x_i^l and x_i^u have a Gaussian distribution.*

Corollary 2.2 *An arbitrary Gaussian switching window x_i cannot have an inclusion relation to another Gaussian switching window x_j, unless the variances of x_i^l and x_j^l are identical, and the variances of x_i^u and x_j^u are identical.*

Consequently, we cannot use Gaussian switching windows directly in our framework for statistical timing analysis with coupling. The Gaussian distribution spans the entire range of real numbers and the point of intersection of the CDFs of two Gaussian may lie far away from their mean values (or the region of interest). In addition, realistic parametric variations have a distribution constrained in a finite region. We therefore consider a truncated Gaussian distribution for the bounding RVs in our approach. The CDF of a truncated Gaussian with mean μ and variance σ^2 as a function of c is assumed to be 0 for all $c \leq \mu - k\sigma$ and is assumed to be 1 for all $c \geq \mu + k\sigma$, for a given k (typically $k \in [3, 7]$). Arithmetic operations on Gaussians are used identically for truncated Gaussians, although they involve approximation. Truncated Gaussian distributions have earlier been assumed for parametric distributions in [3, 20].

Theorem 3 *An arbitrary truncated Gaussian with mean μ_1 and variance σ_1^2 can strictly dominate another truncated Gaussian with mean μ_2 and variance σ_2^2 under the following condition.*

$$\Big(\ max(\mu_1 + k\sigma_1, \mu_2 + k\sigma_2) \ \leq \ \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}\ \Big) \ \bigvee$$
$$\Big(\ min(\mu_1 - k\sigma_1, \mu_2 - k\sigma_2) \ \geq \ \frac{\sigma_2\mu_1 - \sigma_1\mu_2}{\sigma_2 - \sigma_1}\ \Big)$$

The above condition implies that a truncated Gaussian can dominate another if their point of intersection lies outside the $[\mu - k\sigma, \mu + k\sigma]$ range for both the truncated Gaussians. Consequently, we use truncated Gaussian distributions for x_i^l and x_i^u while representing a statistical switching window x_i. We denote such a statistical switching window as a truncated Gaussian switching window. A truncated Gaussian switching window x_i can have an inclusion relation with another truncated Gaussian switching window x_j if both pairs of truncated Gaussians x_i^l, x_j^l and x_i^u, x_j^u satisfy the condition mentioned in Theorem 3 individually. Truncated Gaussian switching windows under mentioned conditions can therefore be used to denote statistical switching windows for statistical timing analysis with coupling. The timing transformation T must ensure this condition for monotonicity and convergence of our approach. It is easy to ensure the condition by adjusting the means (or variances) of the statistical switching window bounds pessimistically. This is done by rewriting the condition in Theorem 3 as inequality conditions on μ_2 (or σ_2). In the rest of the paper, a Gaussian distribution implies a truncated Gaussian distribution, and a Gaussian switching window implies a truncated Gaussian switching window.

4.2 Coupling induced delay pushout as a Gaussian

We now consider the implications of the assumption of a unit normal distribution for the independent random variables $\xi_1, \ldots, \xi_n$, and Gaussian switching windows in evaluating the delay pushout D from its definition in (8). Using the *add (sub)* and *max (min)* operations from [7, 8], the overlap O_{ab} between two Gaussian switching windows x_a and x_b is approximated to a Gaussian with mean μ and variance σ^2. From the definition of the probability of an overlap between statistical switching windows in (5), we have the following. [1]

$$Pr(O_{ab} > 0) \ = \ 1 - \Phi(\frac{0-\mu}{\sigma}) \ = \ \Phi(\frac{\mu}{\sigma}) \quad (22)$$

Consequently, the probability of no overlap between the Gaussian switching windows is given by the following.

$$Pr(O_{ab} \leq 0) \ = \ \Phi(-\frac{\mu}{\sigma}) \quad (23)$$

Evaluating the coefficients $d_1, \ldots, d_n$ of the delay pushout D in (14) is non trivial. The conditional expectations and consequently the coefficients d_ks are functions of $\xi_1, \ldots \xi_n$, and not constants. D is therefore not a linear weighted sum of the ξ_is in reality.

For simplicity, we attain to approximate D as a linear weighted sum of the ξ_is. One approach to achieving this is by approximating each random variable d_k to its mean value. This approach is employed by Wu *et al.* in dynamic range estimations of multiplexing operations in non-linear systems using Polynomial Chaos Expansion [21]. Monte Carlo simulations are used to generate samples of the random variables and are classified as to whether they produce the condition in the conditional expectation or not. The probabilities of the two outcomes are thus computed, and within each value, the mean of the expectation is computed in the usual Monte Carlo fashion. However, Monte Carlo simulations are expensive and not suitable for statistical timing analysis.

From the theory of conditional expectation [22], we know the following for any random variables X and Y.

$$E[E[X|\ Y]] = E[X]$$

Consequently, the mean or the expectation of the conditional expectations in (14) are expressed as the following.

$$E\big[E[\xi_k|\ O > 0]\big] = \ E\big[E[\xi_k|\ O \leq 0]\big] \ = E[\xi_k] \quad (24)$$
$$E\big[E[\xi_i\xi_k|\ O > 0]\big] = \ E\big[E[\xi_i\xi_k|\ O \leq 0]\big] \ = E[\xi_i\xi_k]$$

For $i \neq j$, it is known that $E[\xi_i] = E[\xi_i\xi_j] = 0$ and $E[\xi_i^2] = 1$. The coefficients $d_0, \ldots, d_n$ in the representation of the effective coupling capacitance D from (12) and (14), are consequently simplified to the following.

$$d_i \ = \ d_i^O\Phi(\frac{\mu}{\sigma}) + d_i^N\Phi(-\frac{\mu}{\sigma}) \quad i = 0, 1, \ldots, n \quad (25)$$

[1] It is known that $\Phi(-x) = 1 - \Phi(x)$.

Table 1: % Errors in switching window estimation with reference to Monte Carlo simulations

Circuit	Nodes	% Ξ_l			% Ξ_u			% Ξ_{Av}			# I/N	
		SSTA	STAC	STAC*	SSTA	STAC	STAC*	SSTA	STAC	STAC*	STAC	STAC*
C432	198	3.30	0.00	0.00	-9.2	0.4	-0.01	6.2	0.2	0.007	9.1	9.1
C491	245	0.00	0.00	0.00	-1.3	4.7	0.13	0.7	2.4	0.067	6.6	5.7
C880	445	0.01	-3.90	-0.01	-13.0	13.0	0.01	6.3	8.3	0.010	10.8	8.8
C1355	589	0.01	0.01	0.01	-21.0	2.5	0.01	10.0	1.2	0.007	14.8	14.4
C1908	915	6.00	-5.10	-0.01	-23.0	1.6	0.70	14.0	3.4	0.350	14.7	13.9
C2670	1428	0.00	0.00	0.00	-24.0	7.9	0.01	12.0	4.0	0.004	10.3	9.2
C3540	1721	9.60	-2.30	-0.01	-17.0	8.9	0.10	13.0	5.6	0.056	14.9	12.3
C6288	2450	0.01	0.01	0.01	-22.0	0.0	0.00	11.0	0.0	0.005	50.3	50.9
C7552	3721	0.00	0.00	0.00	-7.0	0.6	0.02	3.5	0.3	0.009	12.4	11.4

An alternate approach to reducing D to a weighted linear sum of random variables is by introducing new random variables that approximate the product of the ξ_is and true d_ks (as a function of the ξ_is). Fang *et al.* employ this approach for approximating the product of two *affine* terms into another affine term in static analysis for fixed-point finite-precision effects in DSP designs [23].

5 Statistical timing with multiple input switching

Gate delay models in static timing analysis that consider MIS compute gate delays as functions of the overlap between the switching windows on the fanin signals of a gate. We can consider a generic MIS gate delay model similar to (7), where *overlap* denotes the overlap between the statistical switching windows on the fanins of a gate and is a random variable. Therefore, given a delay model that considers MIS, we can compute its effects in statistical switching windows. Though analogous to the coupling, MIS does not necessitate iterative timing analysis, and statistical timing with MIS is possible in a single pass of a topologically ordered circuit.

We therefore conclude that our framework for statistical timing with coupling is amicable to incorporating the effects of MIS. Our reason for this conclusion is based on the assumption that the MIS effect does not change causality and consequently the timing transformation T is monotonic under effects due to MIS. This implies convergence to a fixpoint in our approach.

6 Experimental results

In this section, we present statistical timing analysis results for the ISCAS benchmarks, realistic parameters for which are generated from a 0.13μ technology library. Two global sources of variation having a truncated Gaussian distribution (at $\mu \pm 4\sigma$) are considered in addition to a local independent component. Cumulative parametric variations are constrained within 10% of the mean value. We use a simple delay model [20] and a delay pushout model (8) for our experiments. Arrival time at all primary inputs are set to 0. For comparisons, we develop the following timing analysis approaches.

1. *SSTA :* denotes an implementation of a statistical timer based on [8], which does not consider coupling induced delay variation, that is, the delay pushout due to coupling on each wire is assume to be D^N (7).

2. *STAC :* denotes an implementation of a statistical timer based on the above approach that considers coupling induced delay pushouts assuming a deterministic overlap. In this approach, two statistical switching windows x_i ($[x_i^l, x_i^u]$) and x_j ($[x_j^l, x_j^u]$) are treated as static switching windows given by $[\mu_{x_i^l} - 3\sigma_{x_i^l}, \mu_{x_i^u} + 3\sigma_{x_i^u}]$ and $[\mu_{x_j^l} - 3\sigma_{x_j^l}, \mu_{x_j^u} + 3\sigma_{x_j^u}]$ respectively to estimate the deterministic overlap between them. The computed overlap determines the delay pushout. This approach is based on [15].

3. *STAC* :* denotes an implementation of a statistical timer based on our proposed approach. It considers a probabilistic approach to the overlap between two statistical switching windows, and the delay pushout due to coupling is computed as proposed in Section 3.

4. *Monte Carlo (MC) :* simulations are performed for accuracy comparisons. A static timer is implemented that considers coupling induced delay pushouts and is run for 20000 corners in the parameter space to capture the statistical behavior of the switching window at all wires on a given circuit.

We compute the statistical switching window x at the primary output of a given circuit using each of the mentioned approaches. Experimental results show that *SSTA* underestimates the switching window as compared to *MC*. *STAC* is found to give a conservative estimate, while our proposed approach *STAC** accurately estimates the statistical switching window. To give a numerical report, the $(\mu_{x^l} - 3\sigma_{x^l})$ and the $(\mu_{x^u} + 3\sigma_{x^u})$ points obtained from the mentioned approaches are compared. We use the points obtained using *MC* simulations as our reference for accuracy estimations and formally define the error in any approach A as follows.

$$\% \ \Xi_l^A \triangleq \frac{(\mu_{x_A^l} - 3\sigma_{x_A^l}) - (\mu_{x_{MC}^l} - 3\sigma_{x_{MC}^l})}{(\mu_{x_{MC}^l} - 3\sigma_{x_{MC}^l})} \times 100 \quad (26)$$

$$\% \ \Xi_u^A \triangleq \frac{(\mu_{x_A^u} + 3\sigma_{x_A^u}) - (\mu_{x_{MC}^u} + 3\sigma_{x_{MC}^u})}{(\mu_{x_{MC}^u} + 3\sigma_{x_{MC}^u})} \times 100 \quad (27)$$

$$\% \ \Xi_{Av}^A \triangleq \frac{|\%\Xi_l^A| + |\%\Xi_u^A|}{2} \quad (28)$$

Timing analysis accuracy results based on the above metrics are presented in Table 1. *STAC* and *STAC** are iterative approaches and compute timing information on some wires multiple times till convergence. We report the average number of Iterations per Node (*#I/N*) obtained in these approaches. Since SSTA does not consider coupling induced delay pushouts, it is non iterative and is a single pass timer. Consequently, its *#I/N* value is always 1. The average number of iterations per node is found to be 16 and 15 for *STAC* and *STAC** respectively. We present run times for all approaches in Table 2. *STAC** is found to be slower by $1.2X$ in comparison to *STAC*.

It is observed that STAC* improves timing estimation accuracy by as much as 24% in comparison to *SSTA* and up to 13% in comparison to *STAC*. On the average, errors in switching window estimation are reduced by 8.5% in comparison to *SSTA* and by 2.8% in comparison to *STAC*.

Table 2: # Run time comparison for the timers

Circuit	Run Times(s)			
	SSTA	STAC	STAC*	Monte Carlo
C432	0.00	0.01	0.01	21.4
C491	0.00	0.01	0.01	36.8
C880	0.00	0.03	0.03	184.1
C1355	0.00	0.10	0.15	585.7
C1908	0.00	0.20	0.26	1018.0
C2670	0.01	0.25	0.34	1269.0
C3540	0.02	0.46	0.58	2053.0
C6288	0.02	2.35	2.90	11960.4
C7552	0.03	0.87	1.11	4447.0

Results shown are for experiments performed on a Pentium 2.4GHz machine, with 1GB RAM, running Red Hat Linux 9.0.

7 Conclusions and future work

In this paper, we establish a theoretical framework for statistical timing analysis with coupling and multiple input switching. We prove the convergence of our proposed iterative approach and discuss implementation issues under the assumption of a Gaussian distribution for the parameters of variation. The framework is not constrained to Gaussian distributions for the parameters of variation, but an arbitrary distribution could have a complicated *max* operation and estimation of the conditional expectations would be an important consideration.

We do not discuss approaches to reducing iterations (or speeding up approaches) for our timer. Clustering the circuit into strongly connected components and then topologically performing timing analysis with coupling is suggested in [14]. Breaking up feedback edges in the directed graph representation of the circuit to form clusters is suggested. The feedback edges due to coupling are broken based on a metric of timing proximity. In statistical timing with coupling, we propose to use the probability of an overlap between two windows as the metric for timing proximity, that is, edges having a low probability of overlap are broken. Though we do not discuss details on the approaches to speed up iterations, our formulation of statistical timing analysis with coupling is amicable to each of the mentioned speedup approaches.

Experimental results reveal that our approach improves switching window estimation accuracy by up to 24% in comparison to traditional approaches. Since our statistical timing analysis is block based, we are amicable to using the concept of node and edge criticalities introduced in [8]. This makes our approach attractive in guiding timing optimization [24], which is a critical purpose of accurate timing analysis.

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