Efficiency of Linear Supply Function Bidding in Electricity Markets

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***Abstract*—We study the efficiency loss caused by strategic bidding behavior from power generators in electricity markets. In the considered market, the demand of electricity is inelastic, the generators submit their supply functions (i.e., the amount of electricity willing to supply given a unit price) to bid for the supply of electricity, and a uniform price is set to clear the market. We aim to understand how the total generation cost increases under strategic bidding, compared to the minimum total cost. Existing literature has answers to this question without regard to the network structure of the market. However, in electricity markets, the underlying physical network (i.e., the electricity transmission network) determines how electricity flows through the network and thus influences the equilibrium outcome of the market. Taking into account the underlying network, we prove that there exists a unique equilibrium supply profile, and derive an upper bound on the efficiency loss of the equilibrium supply profile compared to the socially optimal one that minimizes the total cost. Our upper bound provides insights on how the network topology affects the efficiency loss.**

I. INTRODUCTION

In power systems, electricity markets aim to balance the demand and the supply at all locations and at all times. Imbalance in the demand and the supply will derail the power system from its normal operating frequency and may cause serious consequences such as blackout [1]. Therefore, electricity markets are crucial for the successful and stable operations of power systems.

In electricity markets, a central coordinator, namely the independent system operator (ISO), is introduced to ensure the balance between supply and demand. The ISO forecasts an inelastic demand profile and dispatch the power generators to satisfy the demand at the minimal total generation cost. Ideally, the ISO should know the generation cost functions of all the generators, and allocate the demand among the generators such that the total generation cost is minimized. In practice, the ISO elicits information about the cost functions from the generators in the form of *supply functions*. Specifically, each generator submits its supply function (i.e., a curve specifying the amount of electricity it is willing to supply given the unit price of electricity) as its bid.

Recent incidents, most notable of which is the California electricity crisis [**?**], suggest the existence of strategic bidding behavior by the generators. A generator may submit a supply function that does not truly reflect its generation cost, in order to maximize its own profit. Subsequently, existing literature has started to look into the efficiency loss due to strategic behavior of generators. Existing works [2]–[3] quantify the efficiency loss by price of anarchy (PoA), defined as the ratio of the total generation cost at the equilibrium to that at the social optimum. By definition, PoA is a number no smaller than 1, and a larger PoA indicates greater efficiency loss.

There has been active research in studying the efficiency of supply function bidding in general markets [2][4] and in electricity markets [5][3][6][7][8]. The usual conclusion is that the efficiency loss is upper bounded, and vanishes as the number of suppliers increases. These works [2]–[8], however, focus on markets with no underlying network structure.

In electricity markets, there is an underlying physical network that limits how the supply can match the demand. For instance, the transmission network topology and the flow limits of transmission lines put constraints on the amounts of electricity injected by the generators into the system. In this work, we aim to study how the network affects the efficiency loss in networked markets.[1] Our first main result suggests that the efficiency loss is still upper bounded when the underlying transmission network is considered. Our second major result provides an upper bound of the PoA, and sheds light into how the network topology affects the upper bound. Our results provide insights in configuring the transmission network and placing the generators.

The rest of this paper is organized as follows. In Section II, we will describe our model of electricity markets and define the supply function equilibrium. We analyze the equilibrium in general electricity networks in Section III, and that in special radial networks in Section IV. Finally, Section V concludes the paper.

II. MODEL

A. Basic Setup

Consider a power system represented as a graph $(\mathcal{N}, \mathcal{E})$. Each node in $\mathcal{N}$ has a generator or a load or both, and each edge in $\mathcal{E}$ is a transmission line connecting two nodes. We denote the set of nodes with a generator by $\mathcal{N}_g$ and

[1] Some related works study the efficiency loss in electricity markets while considering the effect of the transmission network [9][10]. However, they adopt a Cournot competition model, where the suppliers determine the amounts of supply, instead of the more practical form of bidding a supply function.

the set of nodes with a load by $\mathcal{N}_\ell$. We assume that the load is inelastic,[2] and denote the inelastic load profile by $\boldsymbol{d} = (d_j)_{j\in\mathcal{N}_\ell} \in \mathbb{R}_+^{|\mathcal{N}_\ell|}$.[3] The total demand is then given by $D \triangleq \sum_{j\in\mathcal{N}_\ell} d_j$.

Each generator n has a cost $c_n(s_n)$ in providing $s_n \geq 0$ unit of electricity. We make the following standard assumption about cost functions.

Assumption 1: For each generator n, the cost function $c_n(s_n)$ is strictly convex, increasing, and continuously differentiable in $s_n \in [0, +\infty)$.

Due to physical constraints, each generator n's supply s_n must be in a range $[\underline{s}_n, \bar{s}_n]$. As in [3], we assume that no generator is "very big" in its capacity in the following sense.

Assumption 2: No generator has a capacity that is larger than or equal to half of the total demand, namely $\bar{s}_n < \frac{D}{2},\ \forall n \in \mathcal{N}_g$.

The supply profile $\boldsymbol{s} = (s_n)_{n\in\mathcal{N}_g} \in \mathbb{R}_+^{|\mathcal{N}_g|}$ must also satisfy physical constraints of the electrical network. First, in a power system, it is crucial to balance the supply and the demand at all time for the stability of the system [1]. Hence, we need to have

$$\sum_{n\in\mathcal{N}_g} s_n = D. \tag{1}$$

Second, the flow on each transmission line, which depends on the supply profile, cannot exceed the line capacity. Since the direct current (DC) power flow model is commonly used in electricity markets (e.g., by the ISO in economic dispatch and by generators in supply bidding), we use the DC power flow model, where the flow on each line is the linear combination of injections from each node. Then the line flow constraints can be written as follows:

$$-\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}, \tag{2}$$

where $\boldsymbol{f} \in \mathbb{R}_+^{|\mathcal{E}|}$ is the vector of line capacities, $\mathbf{A}_g \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{N}_g|}$ and $\mathbf{A}_\ell \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{N}_\ell|}$ are shift-factor matrices. The shift-factor matrices $\mathbf{A}_\ell$ and $\mathbf{A}_g$ depend on the underlying transmission network topology and the admittance of transmission lines (see [1] and [10] for more details). Note that the bound in (2) is two-sided, because the flows on each power line can go in either direction.

In a regulated and centralized market, the ISO knows the cost functions and can determines the amount of electricity supplied by each generator. It determines the optimal supply profile $\boldsymbol{s}^*$ that minimizes the total generation cost subject to the constraints mentioned above. We summarize the optimization problem to solve as follows:

$$\begin{array}{ll} \max_{\boldsymbol{s}} & \sum_{n\in\mathcal{N}_g} c_n(s_n) \\ s.t. & \sum_{n\in\mathcal{N}_g} s_n = D, \\ & \underline{s}_n \leq s_n \leq \bar{s}_n, \quad \forall n \in \mathcal{N}_g, \\ & -\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}. \end{array} \tag{3}$$

[2]We can easily extend our results to the case with elastic loads. An elastic load can be decomposed as a large inelastic load and a generator whose supply reduces the net load.

[3]We use the notation $\mathbb{R}_+$ to denote the set of positive real numbers.

To avoid triviality, we assume that the feasible set of power generation is non-empty and is not a singleton.

Assumption 3: There exists a strictly feasible allocation of power generation $\boldsymbol{s}$.

B. Deregulated Markets and Supply Function Bidding

In deregulated electricity markets, each generator tells the ISO the amount of electricity it can provide given a unit price of electricity. The mapping from the unit price to the supply is called supply function. We adopt the linear supply function from [3], and define generator n's supply function as:

$$S_n(p, w_n) = w_n \cdot p,$$

where $w_n \in \mathbb{R}_+$ is generator n's strategic action, and $p \in \mathbb{R}_+$ is the unit price of electricity. To clear the market, (i.e., to find the price p satisfies the condition $\sum_{n\in\mathcal{N}_g} S_n(p, w_n) = D$), the ISO sets the price p as follows:

$$p(\boldsymbol{w}) = \frac{D}{\sum_{n\in\mathcal{N}_g} w_n}.$$

where $\boldsymbol{w} = (w_n)_{n\in\mathcal{N}_g} \in \mathbb{R}_+^{|\mathcal{N}_g|}$ is the bidding profile.

Generator n's payoff is its profit defined as follows:

$$\begin{array}{rcl} u_n(w_n, \boldsymbol{w}_{-n}) & \triangleq & p(w_n, \boldsymbol{w}_{-n}) \cdot S_n[p(w_n, \boldsymbol{w}_{-n}), w_n] \\ & - & c_n(S_n[p(w_n, \boldsymbol{w}_{-n}), w_n]), \end{array}$$

where $\boldsymbol{w}_{-n}$ is the action profile of all the generators other than generator n.

Now we can formally define the supply function equilibrium (SFE).

Definition 1: An action profile $\boldsymbol{w}^{**}$ is a supply function equilibrium, if each generator n's action w_n^{**} satisfies[4]

$$\begin{array}{lll} w_n^{**} \in & \arg\max\limits_{w_n} & u_n\left(w_n, \boldsymbol{w}_{-n}^{**}\right) \\ & s.t. & \underline{s}_n \leq S_n\left[p\left(w_n, \boldsymbol{w}_{-n}^{**}\right), w_n\right] \leq \bar{s}_n, \\ & & -\boldsymbol{f} \leq [\mathbf{A}_g]_{*n} \cdot S_n\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_n\right] \\ & & + \sum\limits_{\substack{m\in\mathcal{N}_g \\ m\neq n}} \left\{[\mathbf{A}_g]_{*m} \cdot S_m\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_m^{**}\right]\right\} \\ & & + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}. \end{array}$$

In a SFE, each generator's bid maximizes its own payoff given the others' bids. What is special about our SFE is that the set of feasible bids of each generator depends on the others' bids. Therefore, the SFE is a generalized Nash equilibrium, which is usually harder to analyze than a standard Nash equilibrium [11].

III. Efficiency of Linear Supply Function Bidding

In this section, we will prove that the SFE exists and results in a unique equilibrium supply profile. In addition, we will provide an upper bound of the efficiency loss at SFE, and discuss how this upper bound depends on the transmission network topology.

[4]We denote the nth column of a matrix $\mathbf{A}$ by $[\mathbf{A}]_{*n}$.

Our first main result is the characterization of SFE through a convex programming. Specifically, we show that any SFE results in a unique equilibrium supply profile, which is an optimal solution to a convex optimization problem.

Proposition 1: Any SFE results in an allocation that is the unique solution to the following modified cost minimization problem:

$$\min_{\boldsymbol{s}} \quad \sum_{n\in\mathcal{N}_g} \hat{c}_n(s_n) \tag{4}$$

$$s.t. \quad \sum_{n\in\mathcal{N}_g} s_n = D, \tag{5}$$

$$\underline{\boldsymbol{s}} \le \boldsymbol{s} \le \bar{\boldsymbol{s}}, \tag{6}$$

$$-\boldsymbol{f}_1 \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}_2, \tag{7}$$

where

$$\hat{c}_n(s_n) = \left(1 + \frac{s_n}{D-2s_n}\right)\cdot c_n(s_n) - \int_0^{s_n} \frac{D}{(D-2x)^2}\cdot c_n(x)dx. \tag{8}$$

Proof: Due to space limitation, the proof is in Appendix A of the technical report [12]. ■

Proposition 1 is significant in describing the equilibrium outcome. It shows that although there may be multiple equilibrium bidding profiles, the resulting equilibrium supply profile, which is what we care about the most, is always unique. Moreover, Proposition 1 proves that the equilibrium supply profile is the optimal solution to a convex optimization problem, which provides an efficient way of computing the equilibrium supply profile.

The convex optimization problem in Proposition 1 is crucial in analyzing the PoA. Before we go into detailed analysis, we formally define PoA here.

Definition 2: PoA is defined as

$$\frac{\sum_{n\in\mathcal{N}_g} c_n(s_n^{**})}{\sum_{n\in\mathcal{N}_g} c_n(s_n^{*})},$$

where $(s_n^{**})_{n\in\mathcal{N}_g}$ is the equilibrium supply profile, and $(s_n^{*})_{n\in\mathcal{N}_g}$ is the supply profile that minimizes the total cost.

The PoA is well defined because the equilibrium supply profile is unique.

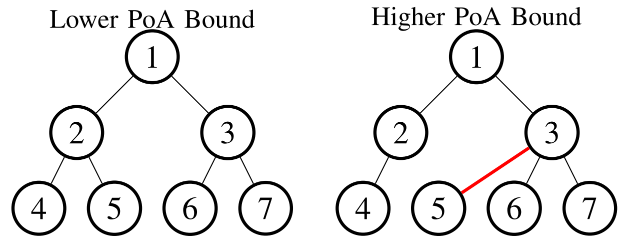
By definition, the PoA is never smaller than 1. A larger PoA indicates that the efficiency loss at the equilibrium is larger. Next, we give an analytical upper bound of the PoA.

Theorem 1: The PoA is upper bounded by $1 + \frac{\Delta}{D-2\Delta}$, where

$$\Delta \triangleq \max_{m\in\mathcal{N}_g} \min\left\{\bar{s}_m, d_m + \sum_{(m,n)\in\mathcal{E}} f_{mn}\right\}. \tag{9}$$

Proof: Due to space limitation, the proof is in Appendix B of the technical report [12]. ■

Theorem 1 gives us an upper bound of the PoA. From the upper bound, we can get insight on the key factors that influence the efficiency loss. First, the efficiency loss can be higher if there is a generator with large generation capacity. This intuition has been obtained in some prior works [5] that do not consider flow limit constraints, and our contribution here is to show that the same intuition also applies to the case with flow limit constraints. Second, another factor that may affect the efficiency loss is how "connected" a generator is. More specifically, the term $d_m + \sum_{(m,n)\in\mathcal{E}} f_{mn}$ can be considered as generator m's weighted degree in the network (with the weights being the capacity of transmission lines) plus its demand. If one generator is much better connected than the others, the efficiency loss can be high. This is a new intuition that was known from prior works.



(a) Network 1 (b) Network 2

Fig. 1. Impact of network topology on the efficiency loss. Assume that all the line capacities are the same. In both networks, the PoA upper bound is determined by node 3, which has the highest connectivity. By connecting node 5 to node 3, the network on the right has a higher PoA bound because node 3 has a higher degree than node 3 on the left.

Our insights shed some light on planning the transmission networks and the locations of generators. We illustrate our insights in Fig. 1, where we show two networks with the same nodes but different network topology.Node 3 on the left has a degree of 3, while the one on the right has a higher degree of 4, because node 5 is added to its set of neighbors. Therefore, the network on the right has a higher PoA bound. This example suggests that it may be beneficial to evenly distribute the connections between generators.

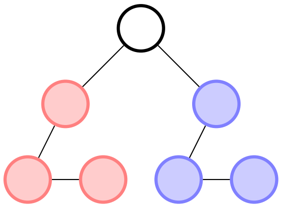
IV. Special Radial Networks

The results in the previous section hold for general electricity networks. In this section, we consider a special class of radial networks, and gain deeper insights for this class of networks.

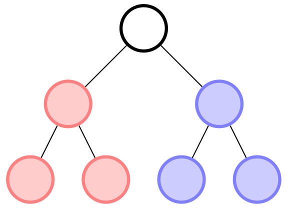
A radial network is a network with no loop. In a radial network, we call each subtree starting from a child of the root a *branch.* We consider a special class of radial networks that have homogeneous nodes and edges within each branch (i.e., the same cost function, the same demand, the same upper and lower bounds of supply, and the same line flow limits). Different branches can have different parameters and cost functions. We proved that at both the socially optimal and equilibrium supply profiles, the supply levels of nodes within a branch are the same, and are independent of the network topology of each individual branch.

We formally state our results for the special radial networks as follows.

Proposition 2: In a radial network with identical nodes and



(a) Network 1



(b) Network 2

Fig. 2. Two radial networks that have different local topologies but the same socially optimal and equilibrium supply profiles. The nodes with the same color are homogeneous, while the nodes with different colors can be different.

lines within each branch, the following statements hold for both socially optimal and equilibrium supply profiles:

- The nodes within a branch have the same supply.
- The only possibly congested line (i.e., a line with a binding line flow constraint) is the one connected to the root.

Proof: Due to space limitation, the proof is in Appendix C of the technical report [12]. ■

One implication of Proposition 2 is that the local network topology in a branch does not matter. For illustration, we show in Fig. 2 two networks with different local network topologies. Based on Proposition 2, these two networks have the same socially optimal and equilibrium supply profiles.

V. Conclusion

In this paper, we studied the efficiency loss of linear supply function bidding in electricity markets. The key feature that sets our work apart from existing works is that we include the flow constraints of the transmission lines in our model. We show that in the resulting networked electricity markets, there exists a unique equilibrium supply profile. We provided an analytical upper bound of the efficiency loss at the equilibrium. Our upper bound suggests that to reduce the efficiency loss, we should evenly distribute the generation capacity and "connectivity" among the generators. Our upper bound also provides a precise definition the connectivity of a generator as its weighted degree in the network (weighted by the transmission line capacity) plus the demand at its location.

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