# Investment and Competition with Positive Externalities in Open Networks


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***Abstract*— A key feature of many information services is that they exhibit positive externalities or network effects, i.e., the value of the service increases with the number of users. Here, we consider an "open" network, in which a given service may be offered by multiple providers and the positive externality depends on the total number of customers served by all providers. Providers compete for customers and may also invest in their networks to improve the externality experienced by their own customers. We model this as a two-stage investment and competition game, where after investing, firms compete via Bertrand competition. In both the socially optimal and the monopoly outcomes, under mild conditions, the firm either does not invest or invests at the maximum level. In addition, a monopolist always underinvests. Interestingly, we find that with competition, there can be at most one firm investing.**


## I. INTRODUCTION

Most information services such as social networks, on-line advertising platforms, on-line games, messaging systems, and even the Internet itself exhibit *positive externalities* or *network effects*, i.e., the value of the service increases with the number of users. Such effects may be due to the ability to reach more users (direct network effects) or due to the fact that more users result in more complementary goods being produced for the given system (indirect network effects) [1]. An example of the former is the ability to find more "friends" on a social network. An example of the latter is the fact that more games may be made for a game platform with more users.

Network effects can impact the competition among firms by enabling a firm with more customers to charge higher prices or further expand its customer base. Indeed there is a large economics literature that looks at the competition among firms which own separate (but similar) networks with the results often being that a single monopolist emerges (e.g., see [2]). In this line of work the underlying network of each firm is "closed," i.e., it is for the exclusive use of a given firm's customers. An example is Facebook, where one has to be a Facebook user to enjoy the positive externality of the network.

Here, we instead consider scenarios where firms compete for users of an "open" network, meaning that the positive externality depends on the customers of *all* of the competing firms. Email service is one example where the client of one service can send emails to other clients of different service providers; thus the value of an email service depends in part on the overall number of customers using any e-mail service.

The model we consider is based on an approach as in [3], [4], [5], [6] for studying competition in the face of negative externality due to congestion effects. As in these works, we assume that there is a pool of infinitesimal customers, whose valuation for service depends on a *delivered price*, which is the sum of service price charged by the firm and a term that models the externality.[1] Our approach differs from this previous work in that this additional term reduces the delivered price instead of increasing it, modeling the fact that due to the network effect, users are willing to pay more for the service.

We study both the monopoly and oligopoly cases. In the monopoly case, we show that under some conditions, the firm either invests zero or invests at the maximum level. The firm invests at the maximum level when the investment cost is low, and stops investing when the investment cost exceeds a threshold. We also show that a profit-maximizing firm stops investing earlier than a welfare-maximizing firm.

In the oligopoly case, we formulate a two-stage game, in which firms invest in their technologies in the first stage and then compete for customers via price competition in the second stage. We show that under positive externality, there can be at most one firm investing and entering the market at an equilibrium. Again, as the investment cost increases, eventually no firm would enter. Interestingly, we show that with competition, firms will stop investing at a lower level of cost than in the monopoly case.

In addition to the aforementioned work on competition with congestion externalities, other related work includes an extensive literature on monopoly pricing in the presence of externalities such as [7], [8]. Another related strand of work is the literature on "club goods", which include both positive and negative externalities, e.g. [9]. In these works the externalities only depend on the members of a given club, i.e. the users of a given firm, as opposed to the open networks we consider. Another work that studies open networks is [10]. Different from us, they focus on both positive and negative externalities and assume that firms do not charge for services.


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[1] In other words we assume that each user's value for the service is separable into two terms, one that depends on the externality and one that does not.

## II. Model and Main Results

We consider a two-stage game where a set $\{1, \ldots, N\}$ of service providers (SPs) first invest simultaneously and then compete for a common pool of customers via Bertrand competition (i.e., price competition). Each SP $i$ first chooses an amount $I_i$ to invest in its service, where we normalize the maximum investment a SP can make to be 1. Given the investment choice of each SP, each SP $i$ then determines the price $p_i$ it will charge for its service. Assuming a linear investment cost, SP $i$'s profit is simply $y_i \cdot p_i - c \cdot I_i$, where $y_i \in \mathbb{R}_{\geq 0}$ is the mass of users from a pool of infinitesimal customers it serves.

The positive externality experienced by the customers of SP $i$ is given by a function $g(Y, I_i)$ where $Y = \sum_{i=1}^{N} y_i$ is the overall mass of customers being served in the market. This models the fact that the network effect seen by the customers of any SP depends on the total number of customers using the underlying network service. However, the magnitude of this effect for SP $i$'s customers also depends on the investment of SP $i$. We make the following assumption about the positive externality.

*Assumption 1 (Positive Externality):* The positive externality $g(y, I)$ is increasing and convex in the investment $I$, is increasing in $y$, and is supermodular in $(y, I)$ (i.e., $g_{yI}(y, I) \geq 0$).[2] In addition, we have

$$\frac{y \cdot g_{yy}(y, I)}{g_y(y, I)} \geq -1.$$

The inequality in the assumption is satisfied when $g(y, I)$ is convex in $y$ or is "not too concave" in $y$.

We assume that customers are infinitesimal and are governed by a downward slopping inverse demand curve $P(y)$, which gives the largest price that a mass of $y$ customers is willing to pay for the service without accounting for the network effect. Given each SP $i$'s service price $p_i$, the mass of users served by all SPs must satisfy

$$p_i \leq g(Y, I_i) + P(Y), \tag{1}$$

such that the users get nonnegative payoffs. Note that the price $p_i$ can be larger than $P(Y)$, due to the additional benefit the users get from the network effect.

It is useful to define $z(y, I) \triangleq g(y, I) + P(y)$ as the "delivered price". We make the following assumption about $z(y, I)$.

*Assumption 2 (Delivered Price):* For any $I \in [0, 1]$, $z(y, I)$ is strictly decreasing and twice differentiable in $y$. In addition, we have

$$-\frac{y g_{yI}(y, I)}{g_I(y, I)} \leq \frac{y z_{yy}(y, I)}{-z_y(y, I)} < 2.$$

### A. Monopoly

Suppose that there is only one firm in the network. This firm may aim to maximize the social welfare or its profit. We drop the subscript $i$, and denote the firm's price by $p$, its investment by $I$, and the number of users it serves by $y$. Then the relationship between the price and the number of users served in (1) reduces to

$$p \leq g(y, I) + P(y). \tag{2}$$

The social welfare is

$$S(y, I) = \int_0^y P(x)dx + g(y, I)y - cI.$$

Based on Assumption 1, the social welfare is increasing in $y$. Hence, the maximum social welfare is achieved when (2) holds with equality. We denote the optimal number of users by $y^*(I, p)$. Due to our assumption, $y^*(I, p)$ is a one-to-one mapping between $y$ and $p$ for each fixed $I$. Therefore, instead of choosing price, it is equivalent to let the firm choose the number $y$ of users and simply set the price to $p = g(y, I) + P(y)$.

Similarly, the firm's profit $py - cI$ is also increasing in $y$. Hence, setting the price is equivalent to setting the number of users to serve.

We show that in the monopoly case, the firm will either invest zero or invest fully (i.e., the optimal $I$ is 0 or 1), whether it is maximizing the social welfare or its own profit.

*Lemma 1:* Whether the firm aims to maximize the social welfare or its own profit, it always invests zero or invests fully.

The firm's investment level depends on the investment cost. There is a threshold of investment cost, below which the firm will invest fully, and above which the firm will invest zero. In addition, the threshold for a welfare-maximizing firm is different from that for a profit-maximizing firm.

*Lemma 2:* As the investment cost $c$ increases, a profit-maximizing firm will stop investing before a welfare-maximizing firm does.

### B. Oligopoly

Suppose that there are more than one firm, all of which aim to maximize their own profit. The firms compete for customers by price. In the case that firm $i$ is the one with the lowest price (i.e., $\{i\} = \arg\min_j p_j$), the entire market is captured by firm $i$, and it will choose the number $y_i$ of users such that

$$p_i = g(y_i, I_i) + P(y_i).$$

Note that we have $y_j = 0$ for all $j \neq i$.

When there are multiple firms with the same price, the market is shared among them, and the users can be divided arbitrarily among them. In other words, any vector $\boldsymbol{y}$ of user division is feasible, as long as it satisfies

$$p_i = g(Y, I_i) + P(Y),$$

and

$$y_i = 0, \forall i \notin \arg\min_j p_j.$$

A subgame perfect equilibrium is a joint investment and pricing decision $(\boldsymbol{I} = (I_1, \ldots, I_N), \boldsymbol{p} = (p_1, \ldots, p_N))$, such

[2] We use $g_{yI}(y, I)$ to denote $g_{yI}(y, I) \triangleq \frac{\partial^2 g(y,I)}{\partial y \partial I}$. Similar notations apply for $g_{yy}(y, I)$, $g_y(y, I)$ and other functions.

that there exists a feasible user division $\boldsymbol{y}$ under which no unilateral deviation is profitable.

*Proposition 1:* If a subgame perfect equilibrium exists, it corresponds to either only one firm fully investing or no firm investing.

Note in this case, even if only one firm invests it still faces potential competition from the other firms that do not invest. This competition can constrain the price that the investing firm charges, which in turn can reduce its incentive to invest, leading to the following result.

*Lemma 3:* As the investment cost $c$ increases, a profit-maximizing firm in an Oligopoly will stop investing at either the same value as a monopolist does or earlier.

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