Robust Supply Function Bidding in Electricity Markets With Renewables

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***Abstract*—We study a two-stage electricity market with renewables. Each energy producer in the market has a portfolio of both renewable and conventional energy generators. In the day-ahead (DA) market, each producer submits a parameterized supply function (i.e., the amounts of energy to produce at various prices) to the independent system operator (ISO), who determines the DA market clearing price and the amounts of DA committed energy by each producer. In the real-time market, each producer tries to fulfill its DA committed energy with (zero-cost) renewables. If the renewable energy is insufficient, the producer uses conventional energy generation and incurs a cost; otherwise, it sells the surplus of renewable energy to the ISO at a predetermined feed-in tariff.**

We study the robust supply function equilibrium (SFE) in this market, where each producer has incomplete information about the other producers' marginal costs and the distribution of its random renewable energy, and performs worst-case optimization against these unknown variables. We fully characterize the unique robust SFE, and study the impact of the feed-in tariff on the equilibrium outcome.

***Index Terms*—Electricity markets, supply function bidding, renewable energy generation, worst-case optimization, robust equilibrium**

I. Introduction

Power systems are undergoing drastic changes due to the penetration of renewable energy. The stochastic nature of the renewable energy brings the challenges to both the system operator (i.e., the independent system operator (ISO)) and the market participants (e.g., the energy producers). From the system operator's perspective, the challenge is to incentivize market participants to invest in renewable energy generation, while keeping the balance of supply and demand given high penetration of intermittent renewables. For the market participants, the challenge is to achieve large profit given the lack of information about the other market participants and the uncertainty of their own renewable energy generation. This paper aims to build a rigorous model to predict the market outcome by market participants with incomplete information, and analyze the impact of the system operator's incentive schemes on the market outcome.

More specifically, we study an electricity market where each energy producer has renewable energy generators, as well as conventional energy generators to mitigate the uncertainty in renewable energy generation. The market operates in two stages, namely the day-ahead (DA) market and the real-time (RT) market. In the DA market, each producer submits a bid to the ISO, without knowing its renewable energy available in the next day. The bid is a parameterized supply function specifying the amounts of energy it is willing to produce at various prices, to the ISO. The ISO then determines the DA electricity price to clear the market, and the amounts of DA committed energy by each producer. The ISO pays each producer based on the DA market clearing price and its DA committed energy.

In the RT market, each producer must fulfill its DA committed energy. Since renewable energy generation is costless, the producer dispatches its (random) renewable energy first. If the renewable energy is insufficient, it uses conventional energy generation to compensate the shortfall, and incurs a cost. If the renewable energy has surplus, it sells the surplus to the ISO at a predetermined *feed-in tariff*. The feed-in tariff is an incentive scheme set by the ISO to encourage the investment in renewable energy.

To analyze the market outcome, we propose a new equilibrium concept, called *robust supply function equilibrium* (SFE). In the robust SFE, each producer has incomplete information about the other producers' cost functions and the distribution of its random renewable energy. Therefore, each producer performs worst-case optimization against these unknown variables. The robust SFE is the robust version of the standard SFE, and may serve as a better prediction of the market outcome when the producers have incomplete information.

Our main contribution is to identify the conditions under which the robust SFE exists, show that the robust SFE is unique when it exists, and give an analytical characterization of the unique robust SFE. Given our characterization of the robust SFE, we are able to study the impact of the feed-in tariff on the equilibrium outcome.

The rest of this paper is organized as follows. We discuss related works in Section **??**. In Section **??**, we will describe our model of electricity markets with renewables. In Section **??**, we define the robust supply function equilibrium. We analyze the equilibrium in Section **??**. Finally, Section **??** concludes the paper.

II. Relation to Prior Work

There are two commonly-used models to analyze the equilibrium of electricity markets. The first model is the Cournot competition model, where each producer submits the amount of electricity to produce (i.e., a quantity) to the ISO [?][?][?]. However, in the Cournot competition model, the producers act quite differently from the way they bid in reality. Therefore, we use another competition model in our paper.

The second commonly-used model is the supply function equilibrium model, where each producer submits the amounts of electricity to produce at different prices (i.e., a curve of price versus quantity) [?][?][?][?][?]. The SFE model is closer to the real bidding formats in electricity markets. However, as far as we know, all the existing works using the SFE model assume that the producers have complete information of the system [?]–[?]. More specifically, they assume that each producer knows the cost functions of all the other producers [?]–[?] and the distribution of the renewable energy [?]. In contrast, we assume that each producer has incomplete information about the other producers' cost functions and the distribution of its own renewable energy. We define and analyze the robust version of the standard SFE, namely the robust SFE.

III. System Model

We consider a two-stage electricity market with $N \geq 3$ energy producers. Each producer has a portfolio of energy sources, including both renewable energy and conventional energy [?]. We could also think of a producer as an aggregate of a renewable energy producer and a conventional energy producer, who form a coalition and bid as a whole into the electricity market [?][1]. These N producers compete to fulfill the total (inelastic) demand D.

A. Operations in The Day-Ahead (DA) Market

In the first stage of the two-stage market, namely the DA market, each producer n submits a supply function to the independent system operator (ISO). As in [?], the supply function is of the following form:

$$S_n(p, w_n) = w_n \cdot p, \tag{1}$$

where $w_n \in \mathbb{R}_+$ is producer n's strategic choice (i.e., its bid), and $p \in \mathbb{R}_+$ is the DA market price determined by the ISO. The supply function indicates the amount $S_n(p, w_n)$ of energy producer n is willing to produce when the price is p.

To clear the market, namely to have

$$\textstyle\sum_{n=1}^{N} S_n(p, w_n) = D, \tag{2}$$

the ISO sets the price as

$$p\left(\boldsymbol{w}\right) = \frac{D}{\sum_{n=1}^{N} w_n}, \tag{3}$$

where $\boldsymbol{w} = (w_1, \ldots, w_N)$ is the bid profile of all the producers.

[1]In this case, we treat the aggregate of these two producers as one player, and ignore the issues such as profit sharing.

Based on the expression of the DA market price, we have

$$S_n\left[p(\boldsymbol{w}), w_n\right] = \frac{w_n}{w_n + \sum_{m \neq n} w_m} \cdot D.$$

We can see that producer n's DA committed energy $S_n\left[p(\boldsymbol{w}), w_n\right]$ is increasing in its bid w_n. Therefore, the bid w_n reflects how "aggressive" producer n is in the bidding process. The producer that submits a higher bid is more aggressive and commits to produce more energy.

In summary, in the DA market, each producer n submits a bid w_n, the ISO determines the DA market price $p(\boldsymbol{w})$, and each producer n commits to produce $s_n = S_n\left[p(\boldsymbol{w}), w_n\right]$ units of energy. The DA payment to producer n is

$$p(\boldsymbol{w}) \cdot S_n\left[p(\boldsymbol{w}), w_n\right].$$

B. Operations in The Real-Time (RT) Market

In the real-time (RT) market following the DA market, each producer n has $\hat{s}_n$ units of renewable energy. The amount $\hat{s}_n$ of available renewable energy is a random variable in the range of $[\underline{s}, \bar{s}]$. Since the renewable energy is costless, the producer always dispatches its renewable energy generator first. There are two possibilities here. If the renewable energy is insufficient for the DA committed energy, namely $\hat{s}_n < s_n$, producer n uses its (costly) conventional energy generation to fulfill the remaining DA committed energy. Assuming that the marginal cost of the conventional energy generation is a constant θ_n, producer n's cost in this case is then

$$\theta_n \cdot (s_n - \hat{s}_n).$$

If the renewable energy exceeds the DA committed energy, namely $\hat{s}_n > s_n$, producer n sells the surplus of renewable energy to the ISO at a unit price $\alpha \geq 0$. In this case, producer n gets a revenue of

$$\alpha \cdot (\hat{s}_n - s_n).$$

In practice, different ISOs set this price α differently. Some ISOs buy the surplus at the locational marginal price (LMP), while some ISOs buy at a higher fixed price to encourage renewable energy generation.[2] The price α of the surplus renewable energy is also called *feed-in tariff*.

In summary, in the RT market, each producer n incurs a cost of

$$c(\theta_n, s_n, \hat{s}_n) = \theta_n \cdot (s_n - \hat{s}_n)^+ - \alpha \cdot (\hat{s}_n - s_n)^+, \tag{4}$$

where $(\cdot)^+ \triangleq \max\{\cdot, 0\}$.

We can see that the key parameter of producer n's cost function is the marginal cost θ_n of conventional energy generation. This parameter θ_n reflects how costly it is for producer n to

[2]In contrast, some ISOs would penalize the producer when its renewable energy generation exceeds its DA committed amount. The reason is that the (potentially) large amount of additional energy injected into the power system may make the system unstable. Assuming that the producer can configure its renewable energy generator and produce any amount up to the available amount $\hat{s}_n$, the producer would never incur a cost when $\hat{s}_n > s_n$. Therefore, the scenario of penalizing positive imbalance in renewable generation can be modeled by setting $\alpha = 0$.

compensate the shortfall of renewable energy generation. We sometimes refer to this parameter θ_n as producer n's type. We write the type space as $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$.

We assume that the range of random renewable energy $[\underline{s}, \bar{s}]$ is the same for all the producers and is common knowledge to them. Similarly, the type space Θ is common knowledge.

C. Performance Benchmark

We measure the performance of the system by the total cost of the sytem (including the producers and the ISO). Since the DA payment $p(\boldsymbol{w}) \cdot S_n\left[p(\boldsymbol{w}), w_n\right]$ and the payment for surplus of renewable energy $\alpha \cdot (\hat{s}_n - s_n)$ are transferred between a producer and the ISO, they do not enter the total cost. Therefore, the total cost is the total generation cost as follows

$$\sum_{n \in \mathcal{N}} \theta_n (s_n - \hat{s}_n)^+. \tag{5}$$

IV. Robust Supply Function Equilibrium

Each producer n's payoff is the DA payment minus the RT cost, which can be written as

$$\begin{aligned} & u_n\left(w_n, \boldsymbol{w}_{-n} | \theta_n, \hat{s}_n\right) \qquad (6) \\ = \;& p\left(w_n, \boldsymbol{w}_{-n}\right) \cdot S_n\left[p\left(w_n, \boldsymbol{w}_{-n}\right), w_n\right] \\ & - c\left\{\theta_n, S_n\left[p\left(w_n, \boldsymbol{w}_{-n}\right), w_n\right], \hat{s}_n\right\}, \end{aligned}$$

where $\boldsymbol{w}_{-n}$ is the bidding profile of all the producers other than n.

From the above equation, we can see that each producer n's payoff $u_n(w_n, \boldsymbol{w}_{-n} | \theta_n, \hat{s}_n)$ depends on its own bid, the other producers' bids, its own type, and the realization of the random renewable energy. Among these parameters, the other producers' bids $\boldsymbol{w}_{-n}$ and the realization of the renewable energy $\hat{s}_n$ are unknown to producer n (at least when it determines the bid in the DA market). Therefore, it is hard for each producer n to maximize its exact payoff $u_n\left(w_n, \boldsymbol{w}_{-n} | \theta_n, \hat{s}_n\right)$ due to the lack of information.

There are usually two approaches, namely the Bayesian approach and the worst-case approach, to deal with the incomplete information. In this paper, since the producers have no statistical information about the other producers and the renewable energy generation, we adopt the worst-case approach. Specifically, each producer n plans against the worst case of the others' types $\boldsymbol{w}_{-n}$ and the realization of renewable energy $\hat{s}_n$, and maximizes its worst-case payoff. Given its type θ_n, each producer n solves the following optimization problem:

$$w_n \in \arg\max_{w_n} \min_{\boldsymbol{w}_{-n}, \hat{s}_n} u_n\left(w_n, \boldsymbol{w}_{-n} | \theta_n, \hat{s}_n\right). \tag{7}$$

Since producer n's bid w_n depends on its type θ_n, we write producer n's bidding strategy as a mapping

$$b_n : \left[\underline{\theta}, \overline{\theta}\right] \to \mathbb{R}_+.$$

In other words, given producer n's type θ_n, its bid is $w_n = b_n(\theta_n)$ according to the bidding strategy b_n.

The producers are different only in their types, and are symmetric in everything else (e.g., the same type space Θ, the same range of renewable energy generation $[\underline{s}, \bar{s}]$). Therefore, we focus on symmetric equilibrium where all the producers have the same bidding strategy (i.e., $b_n = b_m, \ \forall m, n \in \mathcal{N}$. Note, however, that the bids $\{w_n\}_{n \in \mathcal{N}}$ from the producers are in general different, because they have different types $\{\theta_n\}_{n \in \mathcal{N}}$.

As we have discussed before, the type θ_n reflects how costly it is for producer n to compensate the shortfall in renewable energy. Hence, we expect a producer with a higher marginal cost θ_n to bid more conservatively, such that the amount of DA committed energy is smaller and that the potential cost incurred by shortfall in renewable energy is lower. Mathematically, this means that the bidding strategy b_n is a (weakly) decreasing function of the type θ_n.

In summary, we focus on symmetric, (weakly) decreasing bidding strategies. Next, we formally state the conditions under which such bidding strategies constitute a *robust supply function equilibrium*.

Definition 1: The robust supply function equilibrium is a weakly decreasing mapping

$$b : \left[\underline{\theta}, \overline{\theta}\right] \to \mathbb{R}_+,$$

such that for each producer n and for any $\theta_n \in \left[\underline{\theta}, \overline{\theta}\right]$, the bid $b(\theta_n)$ maximizes producer n's worst-case payoff, namely

$$b(\theta_n) \in \arg\max_{w_n \geq 0} \min_{\substack{\hat{s}_n \in [\underline{s}, \bar{s}] \\ \boldsymbol{\theta}_{-n} \in \Theta^{N-1}}} u_n\left[w_n, b(\boldsymbol{\theta}_{-n}) | \theta_n, \hat{s}_n\right], \tag{8}$$

where we abuse the notation and denote

$$b(\boldsymbol{\theta}_{-n}) \triangleq \left[b(\theta_1), \ldots, b(\theta_{n-1}), b(\theta_{n+1}), \ldots, b(\theta_N)\right]$$

the bids of all producers other than n given their types $\boldsymbol{\theta}_{-n}$.

From (**??**), we can see that the robust SFE does not require producer n to know the others' bids or the realization of its renewable energy generation. In fact, all producer n needs to know is the type space Θ and the range of renewable energy $[\underline{s}, \bar{s}]$, which are common knowledge among the producers. Therefore, the robust SFE may be a better equilibrium notion than the standard SFE, in the sense that the robust SFE is more suitable to predict the market outcome when producers have incomplete information.

V. Equilibrium Analysis

In this section, we analyze the robust SFE. We will show that under some mild conditions, there exists a unique robust SFE, and will give the closed-form expression of this unique robust SFE. We will also discuss the impact of the feed-in tariff α on the producers' bids and the total generation cost at the equilibrium.

In the rest of our paper, we make the following assumption about the minimum amount of renewable energy $\underline{s}$.

Assumption 1: The minimum amount of renewable energy $\underline{s}$ satisfies

$$\underline{s} < \frac{D}{N}. \tag{9}$$

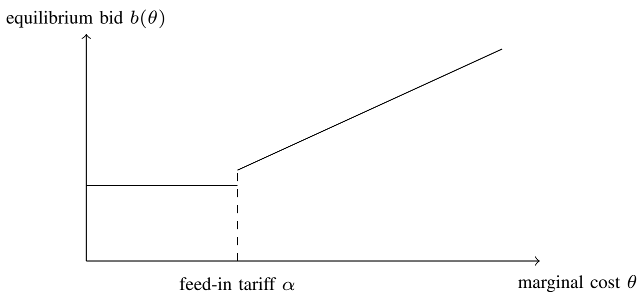

Fig. 1. Illustration of the bidding strategy at the robust supply function equilibrium.

Assumption 1 requires that the realized renewable energy is not guaranteed to fulfill the total demand. If Assumption 1 does not hold (i.e., $\underline{s} \geq \frac{D}{N}$), the renewable energy is always abundant, and hence the total generation cost is always zero. The system always reaches the social optimum (in terms of the total generation cost), leaving no room for design and optimization. Therefore, we focus on the scenario when the renewable energy is not abundant (i.e., $\underline{s} < \frac{D}{N}$).

The robust SFE is fully characterized in the following theorem.

Theorem 1: Suppose that Assumption 1 holds. Then we have the following:

- There exists a robust SFE if and only if 1) , and 2) .
- When robust SFE exists, it is unique, and can be written explicitly as:

$$b(\theta) = \begin{cases} \frac{(N-1)^3}{N^2(N-2)}\alpha D, & \theta \in [\underline{\theta}, \alpha] \\ \theta \frac{(D-\underline{s})^2}{D}, & \theta \in \left(\alpha, \bar{\theta}\right] \end{cases}. \quad (10)$$

Proof: See Appendix A of our technical report [**?**]. ■

Theorem 1 fully characterizes the robust SFE. The bidding strategy is piece-wise linear (see Fig. **??** for illustration). The producers, whose marginal costs of conventional energy generation are no larger than the feed-in tariff (i.e., $\theta \leq \alpha$), submit the same bid. The other producers submit bids that are strictly increasing with their marginal costs of conventional energy generation. In addition, there is a discontinuity in the equilibrium bidding strategy at $\theta = \alpha$. Assumption 1 ensures that at this discontinuity, the bidding strategy jumps upward and hence is increasing.

Theorem 1 leads to several interesting observations, summarized by the following corollaries.

Corollary 1: For each producer with type θ, the equilibrium bid $b(\theta)$ is non-decreasing in the feed-in tariff α.

Proof: This is obvious from the expression of the equilibrium bidding strategy in Theorem 1. ■

In words, Corollary 1 says that the producers bid more conservatively under higher feed-in tariffs. This is because as the feed-in tariff increases, the RT revenue from selling the surplus of renewable energy is higher, as compared to the DA revenue from selling DA committed energy. Therefore, the producers have incentives to bid more conservatively and committ less in the DA market.

Corollary 2: When the feed-in tariff $\alpha \geq \bar{\theta}$, all the producers submit the same equilibrium bid, and have the same amount of DA committed energy $s_n = \frac{D}{N}$.

Proof: From the expression of the equilibrium bidding strategy in Theorem 1, we know that the equilibrium bidding strategy is constant, namely $b(\theta) = \frac{(N-1)^3}{N^2(N-2)}\alpha D$ for all θ. The amount of DA committed energy can be computed straightforwardly based on the DA market price and the supply function. ■

The implication of Corollary 2 is that it may be inefficient (in terms of the total generation cost) to set the feed-in tariff too high. If the feed-in tariff is higher than all the producers' marginal costs of conventional energy generation, the demand will be allocated equally (in the DA market) to all the producers (since they submit the same bid). Ideally, we would like to allocate more demand to the producers with lower marginal costs. In contrast, if we set a lower feed-in tariff, the producers with lower marginal costs will bid more aggressively and commit more in the DA market. This may lead to lower total generation cost.

VI. Conclusion

We studied the robust supply function equilibrium in a two-stage electricity market with renewables. In the day-ahead market, each producer bids its supply function and commits to produce certain amount of energy. In the real-time market, each producer tries to fulfill the day-ahead committed energy by renewable energy: it compensates the shortfall in renewable energy by conventional energy generation with a fixed positive marginal cost, or sells the surplus of renewable energy at a predetermined feed-in tariff. A key feature of our model is that each producer has incomplete information about the other producers' marginal costs of conventional energy generation and the distribution of its own renewable energy. In our proposed robust supply function equilibrium, each producer plans against the worst case of these unknown variables. We characterized the unique robust supply function equilibrium in closed form, and discussed the impact of the feed-in tariff on the market outcome.

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