Efficiency of Linear Supply Function Bidding in Electricity Markets

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Abstract

We study the efficiency loss caused by strategic bidding behavior from power generators in electricity markets. In the considered market, the demand of electricity is inelastic, the generators submit their supply functions (i.e., the amount of electricity willing to supply given a unit price) to bid for the supply of electricity, and a uniform price is set to clear the market. We aim to understand how the total generation cost increases under strategic bidding, compared to the minimum total cost. Existing literature has answers to this question without regard to the network structure of the market. However, in electricity markets, the underlying physical network (i.e., the electricity transmission network) determines how electricity flows through the network and thus influences the equilibrium outcome of the market. Taking into account the underlying network, we prove that there exists a unique equilibrium supply profile, and derive an upper bound on the efficiency loss of the equilibrium supply profile compared to the socially optimal one that minimizes the total cost. Our upper bound provides insights on how the network topology affects the efficiency loss.

I. Introduction

In power systems, electricity markets aim to balance the demand and the supply at all locations and at all times. Imbalance in the demand and the supply will derail the power system from its normal operating frequency and may cause serious consequences such as blackout [1]. Therefore, electricity markets are crucial for the successful and stable operations of power systems.

In electricity markets, a central coordinator, namely the independent system operator (ISO), is introduced to ensure the balance between supply and demand. The ISO forecasts an inelastic demand profile and dispatch the power generators to satisfy the demand at the minimal total generation cost. Ideally, the ISO should know the generation cost functions of all the generators, and allocate the demand among the generators such that the total generation cost is minimized. In practice, the ISO elicits information about the cost functions from the generators in the form of *supply functions*. Specifically, each generator submits its supply function (i.e., a curve specifying the amount of electricity it is willing to supply given the unit price of electricity) as its bid.

Recent incidents, most notable of which is the California electricity crisis [**?**], suggest the existence of strategic bidding behavior by the generators. A generator may submit a supply function that does not truly reflect its generation cost, in order to maximize its own profit. Subsequently, existing literature has started to look into the efficiency loss due to strategic behavior of generators. Existing works [2][3][4] quantify the efficiency loss by price of anarchy

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(PoA), defined as the ratio of the total generation cost at the equilibrium to that at the social optimum. By definition, PoA is a number no smaller than 1, and a larger PoA indicates greater efficiency loss.

There has been active research in studying the efficiency of supply function bidding in general markets [2][5] and in electricity markets [3][4][6][7][8]. The usual conclusion is that the efficiency loss is upper bounded, and vanishes as the number of suppliers increases. These works [2]–[8], however, focus on markets with no underlying network structure.

In electricity markets, there is an underlying physical network that limits how the supply can match the demand. For instance, the transmission network topology and the flow limits of transmission lines put constraints on the amounts of electricity injected by the generators into the system. In this work, we aim to study how the network affects the efficiency loss in networked markets.[1] Our first main result suggests that the efficiency loss is still upper bounded when the underlying transmission network is considered. Our second major result provides an upper bound of the PoA, and sheds light into how the network topology affects the upper bound. Our results provide insights in configuring the transmission network and placing the generators.

The rest of this paper is organized as follows. In Section II, we will describe our model of electricity markets and define the supply function equilibrium. We analyze the equilibrium in general electricity networks in Section III, and that in special radial networks in Section IV. Finally, Section V concludes the paper.

II. Model

A. Basic Setup

Consider a power system represented as a graph $(\mathcal{N}, \mathcal{E})$. Each node in $\mathcal{N}$ has a generator or a load or both, and each edge in $\mathcal{E}$ is a transmission line connecting two nodes. We denote the set of nodes with a generator by $\mathcal{N}_g$ and the set of nodes with a load by $\mathcal{N}_\ell$. We assume that the load is inelastic,[2] and denote the inelastic load profile by $\boldsymbol{d} = (d_j)_{j\in\mathcal{N}_\ell} \in \mathbb{R}_+^{|\mathcal{N}_\ell|}$.[3] The total demand is then given by $D \triangleq \sum_{j\in\mathcal{N}_\ell} d_j$.

Each generator n has a cost $c_n(s_n)$ in providing $s_n \geq 0$ unit of electricity. We make the following standard assumption about cost functions.

Assumption 1: For each generator n, the cost function $c_n(s_n)$ is strictly convex, increasing, and continuously differentiable in $s_n \in [0, +\infty)$.

Due to physical constraints, each generator n's supply s_n must be in a range $[\underline{s}_n, \bar{s}_n]$. As in [4], we assume that no generator is "very big" in its capacity in the following sense.

Assumption 2: No generator has a capacity that is larger than or equal to half of the total demand, namely $\bar{s}_n < \frac{D}{2},\ \forall n \in \mathcal{N}_g$.

The supply profile $\boldsymbol{s} = (s_n)_{n\in\mathcal{N}_g} \in \mathbb{R}_+^{|\mathcal{N}_g|}$ must also satisfy physical constraints of the electrical network. First, in a power system, it is crucial to balance the supply and the demand at all time for the stability of the system [1].

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[1] Some related works study the efficiency loss in electricity markets while considering the effect of the transmission network [9][10]. However, they adopt a Cournot competition model, where the suppliers determine the amounts of supply, instead of the more practical form of bidding a supply function.

[2] We can easily extend our results to the case with elastic loads. An elastic load can be decomposed as a large inelastic load and a generator whose supply reduces the net load.

[3] We use the notation $\mathbb{R}_+$ to denote the set of positive real numbers.

Hence, we need to have

$$\sum_{n\in\mathcal{N}_g} s_n = D. \tag{1}$$

Second, the flow on each transmission line, which depends on the supply profile, cannot exceed the line capacity. Since the direct current (DC) power flow model is commonly used in electricity markets (e.g., by the ISO in economic dispatch and by generators in supply bidding), we use the DC power flow model, where the flow on each line is the linear combination of injections from each node. Then the line flow constraints can be written as follows:

$$-\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}, \tag{2}$$

where $\boldsymbol{f} \in \mathbb{R}_+^{|\mathcal{E}|}$ is the vector of line capacities, $\mathbf{A}_g \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{N}_g|}$ and $\mathbf{A}_\ell \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{N}_\ell|}$ are shift-factor matrices. The shift-factor matrices $\mathbf{A}_\ell$ and $\mathbf{A}_g$ depend on the underlying transmission network topology and the admittance of transmission lines (see [1] and [10] for more details). Note that the bound in (2) is two-sided, because the flows on each power line can go in either direction.

In a regulated and centralized market, the ISO knows the cost functions and can determines the amount of electricity supplied by each generator. It determines the optimal supply profile $\boldsymbol{s}^*$ that minimizes the total generation cost subject to the constraints mentioned above. We summarize the optimization problem to solve as follows:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \textstyle\sum_{n\in\mathcal{N}_g} c_n(s_n) \\ s.t. \quad & \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, \\ & \underline{s}_n \leq s_n \leq \bar{s}_n, \quad \forall n \in \mathcal{N}_g, \\ & -\boldsymbol{f} \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}. \end{aligned} \tag{3}$$

To avoid triviality, we assume that the feasible set of power generation is non-empty and is not a singleton.

Assumption 3: There exists a strictly feasible allocation of power generation $\boldsymbol{s}$.

B. Deregulated Markets and Supply Function Bidding

In deregulated electricity markets, each generator tells the ISO the amount of electricity it can provide given a unit price of electricity. The mapping from the unit price to the supply is called supply function. We adopt the linear supply function from [4], and define generator n's supply function as:

$$S_n(p, w_n) = w_n \cdot p,$$

where $w_n \in \mathbb{R}_+$ is generator n's strategic action, and $p \in \mathbb{R}_+$ is the unit price of electricity. To clear the market, (i.e., to find the price p satisfies the condition $\sum_{n\in\mathcal{N}_g} S_n(p, w_n) = D$), the ISO sets the price p as follows:

$$p(\boldsymbol{w}) = \frac{D}{\sum_{n\in\mathcal{N}_g} w_n}.$$

where $\boldsymbol{w} = (w_n)_{n\in\mathcal{N}_g} \in \mathbb{R}_+^{|\mathcal{N}_g|}$ is the bidding profile.

Generator n's payoff is its profit defined as follows:

$$u_n(w_n, \boldsymbol{w}_{-n}) \triangleq p(w_n, \boldsymbol{w}_{-n}) \cdot S_n[p(w_n, \boldsymbol{w}_{-n}), w_n] - c_n(S_n[p(w_n, \boldsymbol{w}_{-n}), w_n]),$$

where $\boldsymbol{w}_{-n}$ is the action profile of all the generators other than generator n.

Now we can formally define the supply function equilibrium (SFE).

Definition 1: An action profile $\boldsymbol{w}^{**}$ is a supply function equilibrium, if each generator n's action w_n^{**} satisfies[4]

$$
\begin{aligned}
w_n^{**} \in \quad & \arg\max_{w_n} \quad u_n\left(w_n, \boldsymbol{w}_{-n}^{**}\right) \\
& s.t. \quad \underline{s}_n \leq S_n\left[p\left(w_n, \boldsymbol{w}_{-n}^{**}\right), w_n\right] \leq \bar{s}_n, \\
& \qquad -\boldsymbol{f} \leq [\mathbf{A}_g]_{*n} \cdot S_n\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_n\right] \\
& \qquad + \sum_{\substack{m \in \mathcal{N}_g \\ m \neq n}} \left\{[\mathbf{A}_g]_{*m} \cdot S_m\left[p(w_n, \boldsymbol{w}_{-n}^{**}), w_m^{**}\right]\right\} \\
& \qquad + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}.
\end{aligned}
$$

In a SFE, each generator's bid maximizes its own payoff given the others' bids. What is special about our SFE is that the set of feasible bids of each generator depends on the others' bids. Therefore, the SFE is a generalized Nash equilibrium, which is usually harder to analyze than a standard Nash equilibrium [11].

III. Efficiency of Linear Supply Function Bidding

In this section, we will prove that the SFE exists and results in a unique equilibrium supply profile. In addition, we will provide an upper bound of the efficiency loss at SFE, and discuss how this upper bound depends on the transmission network topology.

Our first main result is the characterization of SFE through a convex programming. Specifically, we show that any SFE results in a unique equilibrium supply profile, which is an optimal solution to a convex optimization problem.

Proposition 1: Any SFE results in an allocation that is the unique solution to the following modified cost minimization problem:

$$\min_{\boldsymbol{s}} \quad \sum_{n \in \mathcal{N}_g} \hat{c}_n(s_n) \tag{4}$$

$$s.t. \quad \sum_{n \in \mathcal{N}_g} s_n = D, \tag{5}$$

$$\underline{\boldsymbol{s}} \leq \boldsymbol{s} \leq \bar{\boldsymbol{s}}, \tag{6}$$

$$-\boldsymbol{f}_1 \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{f}_2, \tag{7}$$

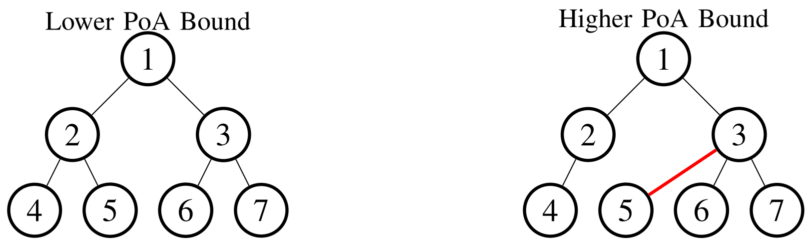
where

$$\hat{c}_n(s_n) = \left(1 + \frac{s_n}{D - 2s_n}\right) \cdot c_n(s_n) - \int_0^{s_n} \frac{D}{(D-2x)^2} \cdot c_n(x) dx. \tag{8}$$

Proof: See Appendix A. ■

Proposition 1 is significant in describing the equilibrium outcome. It shows that although there may be multiple equilibrium bidding profiles, the resulting equilibrium supply profile, which is what we care about the most, is always unique. Moreover, Proposition 1 proves that the equilibrium supply profile is the optimal solution to a convex optimization problem, which provides an efficient way of computing the equilibrium supply profile.

[4] We denote the nth column of a matrix $\mathbf{A}$ by $[\mathbf{A}]_{*n}$.



(a) Network 1 (b) Network 2

Fig. 1. Impact of network topology on the efficiency loss. Assume that all the line capacities are the same. In both networks, the PoA upper bound is determined by node 3, which has the highest connectivity. By connecting node 5 to node 3, the network on the right has a higher PoA bound because node 3 has a higher degree than node 3 on the left.

The convex optimization problem in Proposition 1 is crucial in analyzing the PoA. Before we go into detailed analysis, we formally define PoA here.

Definition 2: PoA is defined as

$$\frac{\sum_{n \in \mathcal{N}_g} c_n(s_n^{**})}{\sum_{n \in \mathcal{N}_g} c_n(s_n^{*})},$$

where $(s_n^{**})_{n \in \mathcal{N}_g}$ is the equilibrium supply profile, and $(s_n^{*})_{n \in \mathcal{N}_g}$ is the supply profile that minimizes the total cost. The PoA is well defined because the equilibrium supply profile is unique.

By definition, the PoA is never smaller than 1. A larger PoA indicates that the efficiency loss at the equilibrium is larger. Next, we give an analytical upper bound of the PoA.

Theorem 1: The PoA is upper bounded by $1 + \frac{\Delta}{D - 2\Delta}$, where

$$\Delta \triangleq \max_{m \in \mathcal{N}_g} \min \left\{ \bar{s}_m, d_m + \sum_{(m,n) \in \mathcal{E}} f_{mn} \right\}. \tag{9}$$

Proof: See Appendix B. ■

Theorem 1 gives us an upper bound of the PoA. From the upper bound, we can get insight on the key factors that influence the efficiency loss. First, the efficiency loss can be higher if there is a generator with large generation capacity. This intuition has been obtained in some prior works [3] that do not consider flow limit constraints, and our contribution here is to show that the same intuition also applies to the case with flow limit constraints. Second, another factor that may affect the efficiency loss is how "connected" a generator is. More specifically, the term $d_m + \sum_{(m,n) \in \mathcal{E}} f_{mn}$ can be considered as generator m's weighted degree in the network (with the weights being the capacity of transmission lines) plus its demand. If one generator is much better connected than the others, the efficiency loss can be high. This is a new intuition that was known from prior works.

Our insights shed some light on planning the transmission networks and the locations of generators. We illustrate our insights in Fig. 1, where we show two networks with the same nodes but different network topology.Node 3 on the left has a degree of 3, while the one on the right has a higher degree of 4, because node 5 is added to its set of neighbors. Therefore, the network on the right has a higher PoA bound. This example suggests that it may be beneficial to evenly distribute the connections between generators.

(a) Network 1 (b) Network 2

Fig. 2. Two radial networks that have different local topologies but the same socially optimal and equilibrium supply profiles. The nodes with the same color are homogeneous, while the nodes with different colors can be different.

IV. Special Radial Networks

The results in the previous section hold for general electricity networks. In this section, we consider a special class of radial networks, and gain deeper insights for this class of networks.

A radial network is a network with no loop. In a radial network, we call each subtree starting from a child of the root a *branch*. We consider a special class of radial networks that have homogeneous nodes and edges within each branch (i.e., the same cost function, the same demand, the same upper and lower bounds of supply, and the same line flow limits). Different branches can have different parameters and cost functions. We proved that at both the socially optimal and equilibrium supply profiles, the supply levels of nodes within a branch are the same, and are independent of the network topology of each individual branch.

We formally state our results for the special radial networks as follows.

Proposition 2: In a radial network with identical nodes and lines within each branch, the following statements hold for both socially optimal and equilibrium supply profiles:

- The nodes within a branch have the same supply.
- The only possibly congested line (i.e., a line with a binding line flow constraint) is the one connected to the root.

Proof: See Appendix C. ■

One implication of Proposition 2 is that the local network topology in a branch does not matter. For illustration, we show in Fig. 2 two networks with different local network topologies. Based on Proposition 2, these two networks have the same socially optimal and equilibrium supply profiles.

V. Conclusion

In this paper, we studied the efficiency loss of linear supply function bidding in electricity markets. The key feature that sets our work apart from existing works is that we include the flow constraints of the transmission lines in our model. We show that in the resulting networked electricity markets, there exists a unique equilibrium supply profile. We provided an analytical upper bound of the efficiency loss at the equilibrium. Our upper bound suggests that to reduce the efficiency loss, we should evenly distribute the generation capacity and "connectivity" among the generators. Our upper bound also provides a precise definition the connectivity of a generator as its weighted degree in the network (weighted by the transmission line capacity) plus the demand at its location.

Appendix A
Proof of Proposition 1

Step 1: At any SFE, there exist at least two generators with strictly positive bids.

Suppose that all the generators bid zero, namely $w_n = 0$ for all n. Then each generator n supply D, resulting in a total supply of $|\mathcal{N}_g| D$. This supply profile violates the constraint that the supply equals the demand. Hence, at least one generator submits strictly positive bids.

Suppose that generator n is the only generator with a strictly positive bid. Then generator n needs to supply

$$D - \frac{w_n}{w_n / \left[\left(|\mathcal{N}_g - 1| \right) D \right]} = - \left(|\mathcal{N}_g - 2| \right) D < 0,$$

which violates the constraint that $s_n \geq 0$. Hence, at least two generators submit strictly positive bids.

Step 2: The payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ of each generator n is strictly convex in w_n at any equilibrium.

We write down the expression of the payoff function as

$$\begin{aligned} u_n(w_n, \boldsymbol{w}_{-n}) &= p(w_n, \boldsymbol{w}_{-n}) \cdot S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] - c_n \left(S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \right) \\ &= \frac{\sum_{m \in \mathcal{N}_g} w_m}{|\mathcal{N}_g| - 1} - w_n - c_n \left(S_n \left[p(w_n, \boldsymbol{w}_{-n}), w_n \right] \right), \end{aligned} \tag{10}$$

where

$$S_n \left(p(w_n, \boldsymbol{w}_{-n}), w_n \right) = D - \frac{w_n}{\sum_{m \in \mathcal{N}_g} w_m} \cdot \left(|\mathcal{N}_g| - 1 \right) D, \tag{11}$$

Since $c_n(s_n)$ is strictly convex and increasing in s_n, and $S_n \left(p(w_n, \boldsymbol{w}_{-n}), w_n \right)$ is strictly convex in w_n when $\sum_{m \neq n} w_m > 0$, the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n when $\sum_{m \neq n} w_m > 0$. In Step 1, we have proved that $\sum_{m \neq n} w_m > 0$ at an equilibrium. Hence, the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n at any equilibrium.

Step 3: Since the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$ is strictly convex in w_n, each generator n's optimization problem is a convex optimization problem. Therefore, $\boldsymbol{w}$ is a SFE if and only if it satisfies the KKT conditions for all n.

We write the Lagrangian multipliers associated with the local constraints (**??**) and (**??**) as $\underline{\mu}_n^{\rm SFE} \in \mathbb{R}_+$ and $\bar{\mu}_n^{\rm SFE} \in \mathbb{R}_+$, respectively. In addition, we write the Lagrangian multipliers associated with the common constraints (**??**) and (**??**) as $\boldsymbol{\lambda}_1^{\rm SFE} \in \mathbb{R}_+^{|\mathcal{E}|}$ and $\boldsymbol{\lambda}_2^{\rm SFE} \in \mathbb{R}_+^{|\mathcal{E}|}$, respectively. Then the action profile $\boldsymbol{w}$ is a SFE if and only if there exists $\underline{\mu}_n^{\rm SFE}$, $\bar{\mu}_n^{\rm SFE}$, $\boldsymbol{\lambda}_1^{\rm SFE}$, and $\boldsymbol{\lambda}_2^{\rm SFE}$ for all n such that the following KKT conditions are satisfied for all n:

$$\frac{\partial u_n(w_n, \boldsymbol{w}_{-n})}{\partial w_n} + \left[\bar{s}_n + \left(|\mathcal{N}_g| - 2 \right) D \right] \underline{\mu}_n^{\rm SFE} - \left(|\mathcal{N}_g| - 2 \right) \bar{\mu}_n^{\rm SFE} + \left(\boldsymbol{\lambda}_1^{\rm SFE} \right)^T \left(\boldsymbol{b}_n + \boldsymbol{f}_1 \right) - \left(\boldsymbol{\lambda}_2^{\rm SFE} \right)^T \left(\boldsymbol{b}_n - \boldsymbol{f}_2 \right) = 0 \tag{12}$$

$$0 \leq \underline{\mu}_n^{\rm SFE} \perp \left[\bar{s}_n + \left(|\mathcal{N}_g| - 2 \right) D \right] w_n + \left(\bar{s}_n - D \right) \sum_{m \in \mathcal{N}_g, m \neq n} w_m \geq 0 \tag{13}$$

$$0 \leq \bar{\mu}_n^{\rm SFE} \perp - \left(|\mathcal{N}_g| - 2 \right) \cdot w_n + \sum_{m \in \mathcal{N}_g, m \neq n} w_m \geq 0 \tag{14}$$

$$\boldsymbol{0} \leq \boldsymbol{\lambda}_1^{\rm SFE} \perp \sum_{n \in \mathcal{N}_g} \left(\boldsymbol{b}_n + \boldsymbol{f}_1 \right) w_n \geq \boldsymbol{0} \tag{15}$$

$$\boldsymbol{0} \leq \boldsymbol{\lambda}_2^{\rm SFE} \perp \sum_{n \in \mathcal{N}_g} \left(\boldsymbol{b}_n - \boldsymbol{f}_2 \right) w_n \leq \boldsymbol{0} \tag{16}$$

By plugging in the expression of the payoff function $u_n(w_n, \boldsymbol{w}_{-n})$, we can rewrite (12) as

$$\begin{aligned} 0 &= \frac{dc_n\left(S_n\left[p(w_n, \boldsymbol{w}_{-n}), w_n\right]\right)}{ds_n} \cdot \left(1 + \frac{S_n\left[p(w_n, \boldsymbol{w}_{-n}), w_n\right]}{\left(|\mathcal{N}_g| - 2\right) D}\right) - p(\boldsymbol{w}) \\ &+ \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \underline{\mu}_n^{\texttt{SFE}} - \left(\sum_{m \in \mathcal{N}_g} \bar{s}_m - D\right) \cdot p(\boldsymbol{w}) \cdot \bar{\mu}_n^{\texttt{SFE}} \\ &+ \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \left(\boldsymbol{\lambda}_1^{\texttt{SFE}}\right)^T \cdot (\boldsymbol{b}_n + \boldsymbol{c}_1) - \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \left(\boldsymbol{\lambda}_2^{\texttt{SFE}}\right)^T \cdot (\boldsymbol{b}_n - \boldsymbol{c}_2) \end{aligned} \tag{17}$$

Step 4: Since the optimization problem (MSW) is a convex optimization problem, $\boldsymbol{s}$ is a solution if and only if it satisfies the KKT conditions.

We write the Lagrangian multipliers associated with the constraints $\mathbf{0} \leq \boldsymbol{s}$ and $\boldsymbol{s} \leq \bar{\boldsymbol{s}}$ in (6) as $\underline{\boldsymbol{\mu}}^{\texttt{MWS}} = (\underline{\mu}_n^{\texttt{MWS}})$ and $\bar{\boldsymbol{\mu}}^{\texttt{MWS}} = (\bar{\mu}_n^{\texttt{MWS}})$, respectively. Similarly, we write the Lagrangian multiplier associated with the constraint $\sum_{n \in \mathcal{N}_g} s_n = D$ in (5) as $p^{\texttt{MSW}}$. Finally, we write the Lagrangian multipliers associated with the constraints $-\boldsymbol{c}_1 \leq \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d}$ and $\mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \leq \boldsymbol{c}_2$ in (7) as $\boldsymbol{\lambda}_1^{\texttt{MWS}}$ and $\boldsymbol{\lambda}_2^{\texttt{MWS}}$, respectively.

The KKT conditions are

$$\frac{dc_n(s_n)}{ds_n} \cdot \left(1 + \frac{s_n}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D}\right) - p^{\texttt{MSW}} - \underline{\mu}_n^{\texttt{MSW}} + \bar{\mu}_n^{\texttt{MSW}} - \left(\boldsymbol{\lambda}_1^{\texttt{MSW}}\right)^T \cdot [\mathbf{A}_g]_{*n} + \left(\boldsymbol{\lambda}_2^{\texttt{MSW}}\right)^T \cdot [\mathbf{A}_g]_{*n} = 0 \tag{18}$$

$$\sum_{n \in \mathcal{N}_g} s_n = D \tag{19}$$

$$\mathbf{0} \leq \underline{\boldsymbol{\mu}}^{\texttt{MSW}} \perp \boldsymbol{s} \geq \mathbf{0} \tag{20}$$

$$\mathbf{0} \leq \bar{\boldsymbol{\mu}}^{\texttt{MSW}} \perp \boldsymbol{s} \leq \bar{\boldsymbol{s}} \tag{21}$$

$$\mathbf{0} \leq \boldsymbol{\lambda}_1^{\texttt{MSW}} \perp (\mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} + \boldsymbol{c}_1) \geq \mathbf{0} \tag{22}$$

$$\mathbf{0} \leq \boldsymbol{\lambda}_2^{\texttt{MSW}} \perp (\mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} - \boldsymbol{c}_2) \leq \mathbf{0} \tag{23}$$

Step 5: Given any SFE $\boldsymbol{w}$, the allocation $\boldsymbol{s}$, where $s_n = \bar{s}_n - \frac{w_n}{p(\boldsymbol{w})}, \forall n$, is a solution to (MSW).

For any SFE $\boldsymbol{w}$, there exists $\underline{\mu}_n^{\texttt{SFE}}$, $\bar{\mu}_n^{\texttt{SFE}}$, $\boldsymbol{\lambda}_1^{\texttt{SFE}}$, and $\boldsymbol{\lambda}_2^{\texttt{SFE}}$ such that the KKT conditions are satisfied.

Define $s_n = \bar{s}_n - \frac{w_n}{p(\boldsymbol{w})}, \forall n$ and the Lagrangian multipliers of the problem MSW as follows

$$\begin{aligned} p^{\texttt{MSW}} &= p(\boldsymbol{w}) \\ &\cdot \left\{1 - \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot \left[\left(\boldsymbol{\lambda}_1^{\texttt{SFE}}\right)^T \cdot (\mathbf{A}_g \cdot \bar{\boldsymbol{s}}_m + \mathbf{A}_\ell \cdot \boldsymbol{d} + \boldsymbol{c}_1) + \left(\boldsymbol{\lambda}_2^{\texttt{SFE}}\right)^T \cdot (\mathbf{A}_g \cdot \bar{\boldsymbol{s}}_m + \mathbf{A}_\ell \cdot \boldsymbol{d} - \boldsymbol{c}_2)\right]\right\} \end{aligned} \tag{24}$$

$$\underline{\mu}_n^{\texttt{MSW}} = \left(\sum_{m \in \mathcal{N}_g} \bar{s}_m - D\right) \cdot p(\boldsymbol{w}) \cdot \bar{\mu}_n^{\texttt{SFE}} \tag{25}$$

$$\bar{\mu}_n^{\texttt{MSW}} = \frac{\sum_{m \in \mathcal{N}_g} \bar{s}_m - D}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \underline{\mu}_n^{\texttt{SFE}} \tag{26}$$

$$\boldsymbol{\lambda}_1^{\texttt{MSW}} = \frac{\left(\sum_{m \in \mathcal{N}_g} \bar{s}_m - D\right)^2}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \boldsymbol{\lambda}_1^{\texttt{SFE}} \tag{27}$$

$$\boldsymbol{\lambda}_2^{\texttt{MSW}} = \frac{\left(\sum_{m \in \mathcal{N}_g} \bar{s}_m - D\right)^2}{\sum_{m \in \mathcal{N}_g, m \neq n} \bar{s}_m - D} \cdot p(\boldsymbol{w}) \cdot \boldsymbol{\lambda}_2^{\texttt{SFE}} \tag{28}$$

It is not difficult to check that the KKT conditions of MSW are satisfied by $\boldsymbol{s}$, $p^{\texttt{MSW}}$, $\underline{\mu}_n^{\texttt{MSW}}$, $\bar{\mu}_n^{\texttt{MSW}}$, $\boldsymbol{\lambda}_1^{\texttt{MSW}}$, and $\boldsymbol{\lambda}_2^{\texttt{MSW}}$.

Step 6: Given a solution $\boldsymbol{s}$ to (MSW), we can construct a SFE $\boldsymbol{w}$.

This can be proved similarly as in Step 5, using the mapping between the Lagrangian multipliers.

APPENDIX B
PROOF OF THEOREM 1

First, we show that for any $s_n \in [\underline{s}_n, \bar{s}_n]$, we have

$$c_n(s_n) \leq \hat{c}_n(s_n) \leq \left(1 + \frac{s_n}{D - 2s_n}\right) \cdot c_n(s_n).$$

We rewrite $\hat{c}_n(s_n)$ here:

$$\begin{aligned} \hat{c}_n(s_n) \quad &= \quad \left(1 + \frac{s_n}{D - 2s_n}\right) \cdot c_n(s_n) \\ &- \quad \int_0^{s_n} \frac{D}{(D-2x)^2} \cdot c_n(x) dx. \end{aligned}$$

It is clear that $\hat{c}_n(s_n) \leq \left(1 + \frac{s_n}{D-2s_n}\right) \cdot c_n(s_n)$ because the integral in the expression of $\hat{c}_n(s_n)$ is nonnegative.

To see $c_n(s_n) \leq \hat{c}_n(s_n)$, we look at

$$\hat{c}_n(s_n) - c_n(s_n) = \frac{s_n}{D - 2s_n} \cdot c_n(s_n) - \int_0^{s_n} \frac{D}{(D-2x)^2} \cdot c_n(x) dx.$$

Note that $\hat{c}_n(0) - c_n(0) = 0$, and that the derivative of $\hat{c}_n(s_n) - c_n(s_n)$ is

$$\frac{s_n}{D - 2s_n} \cdot \frac{dc_n(s_n)}{ds_n} \geq 0,$$

because c_n is increasing and $0 \leq s_n \leq \bar{s}_n < \frac{D}{2}$. Therefore, we have $\hat{c}_n(s_n) - c_n(s_n) \geq 0$ for all $s_n \in [\underline{s}_n, \bar{s}_n]$.

Given the above inequality, we have

$$\begin{aligned} \sum_{n \in \mathcal{N}_g} c_n(s_n^{**}) \quad &\leq \quad \sum_{n \in \mathcal{N}_g} \hat{c}_n(s_n^{**}) \\ &\leq \quad \sum_{n \in \mathcal{N}_g} \hat{c}_n(s_n^*) \\ &\leq \quad \sum_{n \in \mathcal{N}_g} \left(1 + \frac{s_n^*}{D - 2s_n^*}\right) \cdot c_n(s_n^*) \\ &\leq \quad \sum_{n \in \mathcal{N}_g} \left(1 + \frac{\max_{m \in \mathcal{N}_g} s_m^*}{D - 2\max_{m \in \mathcal{N}_g} s_m^*}\right) \cdot c_n(s_n^*) \\ &\leq \quad \left(1 + \frac{\max_{m \in \mathcal{N}_g} s_m^*}{D - 2\max_{m \in \mathcal{N}_g} s_m^*}\right) \cdot \sum_{n \in \mathcal{N}_g} c_n(s_n^*), \end{aligned}$$

where the first and third inequalities come from $c_n(s_n) \leq \hat{c}_n(s_n) \leq \left(1 + \frac{s_n}{D-2s_n}\right) \cdot c_n(s_n)$ for any feasible s_n, and the second inequality follows from the fact that $(s_n^{**})_{n \in \mathcal{N}_g}$ is the optimal solution to the modified cost minimization problem.

We can see that an upper bound of the PoA is

$$1 + \frac{\max_{m \in \mathcal{N}_g} s_m^*}{D - 2\max_{m \in \mathcal{N}_g} s_m^*},$$

which is increasing in $\max_{m\in\mathcal{N}_g} s_m^*$. Hence, our next step is to find an upper bound for $\max_{m\in\mathcal{N}_g} s_m^*$.

Since $\max_{m\in\mathcal{N}_g} s_m^*$ is the maximum of all generators' socially optimal supplies, it must be bounded by the maximum of all generators' feasible supplies. In other words, it is bounded by the optimal value of the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \max_{m\in\mathcal{N}_g} s_m \\ s.t. \quad & \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, \\ & \underline{s}_n \le s_n \le \bar{s}_n, \quad \forall n \in \mathcal{N}_g, \\ & -\boldsymbol{f} \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}. \end{aligned} \tag{29}$$

The optimal value of (29) is upper bounded by the optimal value of another optimization problem with some constraints removed. Specifically, it is upper bounded by the optimal value of the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \max_{m\in\mathcal{N}_g} s_m \\ s.t. \quad & \textstyle\sum_{n\in\mathcal{N}_g} s_n = D, \\ & \underline{s}_n \le s_n \le \bar{s}_n, \quad \forall n \in \mathcal{N}_g. \end{aligned} \tag{30}$$

We can see that the optimal value of (30) is

$$\max_{m\in\mathcal{N}_g} \bar{s}_m.$$

This can be achieved by letting the generator with the largest capacity provides its maximum supply, and the other generators provide any supply profile that satisfies the supply and demand balance constraint.

The optimal value of (29) is also upper bounded by the optimal value of the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \max_{m\in\mathcal{N}_g} s_m \\ s.t. \quad & -\boldsymbol{f} \le \mathbf{A}_g \cdot \boldsymbol{s} + \mathbf{A}_\ell \cdot \boldsymbol{d} \le \boldsymbol{f}. \end{aligned} \tag{31}$$

Since it is hard to solve (31) directly, we work on an equivalent formulation where the variables are the supply profile and the flows on each line $\boldsymbol{p} = (p_{ij})_{\{i,j\}\in\mathcal{E}}$.

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \max_{m\in\mathcal{N}_g} s_m \\ s.t. \quad & -f_{ij} \le p_{ij} \le f_{ij}, \\ & \sum_{(i\to j)\in\ell} \frac{p_{ij}}{B_{ij}} = 0, \text{for any loop } \ell, \\ & s_i - d_i = \sum_{j:i\to j} p_{ij} - \sum_{k:k\to i} p_{ki}, \forall i \in \mathcal{N}. \end{aligned} \tag{32}$$

Note that the constraints are flow limit constraints (the first constraint), voltage angle constraints (the second one), and energy conservation at each node (the third one). We further relax the problem by removing the voltage angle constraint, and get:

$$\begin{aligned} \max_{\boldsymbol{s}} \quad & \max_{m \in \mathcal{N}_g} s_m \\ s.t. \quad & -f_{ij} \le p_{ij} \le f_{ij}, \\ & s_i - d_i = \sum_{j: i \to j} p_{ij} - \sum_{k: k \to i} p_{ki}, \forall i \in \mathcal{N}. \end{aligned} \tag{33}$$

Now we can analytically calculate the optimal value of (33) as

$$\max_{m \in \mathcal{N}_g} \left\{ d_m + \sum_{(m,n) \in \mathcal{E}} f_{mn} \right\}.$$

In summary, we have proved that the optimal value of (29) is upper bounded by

$$\begin{aligned} \Delta \quad & \triangleq \quad \min \left\{ \max_{m \in \mathcal{N}_g} \bar{s}_m, \max_{m \in \mathcal{N}_g} \left\{ d_m + \sum_{(m,n) \in \mathcal{E}} f_{mn} \right\} \right\} \\ & = \quad \max_{m \in \mathcal{N}_g} \min \left\{ \bar{s}_m, d_m + \sum_{(m,n) \in \mathcal{E}} f_{mn} \right\}, \end{aligned}$$

which yields the upper bound of the PoA $1 + \frac{\Delta}{D - 2\Delta}$ in Theorem 1.

APPENDIX C
PROOF OF PROPOSITION 2

For a special radial network, the social welfare maximization problem can be simplified as

$$\min_{\boldsymbol{s}} \quad \sum_{k=0}^{K} \sum_{n \in \mathcal{N}^k} c^k(s_n) \tag{34}$$

$$s.t. \quad \sum_{k=0}^{K} \sum_{n \in \mathcal{N}^k} s_n = D, \tag{35}$$

$$\underline{s}^k \le s_n \le \bar{s}^k, \forall k, \forall n \in \mathcal{N}^k, \tag{36}$$

$$-f^k \le \sum_{n: \ell \in \mathcal{E}_n} (s_n - d^k) \le f^k, \forall k, \forall \ell \in \mathcal{E}^k. \tag{37}$$

Before proving the proposition, we first prove a useful monotonicity property of the optimal supply profile.

Lemma 1: At the optimal supply profile, on any path from the root to a leaf node, the supplies of the nodes on this path must satisfy either one of the two conditions:

- all the supplies are no smaller than the demand d^k, and are non-increasing in the node's distance from the root, namely $s_i \ge s_j \ge d^k$ for any $i \prec j$; or
- all the supplies are no larger than the demand d^k, and are non-decreasing in the node's distance from the root, namely $s_i \le s_j \le d^k$ for any $i \prec j$.

Proof:

We choose an arbitrary path in branch k. If all the nodes in this path have the optimal supply equal to the demand (i.e., $s_n = d^k, \forall n \in \mathcal{N}^k$), the monotonicity property is immediately satisfied. In the rest of the proof, we consider the cases where at least one node has an optimal supply that is different from the demand d^k.

Among all the nodes with an optimal supply different from the demand d^k, we pick one that has the longest distance from the root (if there are multiple nodes with the same longest distances, we pick an arbitrary one). We suppose that this node is node n, namely

$$n \in \arg \max_{m \in \mathcal{N}^k : s_m \neq d^k} |\mathcal{N}_m| . \tag{38}$$

There are two possibilities, $s_n > d^k$ and $s_n < d^k$. We first consider the case of $s_n > d^k$.

First, all the nodes $i \succ n$ have $s_i = d^k$. Hence, we have $s_i \geq s_j \geq d^k$ for any i, j that are descendants of n.

Second, we claim that any node $m \prec n$ must have $s_m \geq s_n$. Assume that the contrary is true: namely, there exists nodes that are ancestors of n and have supply smaller than s_n. Suppose that among these nodes, the one furthest away from the root is node m. We define a new supply profile $\boldsymbol{s}'$, where $s'_n = s_n - \varepsilon$, $s'_m = s_m + \varepsilon$, and $s'_i = s_i, \forall i \neq m, n$. Here $\varepsilon > 0$ is a positive number small enough such that $s'_m < s'_n$. Clearly, this new supply profile satisfies the generator capacity constraints. We now check the transmission line flow constraints.

For any line ℓ "above" node m, we have $\sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i) = \sum_{i:\ell \in \mathcal{N}_i} (s'_i - d_i)$, because $\ell \in \mathcal{N}_m$ and $\ell \in \mathcal{N}_n$. For any line ℓ "below" node n, we have $\sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i) = \sum_{i:\ell \in \mathcal{N}_i} (s'_i - d_i)$, because we did not change the supply of those nodes. For any line ℓ between node m and node n, we have

$$\sum_{i:\ell \in \mathcal{N}_i} (s'_i - d_i) = \sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i) - \varepsilon < \sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i), \tag{39}$$

because $\ell \in \mathcal{N}_n$ and $\ell \notin \mathcal{N}_m$. Since node m is the furthest node with supply lower than d^k, we must have $\sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i) > 0$. Therefore, we can find a ε small enough such that $0 \leq \sum_{i:\ell \in \mathcal{N}_i} (s'_i - d_i) < \sum_{i:\ell \in \mathcal{N}_i} (s_i - d_i) \leq f^k$.

In summary, we can find a ε small enough such that $\boldsymbol{s}'$ is feasible. Since all the nodes in branch k have the same convex cost function, the supply profile $\boldsymbol{s}'$ results in a lower total cost than $\boldsymbol{s}$. This is contradictory to the fact that $\boldsymbol{s}$ is the optimal supply. This proves our claim that any node $m \prec n$ must have $s_m \geq s_n$.

We can repeat the above procedure for the neighboring ancestor of node n, for the neighboring ancestor of that node, and so on. As a result, we have $s_i \geq s_j \geq d^k$ for any $i \prec j$.

Using the same argument, we can prove that if node n has an optimal supply $s_n < d^k$, then we have $s_i \leq s_j \leq d^k$ for any $i \prec j$. ■

Now we prove Proposition 2 based on the monotonicity of the optimal supply profile.

First, in one branch, there cannot be two paths with supplies lower than the demand on one path and supplies larger than the demand on the other path. This is because any two paths in same branch must share some nodes close to the root (two disjoint paths would be in two different branches). Any node n shared by such two paths must have $s_n < d^k$ and $s_n > d^k$, which is impossible. Hence, in one branch, the supplies must be no greater than the demand or no smaller than the demand for all the nodes.

We prove the proposition for the case where all the nodes have supplies no smaller than the demand. The same logic applies to the proof for the case where all the nodes have supplies no larger than the demand.

Since $s_n \geq d^k$, we have $\sum_{n:\ell \in \mathcal{E}_n} (s_n - d_n) \geq 0 > -f^k$ for any line ℓ, and hence $\lambda_{1,\ell} = 0$ for any line ℓ. Contrary to the claim in the proposition, we assume that there is a line ℓ that is not connected to the root and has a binding flow limit constraint. To ensure that the flow constraints for the lines above line ℓ are satisfied, we must have $s_n \leq d^k$ for any n that is above line ℓ. Hence, we have $s_n = d^k$ for any n that is above line ℓ. Due to the monotonicity property, we must have $s_n = d^k$ for all the nodes in this branch. However, in this case, the flow constraint for line ℓ cannot be binding. This leads to a contradiction. Hence, the only possible congested line is the one that connects to the root.

To prove that all the nodes have the same optimal supply, we look at the KKT conditions. Define $p \in \mathbb{R}$ as the Lagrangian multiplier associated with (35), $\underline{\mu}_n, \bar{\mu}_n \in \mathbb{R}_+$ as the Lagrangian multipliers associated with (36), and $\lambda_{1,\ell}, \lambda_{2,\ell} \in \mathbb{R}_+$ as the Lagrangian multipliers associated with (37). Then we can write down the KKT conditions as follows:

$$\frac{dc^k(s_n)}{ds_n} - p - \underline{\mu}_n + \bar{\mu}_n - \sum_{\ell \in \mathcal{E}_n} \lambda_{1,\ell} + \sum_{\ell \in \mathcal{E}_n} \lambda_{2,\ell} = 0, \ \forall k, \forall n \in \mathcal{N}^k, \tag{40}$$

$$\sum_{k=0}^{K} \sum_{n \in \mathcal{N}^k} s_n = D \tag{41}$$

$$0 \leq \underline{\mu}_n \perp s_n \geq \underline{s}^k, \ \forall n, \tag{42}$$

$$0 \leq \bar{\mu}_n \perp -s_n \geq -\bar{s}^k, \ \forall n, \tag{43}$$

$$0 \leq \lambda_{1,\ell} \perp \sum_{n: \ell \in \mathcal{E}_n} (s_n - d_n) \geq -f^k, \ \forall k, \forall n \in \mathcal{N}^k, \tag{44}$$

$$0 \leq \lambda_{2,\ell} \perp -\sum_{n: \ell \in \mathcal{E}_n} (s_n - d_n) \geq -f^k, \ \forall k, \forall n \in \mathcal{N}^k. \tag{45}$$

We have shown that all the lines, except the one connecting to the root, have non-binding flow limit constraints. We index the line connecting to the root by ℓ^*. Then we have $\lambda_{2,\ell} = 0$ for all $\ell \neq \ell^*$. Since we are in the case where Hence, the first-order condition can be simplified to

$$\frac{dc^k(s_n)}{ds_n} = p + \underline{\mu}_n - \bar{\mu}_n - \lambda_{2,\ell^*}, \forall n \in \mathcal{N}^k. \tag{46}$$

We look at the root node m of branch k. If $s_m = \underline{s}^k$, due to the monotonicity property, we must have $s_n = \underline{s}^k$ for all $n \succ m$. Then the proposition is proved.

If $s_m \in \left(\underline{s}^k, \bar{s}^k\right)$, we have $\underline{\mu}_m = \bar{\mu}_m = 0$, and $\frac{dc^k(s_m)}{ds_m} = p - \lambda_{2,\ell^*}$. For any $n \succ m$, since $s_n \leq s_m$, we have $\bar{\mu}_n = 0$, and $\frac{dc^k(s_n)}{ds_n} = p + \underline{\mu}_n - \lambda_{2,\ell^*} \geq p - \lambda_{2,\ell^*} = \frac{dc^k(s_m)}{ds_m}$. Since c^k is strictly convex and $s_n \leq s_m$, the only possible solution is that $\underline{\mu}_n = 0$, which results in $s_n = s_m$.

If $s_m = \bar{s}^k$, we have $\underline{\mu}_m = 0$, and $\frac{dc^k(s_m)}{ds_m} = p - \bar{\mu}_m - \lambda_{2,\ell^*}$. For any $n \succ m$, if $s_n < \bar{s}^k$, we have $\bar{\mu}_n = 0$, and $\frac{dc^k(s_n)}{ds_n} = p + \underline{\mu}_n - \lambda_{2,\ell^*} \geq \frac{dc^k(s_m)}{ds_m} = p - \bar{\mu}_m - \lambda_{2,\ell^*} = \frac{dc^k(s_m)}{ds_m}$. But since c^k is strictly convex and $s_n < s_m$, we must have $\frac{dc^k(s_n)}{ds_n} < \frac{dc^k(s_m)}{ds_m}$. This leads to a contradiction. Hence, we must have $s_n = \bar{s}^k$ for any $n \succ m$.

In conclusion, all the nodes must have the same optimal supply.

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