Homework 3 (EECS 333) Introduction to Communication Networks

Prof. Dongning Guo

Due in class on April 13, 2007 (Friday)

Reading assignment

Sections 3.8–9 and 5.1, 5.4-6 in Leon-Garcia/Widjaja.
We will also cover Sections 5.2–3 later.

Classnotes

All handouts, class notes, the homeworks and their solutions are found on the course website.

Problem 1  Signal Attenuation

Consider communication through a cable at 1 MHz, where the signal power is attenuated by 0.7 dB per kilometer.

1. How long can a link be if an attenuation of 20 dB can be tolerated?
2. Repeat question 1 assuming we use an alternative cable which has an attenuation of 50% each kilometer.

Problem 2  Rate, Reliability and Capacity

Consider a low-pass communications system with a 1 MHz bandwidth.

1. (Rate.) What bit rate is attainable if one chooses to use 4-level pulses?
2. (Reliability.) Let 1 pulse of two possible levels be sent per microsecond. Suppose the noise is additive Gaussian and the signal-to-noise ratio (SNR) is 10 dB. What is the minimum probability of misreading the level because of the noise? Give a numerical result with justification.
   [Hint: You can consider the following model:
   \[ Y = X + \sigma W \]
   where \( X \) takes \(-1\) and \(+1\) equally likely, and \( W \) is a standard Gaussian random variable, i.e., the probability density function of \( W \) is
   \[ p(w) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{w^2}{2} \right]. \tag{1} \]
   The noise variance \( \sigma^2 \) is the inverse of the SNR. You may find the result through the error function \( \text{erfc}(\cdot) \).
3. (Capacity.) Again consider the original low-pass communications system with a 1 MHz bandwidth. What is the Shannon capacity of this channel if the SNR is 10 dB? Compare the result with the rate in part 1. What does the comparison tell us in terms of the rate and reliability?
   [Hint: Recall that the Shannon capacity of a channel with bandwidth \( W \), noise spectrum density \( N_0 \) and power \( P \) is
   \[ C = W \log_2 \left( 1 + \frac{P}{WN_0} \right) \]
   where \( P/(WN_0) \) is the SNR.]
Problem 3  Bit-stuffing Overhead

Suppose the data to be transmitted is a (long) sequence of independent bits equally likely to be 0 and 1. Find the exact average bit stuffing overhead for inserting a 0 following every TWO consecutive 1’s.

[Hint: Suppose you are at the beginning or have just stuffed a “0”. Let $x$ be the average number of bits you have to read before another bit-stuffing takes place. How is it related to the overhead? In order to find $x$, consider a 3-state machine that describes the state of the bit-stuffing algorithm. The state can be the number of consecutive 1’s seen so far since the last occurrence of 0 (including a stuffed “0”).]

Problem 4  Extended Hamming Code

Problem 3.69 in Leon-Garcia.

Problem 5  Block Error Probability

Problem 5.15 (a) (b) in Leon-Garcia.

Problem 6  Block Size Matters

Problem 5.15 (c) (d) (e) in Leon-Garcia.

[Note: Altogether Problem 5.15 is worth 20 points.]