A Unified Approach to Power Control for Multiuser Detectors

Farhad Meshkati, Dongning Guo, H. Vincent Poor, Stuart C. Schwartz and Narayan B. Mandayam

Abstract— **In this paper, a unified approach to power control is proposed for a large family of receivers which includes the matched filter, decorrelator, and minimum mean square error detectors as well as the individually and jointly optimal multiuser detectors. The proposed algorithm exploits the linear relationship that has been shown to exist between the transmit power and the output signal to noise plus interference ratio in large systems. Simulation is used to show the convergence of the proposed power control scheme and demonstrate its performance in finite-size systems.**

Keywords— **Power control, multiuser detection, multiuser efficiency, large-system analysis.**

I. Introduction

Power control plays an important role in wireless CDMA networks. It is used for interference management and resource allocation. In the uplink, for example, the purpose of power control is for each mobile to transmit just enough power to achieve the required quality of service without causing unnecessary interference in the system. Wideband CDMA is the main multiaccess scheme for third generation cellular systems. While second generation systems (e.g. IS-95) were mainly designed to carry voice services, data applications have become an important part of the traffic carried by third generation systems. Data services have different characteristics as compared to voice services. Namely, they are less tolerant to error but are not as sensitive to delay. Power control for voice services has been studied extensively (see [1–3]). The most common approach is for each user to adjust its transmit power to achieve a target SIR (signal to interference plus noise ratio), which is determined by the desired voice quality. In [4–8], game theory has been used to study power control for data networks. Taking this approach, it is shown in [4, 6] that Nash equilibrium is achieved when all the users aim for a target SIR, γ^*. This is true even when the matched filter is replaced by the decorrelator or the minimum mean square error (MMSE) receiver as shown in [7]. The Nash equilibrium is the set of transmit powers for which no user could improve its utility (which is a measure of user's happiness) given the transmit powers of other users.

Multiuser receivers have been shown to provide significant improvements in terms of system capacity as compared to the conventional matched filter [9]. As the demand for higher data rates increases, the use of multiuser receivers becomes more compelling. Multiuser receivers are expected to be deployed in future wireless systems, especially in the uplink. Because of this, power control for multiuser detectors has attracted attention in recent years. In particular, power control algorithms for the MMSE detector and successive interference cancellation (SIC) receiver have been proposed in [10, 11] for voice-oriented systems. It is shown in [10] that for the MMSE receiver, similar to the matched filter, the output SIR is measured for each user and then the user's transmit power is adjusted to achieve the target SIR. So, after iteration n, the transmit power of user k is updated as $p_k(n+1) = \frac{\gamma^*}{\gamma_k(n)} p_k(n)$, where γ^* is the target SIR and $\gamma_k(n)$ is the output SIR for the k^{th} user after the n^{th} iteration. A similar approach is shown to also work for the SIC receiver [11].

So far, power control for multiuser receivers has been considered separately for each receiver type and no general approach has been proposed. In this paper, we present a unified approach to power control that is applicable to a large family of multiuser detectors. Our approach is based on the large-system results in [12]. A linear relationship between the input power and the output SIR in the large-system limit has been shown to exist for a family of multiuser detectors that includes many well-known receivers such as the matched filter, decorrelator, and MMSE detectors as well as the jointly and individually optimal detectors [9]. This linear relationship, which is characterized by the multiuser efficiency, is exploited in obtaining the proposed power control algorithm. The algorithm, which is a unifying approach to power control, can be implemented in both voice and data oriented networks with multiuser detectors (as well as the matched filter). Convergence and performance of the proposed scheme in finite-size systems are also demonstrated using simulation.

The organization of this paper is as follows. In Section II, we provide some relevant background on multiuser detection. Power control for both data and voice networks is discussed in Section III. The description of our proposed power control algorithm is given in Section IV. We present simulation results in Section V and give conclusions in Section VI.

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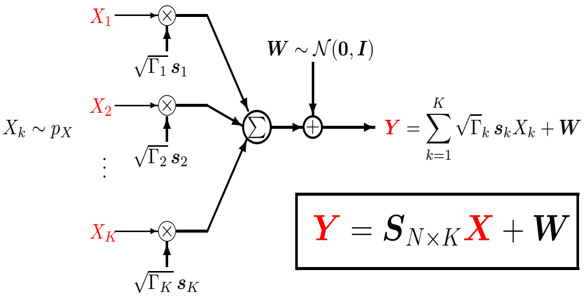

Fig. 1. System Model for Uplink CDMA

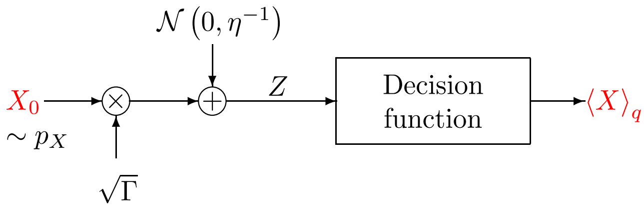

Fig. 2. Equivalent Single-User Channel for the Multiuser Channel

II. Multiuser Detection

We consider the uplink of a synchronous DS-CDMA system with K users and a processing gain of N. Let p_k, h_k, and γ_k represent the transmit power, channel gain and output SIR, respectively, for user k. Also, define

$$\Gamma_k = \frac{p_k h_k}{\sigma^2} \tag{1}$$

as the received signal to noise ratio (SNR) for user k where σ^2 here is the background noise power (including other cell interference). The received signal (after chip-matched filtering) sampled at the chip rate over one symbol duration can be represented as

$$\mathbf{Y} = \sum_{k=1}^{K} \sqrt{\Gamma_k} X_k \mathbf{s_k} + \mathbf{W} \tag{2}$$

where $\mathbf{s_k}$ and X_k are the spreading sequence and transmitted symbol of user k, respectively. We assume random spreading sequences for all users. X_k's are assumed to be independent and identically distributed (i.i.d.) with distribution p_X. In (2), $\mathbf{W}$ is the normalized noise vector which is Gaussian with mean $\mathbf{0}$ and covariance $\mathbf{I}_{N\times N}$. The system model is shown in Fig. 1.

For any given detector, the multiuser efficiency, η_k, for user k, quantifies the performance loss due to the existence of other users in the system. In general, η_k depends on the received SNRs, the spreading sequences as well as the type of detector. However, in the asymptotic case of large systems, the dependence on the spreading sequences disappears and the received SNRs affect η only through their distribution. By a large system, we refer to the limit where the number of users and the spreading factor in a CDMA system both tend to infinity but with a fixed ratio. In [12], it is argued that every multiuser detector can be regarded as a conditional mean estimator (CME) for certain postulated CDMA channel and input distribution. In other words, a multiuser detector is a generalized CME, which is optimal for the postulated system but suboptimal for the actual system.

In [12], the large-system input-output relationship of a class of generalized CME that corresponds to a particular postulated system is studied. The postulated system considered has an arbitrary input distribution, q_X, and a postulated CDMA channel that differs from the actual channel only in the noise variance, i.e.

$$\mathbf{Y} = \sum_{k=1}^{K} \sqrt{\Gamma_k} X_k \mathbf{s_k} + \varrho \mathbf{W} \tag{3}$$

where X_k's have distribution q_X and ϱ is the postulated channel parameter. The output of the generalized CME for such a system can be written as $\langle \mathbf{X} \rangle_q = \mathbb{E}_q\{\mathbf{X}|\ \mathbf{Y}, \mathbf{s}_1, ..., \mathbf{s}_K\}$, where $\mathbf{X} = [X_1, ..., X_K]^T$ and q represents the probability laws of the postulated system. This family of receivers contains many popular receivers such as the conventional matched filter, decorrelator, and MMSE detectors as well as the jointly and individually optimal detectors. For example, if q_X is the standard normal distribution, the CME becomes the matched filter when $\varrho \to \infty$. It is shown in [12] that for this class of receivers, the large-system multiuser efficiency is obtained by solving the following joint equations:

$$\eta^{-1} = 1 + \beta\, \mathbb{E}\{\Gamma \cdot \mathcal{E}(\Gamma; \eta, \xi)\}, \tag{4}$$
$$\xi^{-1} = \varrho + \beta\, \mathbb{E}\{\Gamma \cdot \mathcal{V}(\Gamma; \eta, \xi)\}, \tag{5}$$

where $\beta = \lim_{K,N\to\infty} \frac{K}{N}$. The expectations are taken with respect to the received SNR distribution, p_Γ. In (4) and (5), $\mathcal{E}$ and $\mathcal{V}$ are functions that can be easily computed given the distribution of the transmitted symbols and the type of receiver (see [12]).

In the large-system limit, the multiuser channel can be modelled as a single-user channel shown in Fig. 2. For any detector in this family, the large-system output SIR is then given by

$$\gamma_k = \eta_k \Gamma_k = \frac{\eta_k h_k}{\sigma^2} p_k\ , \tag{6}$$

where η_k is the multiuser efficiency of user k which satisfies (4) and (5). Hence, there exists a linear relationship between the output SIR and the transmit power for each user. This is mainly due to the fact that in a large system, as the user's transmit power changes, the interference seen by that user essentially stays the same as long as the overall distribution of the received powers remains the same. Note that although (6) is true only in the large-system limit, it is a very good approximation for most finite-size systems of practical interest.

III. Power Control for Voice and Data Networks

Power control for voice services has been studied extensively. It is well known that for voice-oriented systems, the optimal power control strategy is for all users to target a specific SIR. Power control for data networks, on the other hand, has not been explored as much. One approach that has been successful in providing insights into design of power control schemes for data networks has been the game theoretic approach proposed in [4]. In this approach, power control is modelled as a non-cooperative game in which each user tries to maximize its own utility. The utility is defined as the ratio between user's throughput and transmit power. For the matched filter, it has been shown in [4,6] that Nash equilibrium is achieved when all users aim for a target SIR, γ^*, which is dependent on the packet size and modulation. This is true even when the matched filter is replaced by the decorrelator or MMSE detectors (see [7]). The results can also be extended to multiple antenna systems as shown in [8].

As described in Section II, for a large family of receivers, there exists a linear relationship between the user's output SIR and transmit power. This linear relationship, characterized by multiuser efficiency, applies to many popular receivers such as the matched filter, decorrelator, and MMSE detectors as well as the individually and jointly optimal multiuser detectors. Based on (4) and (5), the multiuser efficiency is dependent on the received powers only through their distribution. This means that in the asymptotic case of large systems, η_k is essentially independent of p_k, and hence we have

$$\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k} \ . \quad (7)$$

If we consider a data network similar to the one considered in [4], then based on (7) and following the same reasonings as those in [6–8], we can say that in the large-system limit, the Nash equilibrium is achieved when all users aim for γ^* as their output SIR. This is true for the family of receivers to which (4) and (5) apply. Here, the Nash equilibrium is the set of transmit powers for which no user would be able to improve its utility given the transmit powers of other users.

It is seen from the above discussion that, in both voice and data systems, it is desirable for each user to aim for a target SIR, γ^*, independent of the type of the receiver. The difference is that for voice services, the target SIR is determined by the desired quality of voice whereas for data services, the target SIR depends on the modulation and packet size.

IV. The Unified Power Control Algorithm

In this section, we present a unified approach to power control that can be applied to a large family of receivers. The objective of the algorithm is for each user to achieve an output SIR equal to γ^*. To do this, we exploit the linear relationship between the output SIR and transmit power, given by (6). We use multiuser efficiency as the unifying concept for our algorithm. The description of the proposed algorithm is as follows.

The Unified Power Control (UPC) Algorithm:

1. *n=0*, start with initial powers $p_1(0), ..., p_K(0)$.
2. Based on the power profile, calculate the multiuser efficiency, $\eta_k(n)$, using (4) and (5).
3. Update the powers using $p_k(n+1) = \frac{1}{\eta_k(n)}(\frac{\gamma^* \sigma^2}{h_k})$ for $k = 1, ..., K$.
4. *n=n+1*, stop if convergence has reached; otherwise, go to Step 2.

Note that in Step 2, while finding an analytical expression for η_k is difficult for most multiuser detectors, η_k can be easily obtained using numerical methods. Also, if all the users have the same type of receiver, η_k will be independent of k and, hence, (4) and (5) need to be solved only once per iteration. The above algorithm is applicable to a large family of receivers which includes many popular receivers such as the matched filter, decorrelator, and MMSE detectors as well as jointly and individually optimal multiuser detectors. It should be noted that the power control algorithm given in [1] cannot be easily applied to the optimal multiuser receivers since finding the output SIR in this case is not straightforward. Proving the convergence of our proposed algorithm for finite-size systems is difficult. First of all, (4) and (5) are asymptotic results. Secondly, even if we assume that these expressions are applicable to finite-size systems, the coupled nature of the equations and their complexity make it difficult to prove whether our proposed algorithm converges. Because of these reasons, we use simulation to investigate the convergence and performance of the proposed power control scheme. We see in the next section that the UPC algorithm does indeed converge in finite-size systems and the convergence is quite rapid.

V. Simulation Results

In this section, we consider the uplink of a DS-CDMA system with processing gain 32 and 8 users (i.e. $N = 32$ and $K = 8$). The channel gain for user k is given by $h_k = 0.1 d_k^{-4}$ where d_k is the distance of the user k from the base station and is given by $d_k = 100 + 10k$ in

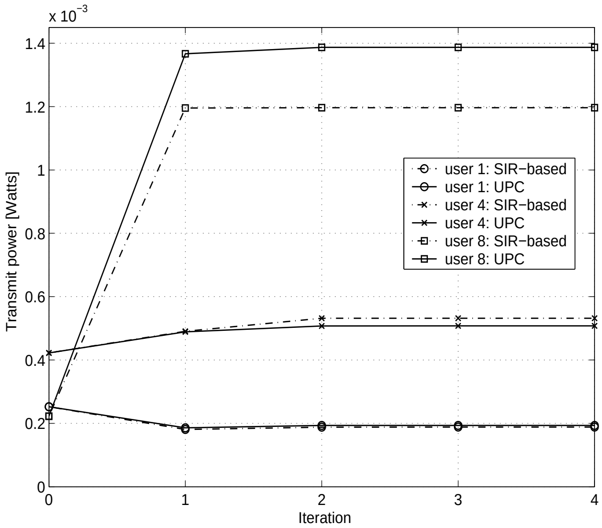

Fig. 3. Users' Transmit Power for MMSE Detector ($N = 32$ and $K = 8$)

meters. We simulate the power control scheme described in the previous section with the target SIR being equal to 8.1dB. This is the SIR at the Nash equilibrium in a data network with 100 bits per packet (see [7]).

We first consider the MMSE receiver and compare the performance of our proposed UPC algorithm with the scheme presented in [10], which we call the SIR-based scheme. In Fig. 3, we plot the transmit powers for users 1,4 and 8 at the end of each iteration for a fixed set of spreading sequences. It is seen from the figure that the UPC algorithm not only converges but also is in close agreement with the conventional SIR-based method. We repeat this for 1000 realizations of the spreading sequences and initial power vector. The summary of results is given in Table I. The steady state values of transmit powers are the same in UPC scheme for all 1000 realizations because multiuser efficiency is independent of the spreading sequences. For the SIR-based method, the steady state values depend on the particular set of spreading sequences. However, their averages are very close to the values obtained by the UPC algorithm. We have also calculated the difference between the steady state power values of each user for the above two cases and found the standard deviation of this difference, ϵ_{diff} (see Table I). Based on this, we can say that the UPC steady state power values are within 1dB of the true values more than 95% of the time.

Since long spreading codes are used for the uplink, the set of spreading sequences is different at each power update. Fig. 4 shows the transmit powers for users 1,4 and 8 when a different set of spreading codes is used for each iteration. It is seen that because the UPC method is independent of the spreading codes, the power values stay constant after the first couple of iterations. The power values obtained by the SIR-based approach, on the other hand, fluctuate around the large-system values as the spreading codes change from one iteration to another. Their time averages, however, are very close to the power values obtained by the UPC scheme.

Next, we look at power control for the maximum likelihood (ML) multiuser detector which is the jointly optimal detector (see [9]). We use the UPC algorithm to update each user's power. The transmit powers for users 1,4, and 8 are shown in Fig. 5. The convergence is clear from the figure. It is seen that the steady state power values are smaller than those for the MMSE detector. This makes sense because the ML receiver is more effective in demodulating each user's signal and hence re-

TABLE I

Summary of Results for the MMSE Detector ($N = 32$ and $K = 8$)

User #	$p_{final}[mW]$ (UPC)	$p_{final}^{avg}[mW]$ (SIR-based)	$\epsilon_{diff}[mW]$
1	0.193	0.189	0.0232
2	0.274	0.268	0.0338
3	0.377	0.369	0.0460
4	0.508	0.501	0.0663
5	0.669	0.656	0.0816
6	0.866	0.857	0.112
7	1.104	1.086	0.144
8	1.387	1.366	0.178

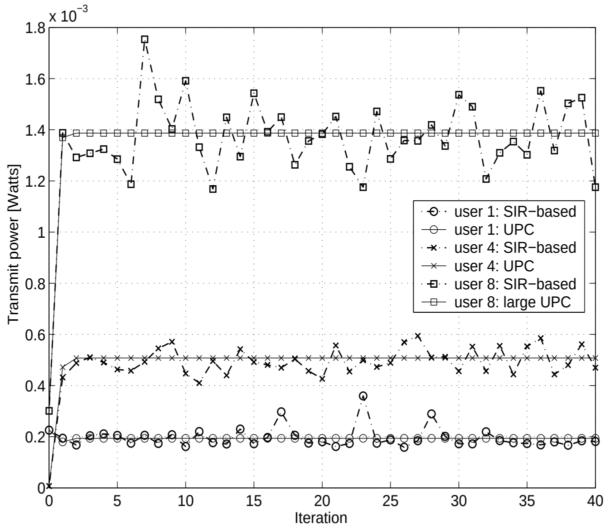

Fig. 4. Users' Transmit Power for MMSE Detector ($N = 32$ and $K = 8$)

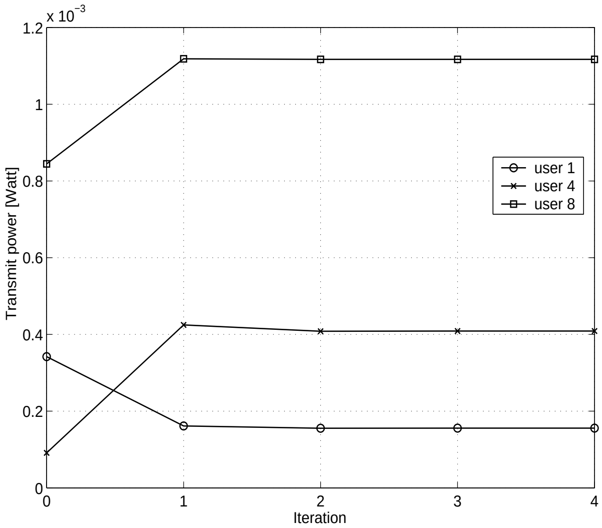

Fig. 5. Users' Transmit Power for Maximum Likelihood Detector using UPC Algorithm ($N = 32$ and $K = 8$)

quires less power to achieve the same target SIR.

VI. Conclusion

In this paper, we have presented a unified approach to power control in CDMA systems. The proposed approach exploits the linear relationship between the transmit power and output SIR in the large-system limit. The proposed power control algorithm is applicable for a large family of receivers including the matched filter, decorrelator, and MMSE detectors as well as the individually and jointly optimal multiuser detectors. We have used simulations to show the convergence and performance of the unified power control algorithm in finite-size systems of practical interest.

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