

Optimization of Training and Feedback with Beamforming for MIMO Channel

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Abstract— We examine the capacity of beamforming over a block Rayleigh fading Multi-Input/Multi-Output (MIMO) channel with finite training for channel estimation and limited feedback. A fixed-length packet is assumed, which is spanned by T training symbols, B feedback bits, and the data symbols. The training symbols are used to obtain a Minimum Mean Squared Error (MMSE) estimate of the channel matrix. Given this estimate, the receiver selects a transmit beamforming vector from a codebook containing 2^B *i.i.d.* random vectors, and relays the corresponding B -bit index back to the transmitter. We derive bounds on the capacity. For a large number of transmit antennas N_t , the optimal T , which maximizes the capacity, increases as $N_t/\log N_t$ while the optimal B increases as $N_t/\log^2 N_t$.

I. INTRODUCTION

Adding multiple antennas at the transmitter and receiver creates a Multi-Input/Multi-Output (MIMO) channel whose capacity is increased substantially over a Single-Input/Single-Output channel [1], [2]. The gain in capacity depends on the number of antennas and also whether either the transmitter or receiver is able to track the channel. In practice, the receiver estimates channel coefficients from a known training sequence. References [3], [4] studied the effect of training on the channel capacity and how much training is needed to achieve a target rate. In some situations, the transmitter may be able to obtain channel information from the receiver via a low-rate feedback channel. Several feedback schemes have been proposed and analyzed recently [5]–[13]. Here we study the capacity of beamforming for a MIMO channel with limited feedback *and* training. A Multi-Input/Single-Output (MISO) channel was considered in [14].

We consider an *i.i.d.* block Rayleigh fading channel with N_t transmit and N_r receive antennas. The $N_t N_r$ channel parameters are stationary within each block, and are independent from block to block. The block size is assumed to be constant, and the transmitted codewords span many blocks, so that the maximum achievable rate is the ergodic capacity. Each coherence block contains T training symbols and D data symbols. Furthermore, we assume that after transmission of the training symbols, the transmitter waits for the receiver to relay B bits over a feedback channel, which specify a

particular beamforming vector. This delay, in addition to the T training symbols, is counted as part of the packet overhead.

We assume that the receiver computes a Minimum Mean Squared Error (MMSE) estimate of the channel, based on the training symbols, and uses the noisy channel estimate to choose a transmit beamforming vector. A *Random Vector Quantization (RVQ)* scheme is assumed [15] in which the beamformer is selected from a codebook consisting of 2^B random vectors, which are independent and isotropically distributed, and known *a priori* at the transmitter and receiver. The associated codebook index is relayed using B bits via a noiseless feedback channel to the transmitter. The capacity of this scheme with perfect channel estimation is analyzed in [10], [11]. It is shown in [10], [11] that the RVQ codebook is optimal (i.e., maximizes the capacity) in the large system limit in which N_t , N_r , and B tend to infinity with fixed feedback bits per degree of freedom $\bar{B} = B/N_t$ and $\bar{N}_r = N_r/N_t$. RVQ has been observed to give excellent performance for systems with small N_t [16].

The capacity with MMSE channel estimates at the receiver (with or without limited feedback) is unknown. We derive upper and lower bounds on the capacity with RVQ and limited training, which are implicit functions of the number of training symbols T and feedback bits B . Given a fixed block size, or packet length L , we then optimize the capacity bounds over B and T . Namely, small T leads to a poor channel estimate, which decreases capacity, whereas large T leads to an accurate channel estimate, but leaves few symbols in the packet for transmitting the message. This tradeoff has been studied in [3], [17] for MIMO channels without feedback. Here there is also an optimal amount of feedback B , which increases with the training interval T . That is, more feedback is needed to quantize more accurate channel estimates. A similar optimization problem was studied for MISO channel in [14]. However, those results for MISO channel can not be generalized to MIMO channel.

We show that the optimal T/N_t and B/N_t , which maximize the *capacity* tend to zero at the rate of $1/\log(N_t)$ and $1/\log^2(N_t)$, respectively as $L \rightarrow \infty$ with fixed L/N_t . For large N_t , the packet overhead devoted to feedback should be less than that devoted to training and the ratio between the optimal feedback and training lengths tends to zero as

$1/\log(N_t)$. This is in contrast to the results for MISO channel in which the ratio between the optimal feedback and training lengths approximately equals one for large N_t [14]. The difference is due to the fact that the amount of feedback required to achieve a target rate in a MISO channel is less than that in a MIMO channel.

II. SYSTEM MODEL

We consider a point-to-point *i.i.d.* block fading MIMO channel with N_t transmit antennas and N_r receive antennas. We assume a rich scattering environment in which the channel gains across transmit antennas are independent and Rayleigh distributed. The i th $N_r \times 1$ received vector of a particular block is given by

$$\mathbf{r}(i) = \mathbf{H}\mathbf{v}b(i) + \mathbf{n}(i) \quad \text{for } 1 \leq i \leq D \quad (1)$$

where \mathbf{H} is an $N_r \times N_t$ channel matrix whose elements are independent, complex Gaussian random variables with zero mean and unit variance, \mathbf{v} is an $N_t \times 1$ unit-norm beamforming vector, b is the transmitted symbol with unit variance, \mathbf{n} is additive white Gaussian noise with covariance $\sigma_n^2 \mathbf{I}$, and D is the number of data symbols in a block.

In prior work [10], [11], we have analyzed the channel capacity with perfect channel knowledge at the receiver, but with *limited* channel knowledge at the transmitter. Specifically, a quantized beamforming vector is relayed from the receiver to the transmitter, given by

$$\mathbf{v}_H = \arg \max_{\mathbf{v}_j \in \mathcal{V}} \left\{ \log(1 + \rho \|\mathbf{H}\mathbf{v}_j\|^2) \mid \mathbf{H} \right\} \quad (2)$$

where $\rho = 1/\sigma_n^2$ is a background signal-to-noise ratio (SNR), and $\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_{2^B}\}$ is the quantization codebook, which is known at both the transmitter and receiver *a priori*. The (uncoded) index corresponding to the best beamforming vector (i.e., which maximizes the achievable rate) is relayed to the transmitter via an error-free feedback link. The capacity depends on the beamforming codebook \mathcal{V} and B . As $B \rightarrow \infty$, the \mathbf{v}_H that maximizes the capacity is the eigenvector of $\mathbf{H}^\dagger \mathbf{H}$, which corresponds to the maximum eigenvalue.

We have shown in [10], [11] that RVQ, in which the codebook vectors are independent and isotropically distributed, is optimal (i.e. maximizes capacity) in the large system limit in which $(B, N_t, N_r) \rightarrow \infty$ with fixed normalized feedback $\bar{B} = B/N_t$ and $\bar{N}_r = N_r/N_t$. The resulting capacity was shown to grow as $\log(\rho N_t)$. Although, strictly speaking, RVQ is suboptimal for a finite-size system, numerical results show that it gives excellent performance [16].

In addition to limited channel information at the transmitter, here we also account for channel estimation error at the receiver. Letting $\hat{\mathbf{H}}$ be the estimated channel matrix, we have

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{w} \quad (3)$$

where \mathbf{w} is the error matrix whose elements are *i.i.d.* with zero mean and variance σ_w^2 . Here we assume that the receiver computes the MMSE estimate of \mathbf{H} . As a result, $\hat{\mathbf{H}}$ and \mathbf{w} are independent and $\hat{\mathbf{H}}$ has zero mean and covariance $(1 - \sigma_w^2) \mathbf{I}$.

The receiver then selects $\mathbf{v}_{\hat{\mathbf{H}}}$, assuming that $\hat{\mathbf{H}}$ is the actual channel, i.e.,

$$\mathbf{v}_{\hat{\mathbf{H}}} = \arg \max_{\mathbf{v}_j \in \mathcal{V}} \left\{ \log(1 + \rho \|\hat{\mathbf{H}}^\dagger \mathbf{v}_j\|^2) \mid \hat{\mathbf{H}} \right\}. \quad (4)$$

The quality of the channel estimate depends on the number of training symbols T , and so does the capacity.

In what follows, we assume that the forward and feedback links are time-division multiplexed, and each block consists of T training symbols, B feedback bits, and D data symbols. Given that the size of each block is L symbols, we have the constraint

$$L = T + \mu B + D \quad (5)$$

where μ is a conversion factor, which relates bits to symbols. We would like to maximize the capacity of beamforming, which is the maximum mutual information between b and \mathbf{r} ,

$$\max_{T, B} \{C = E[\max_{p_b} I(\mathbf{r}; b \mid \hat{\mathbf{H}}, \mathbf{v}_{\hat{\mathbf{H}}})]\} \quad (6)$$

subject to (5), where p_b is the probability density function (pdf) for the transmitted symbol b .

III. CAPACITY OPTIMIZATION

Determining the ergodic capacity of RVQ beamforming with channel estimation in (6) appears to be intractable, so instead we derive upper and lower bounds, which are functions of D , B , and T . Similar to [14], conditioning on the actual channel matrix, in addition to the estimate, gives the upper bound

$$C \leq E[\max_{p_b} I(\mathbf{r}; b \mid \hat{\mathbf{H}}, \mathbf{v}_{\hat{\mathbf{H}}}, \mathbf{H})] \quad (7)$$

$$= E[\log(1 + \rho \|\mathbf{H}\mathbf{v}_{\hat{\mathbf{H}}}\|^2)] \quad (8)$$

$$\leq \log(1 + \rho E[\|\mathbf{H}\mathbf{v}_{\hat{\mathbf{H}}}\|^2]) \quad (9)$$

where we use the fact that the maximizing pdf in (7) is Gaussian, and apply Jensen's inequality (9). Substituting (3) into the expectation in (9) gives

$$E[\|\mathbf{H}\mathbf{v}_{\hat{\mathbf{H}}}\|^2] = \sigma_w^2 + E[\eta] \quad (10)$$

where

$$\eta \triangleq \mathbf{v}_{\hat{\mathbf{H}}}^\dagger \hat{\mathbf{H}}^\dagger \hat{\mathbf{H}} \mathbf{v}_{\hat{\mathbf{H}}} \quad (11)$$

and we use that fact that $\mathbf{v}_{\hat{\mathbf{H}}}$ and \mathbf{w} are independent. Substituting (10) into (9) gives the capacity upper bound

$$C \leq C_u = \log(1 + \rho \sigma_w^2 + \rho E[\eta]). \quad (12)$$

Similar to [14], [18], the capacity lower bound can be derived by assuming that the sum of estimation error and AWGN noise is Gaussian. Thus,

$$C \geq E \left[\log \left(1 + \frac{1}{\sigma_w^2 + \sigma_n^2} \eta \right) \right] \quad (13)$$

$$\geq C_l = (1 - c_{N_t}) \log \left(1 + \frac{\rho}{1 + \rho \sigma_w^2} E[\eta] \right) \quad (14)$$

where

$$c_{N_t} \triangleq \frac{\sigma_n}{2E[\eta]}. \quad (15)$$

and σ_η is the standard deviation for η . Eq. (14) is obtained by applying the inequality derived in [19]. We note that both the upper and lower bounds are functions of σ_w^2 and $E[\eta]$.

To estimate the channel matrix, we compute the MMSE estimate from training symbols. Reference [3] shows that using the set of beamforming vectors, which achieves the Welch bound, for training, minimizes the mean square error. The variance of the estimation error is given by [3]

$$\sigma_w^2 = \begin{cases} 1 - \frac{\bar{T}}{1+\rho^{-1}}, & \bar{T} < 1 \\ \frac{1}{1+\rho\bar{T}}, & \bar{T} \geq 1 \end{cases}. \quad (16)$$

where the normalized training length $\bar{T} = T/N_t$.

To evaluate and maximize both bounds, we need to evaluate $E[\eta]$, which is given by

$$E[\eta] = E_{\hat{\mathbf{H}}} E_{\mathcal{V}} \left[\max_{1 \leq j \leq 2^B} \{ \mathbf{v}_j^\dagger \hat{\mathbf{H}}^\dagger \hat{\mathbf{H}} \mathbf{v}_j \} | \hat{\mathbf{H}} \right]. \quad (17)$$

Since the RVQ beamforming vectors \mathbf{v}_j 's, are *i.i.d.*, the corresponding received powers $\mathbf{v}_j^\dagger \hat{\mathbf{H}}^\dagger \hat{\mathbf{H}} \mathbf{v}_j$, $j = 1, \dots, 2^B$, are also *i.i.d.* However, the pdf for $\mathbf{v}_j^\dagger \hat{\mathbf{H}}^\dagger \hat{\mathbf{H}} \mathbf{v}_j$ for given $\hat{\mathbf{H}}$ is difficult to obtain [20]. As a result, computing $E[\eta]$ analytically for any N_t , N_r and B is not tractable. Maximizing the capacity bounds over training and feedback lengths for a finite-size system therefore remains an open research problem. This motivates the asymptotic analysis in the next section. In Section IV, we show Monte Carlo simulation results for specific set of parameters.

A. Asymptotic Analysis

As $N_t \rightarrow \infty$ with fixed $\bar{N}_r = N_r/N_t$ and normalized feedback bits $\bar{B} = B/N_t$,

$$\frac{1}{N_t} \eta \rightarrow (1 - \sigma_w^2) \gamma_{\text{rvq}}^\infty \quad (18)$$

almost surely [21], where the asymptotic RVQ received power $\gamma_{\text{rvq}}^\infty$ is given by the following equations [11]. For $0 \leq \bar{B} \leq \bar{B}^*$, $\gamma_{\text{rvq}}^\infty$ satisfies

$$(\gamma_{\text{rvq}}^\infty)^{\bar{N}_r} e^{-\gamma_{\text{rvq}}^\infty} = 2^{-\bar{B}} \left(\frac{\bar{N}_r}{e} \right)^{\bar{N}_r} \quad (19)$$

and for $\bar{B} \geq \bar{B}^*$,

$$\gamma_{\text{rvq}}^\infty = (1 + \sqrt{\bar{N}_r})^2 - \exp \left\{ \frac{1}{2} \bar{N}_r \log(\bar{N}_r) - (\bar{N}_r - 1) \right. \\ \left. \times \log(1 + \sqrt{\bar{N}_r}) + \sqrt{\bar{N}_r} - \bar{B} \log(2) \right\} \quad (20)$$

where

$$\bar{B}^* = \frac{1}{\log(2)} \left(\bar{N}_r \log(\sqrt{\bar{N}_r}) - \bar{N}_r \log(1 + \sqrt{\bar{N}_r}) + \sqrt{\bar{N}_r} \right). \quad (21)$$

We note that $\gamma_{\text{rvq}}^\infty$ is a function of only \bar{B} and \bar{N}_r . The limit in (18) implies that

$$E[\eta] = (1 - \sigma_w^2) \gamma_{\text{rvq}}^\infty N_t + \kappa_{N_t} \quad (22)$$

where $\kappa_{N_t}/N_t \rightarrow 0$ as $N_t \rightarrow \infty$. Determining κ_{N_t} explicitly is difficult and is an open problem. Substituting (22) and (16) into (12) and (14) gives the upper bound C_u and the lower bound

C_l as functions of \bar{T} and \bar{B} . With D transmitted symbols in an L -symbol packet, the effective capacity $\mathcal{C} = (\bar{D}/\bar{L})C$ where the normalized data symbols $\bar{D} = D/N_t$ and the normalized packet length $\bar{L} = L/N_t$.

We would like to maximize the associated bounds

$$\max_{\bar{T}, \bar{B}, \bar{D}} C_l = \frac{\bar{D}}{\bar{L}} C_l, \quad (23)$$

$$\max_{\bar{T}, \bar{B}, \bar{D}} C_u = \frac{\bar{D}}{\bar{L}} C_u, \quad (24)$$

$$\text{subject to } \bar{T} + \mu \bar{B} + \bar{D} = \bar{L}. \quad (25)$$

The solutions to these optimization problems lead to the following Theorem, which characterizes the asymptotic behavior of the *actual* capacity \mathcal{C} .

Theorem 1: Let $\{\bar{T}^o, \bar{B}^o, \bar{D}^o\} = \arg \max_{\{\bar{T}, \bar{B}, \bar{D}\}} \mathcal{C}$ subject to (25) and \mathcal{C}^o be the maximized capacity. As $(N_t, N_r) \rightarrow \infty$ with fixed $\bar{N}_r = N_r/N_t$,

$$\bar{T}^o \log(N_t) \rightarrow \bar{L}, \quad (26)$$

$$\bar{B}^o \log^2(N_t) \rightarrow \frac{\bar{L}^2 \log(2)}{2\mu^2 \bar{N}_r}, \quad (27)$$

$$\frac{\bar{D}^o}{1 - \frac{1}{\log(N_t)} - \left(\frac{\bar{L} \log(2)}{2\bar{N}_r \mu} \right) \frac{1}{\log^2(N_t)}} \rightarrow \bar{L} \quad (28)$$

and the capacity satisfies

$$C^o - \log(\rho N_t) + \log(\log(N_t)) \rightarrow \xi \quad (29)$$

where

$$\xi^* - \log(1 + \rho) \leq \xi \leq \xi^* \quad (30)$$

and $\xi^* = \log(\rho \bar{L} \bar{N}_r) - \log(\rho + 1) - 1$.

As N_t tends to infinity, the capacity with limited training and feedback increases as $\log(\rho N_t) - \log(\log(N_t))$. Compared with the MISO channel for large N_t [14], the optimized rate for the MIMO channel is larger by $\log(\log(N_t))$. However, the optimal training length for both MISO and MIMO channels increases to infinity at the same rate, which is $N_t/\log(N_t)$. The optimal feedback lengths for MIMO channel and MISO channel [14] increase to infinity at the rates of $N_t/\log^2 N_t$ and $N_t/\log N_t$, respectively.

From Theorem 1, the ratio between the optimal feedback and training lengths for MIMO channel tends to zero

$$\frac{\mu \bar{B}^o}{\bar{T}^o} \asymp \frac{\bar{L} \log(2)}{2\mu \bar{N}_r \log(N_t)} \rightarrow 0 \quad (31)$$

where $a(N_t) \asymp b(N_t)$ is equivalent to $a(N_t)/b(N_t) \rightarrow 1$ as $N_t \rightarrow \infty$. This differs from the result for the MISO channel in which $\mu \bar{B}^o/\bar{T}^o \rightarrow 1$. The difference can be attributed to the fact that the amount of normalized feedback needed for the MIMO channel is less than that needed for the MISO channel to achieve a desired asymptotic rate as N_t increases. Eq. (31) implies that the fraction of the packet dedicated to training should be more than that dedicated to feedback as N_t and N_r/N_t increase.

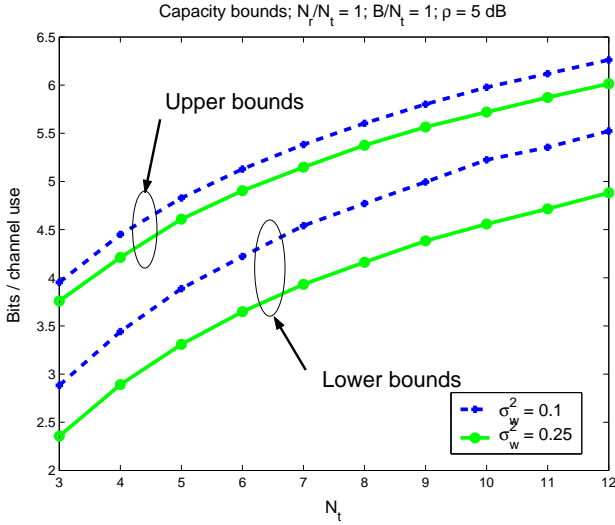


Fig. 1. Capacity bounds (bits/channel use) (12) and (14) versus number of transmit antennas with $\sigma_w^2 = 0.1$ and 0.25 .

IV. NUMERICAL RESULTS

Fig. 1 shows the upper and lower bounds (12) and (14) with $\sigma_w^2 = 0.1$ and 0.25 , $\bar{B} = 1$, $\bar{N}_r = 1$, and $\rho = 5$ dB. To obtain c_{N_t} and $E[\eta]$ in (12) and (14), we use Monte Carlo simulations. Both bounds on the capacity with beamforming grow logarithmically with N_t as expected. The gap between the upper and lower bounds narrows as the variance for estimation error σ_w^2 decreases. With $\sigma_w^2 = 0$, the upper bound is equal to the lower bound, and is the actual capacity. We note that the gap between the bounds also decreases with larger \bar{B} . Since RVQ requires exhaustive search, and the number of entries in the codebook grows exponentially with the number of antennas, simulation results are not shown for $N_t > 12$.

We simulate the capacity lower bound with a range of possible values of B and T and select B_l^o and T_l^o , which maximize the lower bound. Fig. 2 shows $\{\bar{T}_l^o, \bar{B}_l^o, \bar{D}_l^o\}$ normalized by the packet length L versus N_t for $\bar{N}_r = 2$, $\bar{L} = 50$, $\mu = 1$, and $\rho = 5$ dB. As predicted by Theorem 1, both the optimal \bar{T} and \bar{B} decrease to zero, and \bar{D} increases to \bar{L} as $N_t \rightarrow \infty$. The associated lower bound is shown in Fig. 3 with a dashed line. We also compare the optimized bound to a lower bound with a heuristic choice of parameters ($\bar{B} = 1$ and $\bar{T} = 1.5$) shown by a dashed-dot line. For $N_t = 3$, the bound with optimal parameters is approximately 10% greater than that with the heuristic choice. Also shown in Fig. 3 is the capacity with perfect channel knowledge at the transmitter and receiver. The performance with limited feedback and training is substantially less than that with perfect channel knowledge. For $N_t = 3$, the capacity with perfect channel knowledge is about 40% larger than the rate with optimized feedback and training lengths. The solid curve with dots is the capacity with perfect channel estimation and B_l^o feedback bits. Here we see a substantial gain relative to the solid line, since with perfect channel knowledge the receiver does not require training overhead.

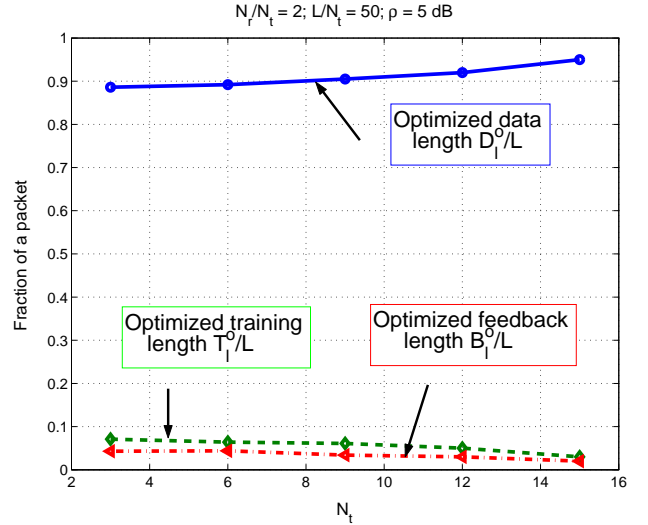


Fig. 2. $\{\bar{T}_l^o/\bar{L}, \bar{B}_l^o/\bar{L}, \bar{D}_l^o/\bar{L}\}$ versus number of transmit antennas N_t .

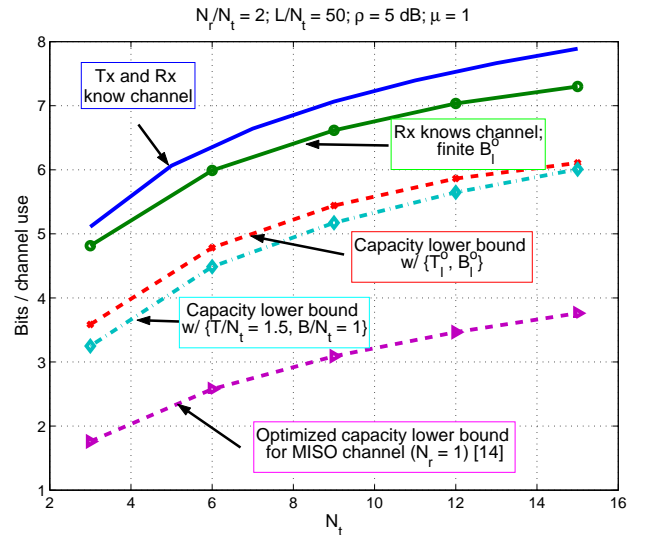


Fig. 3. Achievable rate versus number of transmit antennas N_t with different assumptions about channel knowledge at the receiver and transmitter. Also shown is the optimized capacity lower bound for MISO channel [14]

We also include in the figure the capacity lower bound for MISO channel with optimized training and feedback lengths [14], which is shown to be substantially lower than that for MIMO channel with $\bar{N}_r = 2$. As \bar{N}_r increases, we expect the gap between the optimized lower bounds for MISO and MIMO channels to increase.

Fig. 4 shows the capacity lower bound versus total overhead $(\bar{T} + \mu\bar{B})/\bar{L}$. The capacity is zero when $\bar{T} + \bar{B} = 0$, since the estimate is uncorrelated with the channel, and when $\bar{T} + \bar{B} = \bar{L}$, since $\bar{D} = 0$. The solid line corresponds to optimized parameters with $\bar{L} = 10$, $N_t = 9$, $\bar{N}_r = 2$, $\mu = 1$, and $\rho = 5$ dB. Different curves correspond to different ratios between \bar{T} and \bar{B} . With equal amounts of training and feedback, the rate is almost equal to that with optimized parameters with the peak

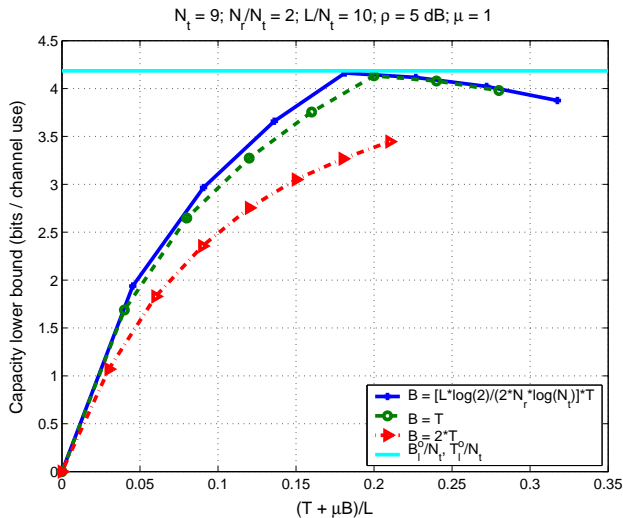


Fig. 4. Lower bound on capacity versus normalized training and feedback $(\bar{T} + \mu\bar{B})/\bar{L}$.

achieved when $(\bar{T} + \bar{B})/\bar{L} = 0.2$. Allocating overhead according to Theorem 1, i.e., $\mu\bar{B}/\bar{T} = \bar{L} \log(2)/(2\mu\bar{N}_r \log(N_t))$ performs marginally better than allocating equal training and feedback. As N_t increases, we expect the gap between the two overhead allocations to increase. The performance degrades when \bar{B} deviates significantly from \bar{T} (e.g., $\bar{B} = 2\bar{T}$). The three curves shown are not extended to $(\bar{T} + \bar{B})/\bar{L} = 1$ since simulation complexity increases exponentially with \bar{B} due to RVQ's exhaustive search.

V. CONCLUSION

We have presented bounds on the capacity of a MIMO block Rayleigh fading channel with beamforming, assuming limited training and feedback. For a large number of transmit antennas, we have characterized the optimal amount of training and feedback as a fraction of the packet duration, assuming linear MMSE estimation of the channel, and an RVQ codebook for quantizing the beamforming vector. Our results show that when optimized, the fractions of the packet devoted to training and feedback tend to zero at the rates of $1/\log N_t$ and $1/\log^2 N_t$, respectively, as $N_t \rightarrow \infty$. This is in contrast to the MISO channel in which both training and feedback lengths tend to zero at the rate of $1/\log N_t$ [14]. We showed that allocating packet overhead according to the ratio $\mu\bar{B}/\bar{T} = \bar{L} \log(2)/(2\mu\bar{N}_r \log(N_t))$, which is obtained by the asymptotic analysis, can achieve close to the optimal performance for a finite-size system.

Although the pilot-based scheme considered is practical, it is most likely suboptimal. Namely, in the absence of feedback such a pilot-based scheme is strictly suboptimal, although it is nearly optimal at high SNRs [3]. With feedback the capacity of the block fading MIMO channel considered (i.e., no channel knowledge at the receiver and transmitter) is unknown. Extensions of the model presented here include allocating different powers for the training and data portions.

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