

Adaptive Allocation of Pilot and Data Power for Time-Selective Fading Channels with Feedback

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Abstract—We consider data transmission through a time-selective (correlated) flat Rayleigh fading channel under an average power constraint. The channel is estimated at the receiver with a pilot signal, and the estimate is fed back to the transmitter. The estimate is used for coherent demodulation, and to adapt the data and pilot powers. We start with a block fading channel in which the channel gain changes according to a Gauss-Markov process. The channel estimate is updated during each coherence block with a Kalman filter, and optimizing the data and pilot powers is formulated as a dynamic program. We then study a continuous limit in which the coherence time tends to zero, and the correlation between successive channel gains tends to one, so that the channel process becomes a diffusion process. In this limit it is shown that the optimal pilot power control policy is “bang-bang”, i.e., depending on the current system state (channel estimate and associated error variance) the pilot power is either the maximum allowable, or zero. The associated regions of the state space are illustrated numerically for specific system values. This example shows that the achievable rate with the optimized training policy provides substantial gains relative to constant training power at low SNRs.

I. INTRODUCTION

The achievable rate for a time-selective fading channel depends on what channel state information (CSI) is available at the receiver and transmitter. Namely, CSI at the receiver can increase the rate by allowing coherent detection, and CSI at the transmitter allows adaptive rate and power control (e.g., see [1, Ch. 6]). Obtaining CSI at the receiver and/or transmitter requires overhead in the form of a pilot signal and feedback.

We consider a time-selective flat Rayleigh fading channel, which is unknown at both the receiver and transmitter. The transmitter divides its average power between a pilot, used to estimate the channel at the receiver, and the data. Given an average transmitted power constraint, our problem is to optimize the instantaneous pilot and data powers over the channel realization. Our objective function is a lower bound on the achievable rate. Similar problems have been considered in previous work, e.g., [2]; however, our model differs in two key respects. First, the channel evolves as a correlated process with known statistics. Second, the estimated CSI is assumed to be available at the transmitter (i.e., through a noiseless feedback channel). The transmitter uses this CSI to control the data and pilot powers. Namely, because the channel is correlated

in time, adapting the pilot power with the estimated channel state can increase the achievable rate.

We start with a correlated block fading model in which the sequence of channel gains is Gauss-Markov with known statistics. The channel estimate is updated at the beginning of each block using a Kalman filter, and determines the power for the data, and the power for the pilot symbols in the succeeding coherence block. We show that optimal power control policies are specified implicitly through a Bellman equation [10]. Other dynamic programming formulations of power control problems have been presented in [11]–[14], although in that work the channel is either known perfectly (perhaps with a delay), or is unknown and not measured.

Because an analytical solution to the Bellman equation appears to be difficult to obtain, we study a continuous limit in which the channel coherence time, or block length, tends to zero, and the correlation between successive blocks tends to one. In this limit, the Gauss-Markov channel becomes a continuous-time Ornstein-Uhlenbeck process [5], and the Bellman equation becomes a partial differential equation (PDE). A diffusion equation is also derived, which describes the evolution of the state (channel estimate and the associated error variance), given a power allocation policy. In this limit, we show that given a peak power constraint for the pilot power, the optimal pilot power control policy is “bang-bang”: the pilot power is either the maximum allowable or zero, depending upon the current state. Also, the optimal data power control policy is found to be a variation of waterfilling [1].

The boundary of the state space, which specifies the optimal pilot power control, can be obtained by solving the corresponding PDE. This problem falls into a class of *free boundary problems* [6], [7], for which obtaining a numerical solution is challenging. We reformulate this problem as a variational inequality [7], which can be solved numerically as a quadratic optimization problem. This technique is used to compare achievable rates with the optimal pilot power control policy with constant pilot power for a particular set of system parameters corresponding to fast fading. Our results show that the optimal policy offers substantial gains at low SNRs. Furthermore, this improvement increases as the channel correlation increases (i.e., the channel varies more slowly).

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II. CORRELATED BLOCK FADING MODEL

We start with a block fading channel model in which each coherence block contains M symbols, consisting of T pilot symbols and D data symbols. The vector of channel outputs for coherence block i is given by

$$\mathbf{y}_i = h_i \begin{pmatrix} \sqrt{P_{i;T}} \mathbf{s}_{i;T} \\ \sqrt{P_i} \mathbf{s}_i \end{pmatrix} + \mathbf{z}_i \quad (1)$$

where $\mathbf{s}_{i;T}$ and \mathbf{s}_i are, respectively, vectors containing the pilot and data symbols, each with unit variance, and $P_{i;T}$ and P_i are the associated pilot and data powers. The noise \mathbf{z}_i contains circularly symmetric complex Gaussian (CSCG) random variables, and is white with covariance $\sigma_z^2 \mathbf{I}$. The channel gain h_i is also CSCG, is constant within the block, and evolves from block to block according to a Gauss-Markov process, i.e.,

$$h_{i+1} = r h_i + \sqrt{1-r^2} w_i \quad (2)$$

where w_i is an independent CSCG random variable with mean zero and variance σ_h^2 , and $r \in [0, 1]$ determines the correlation between successive blocks. We will assume that r and σ_h^2 are known at the receiver.

The training energy per symbol in block i is defined as $\epsilon_i = \alpha P_{i;T}$, where $\alpha = T/M$. In what follows, it will be convenient to write $P_{i;T}$ as ϵ_i/α .

The receiver updates the channel estimate during each coherence block with a Kalman filter [8], using the pilot symbols, and relays the estimate back to the transmitter. The feedback occurs between the pilot and data symbols, and is assumed to occupy an insignificant fraction of the coherence time. The channel estimate \hat{h}_i and estimation error $\theta_i = E(|h_i|^2) - E(|\hat{h}_i|^2)$ evolve according to the following updates:

$$\hat{h}_{i+1} = r \hat{h}_i + g_{i+1} \sqrt{\frac{\epsilon_{i+1}}{\alpha}} T e_{i+1|i} + g_{i+1} \mathbf{s}_{i+1;T}^H \mathbf{z}_{i+1} \quad (3)$$

$$\theta_{i+1} = \frac{\sigma_z^2 \theta_{i+1|i}}{\epsilon_i M \theta_{i+1|i} + \sigma_z^2} \quad (4)$$

where

$$g_i = \sqrt{\frac{\epsilon_i}{\alpha}} \frac{\theta_i}{\sigma_z^2} \quad (5)$$

$$e_{i+1|i} = h_{i+1} - r \hat{h}_i \quad (6)$$

$$\theta_{i+1|i} = r^2 \theta_i + (1-r^2) \sigma_h^2. \quad (7)$$

It is straightforward to show that the channel estimate \hat{h}_i in (3) does not depend on T . Hence the data rate is maximized by letting $T \rightarrow 0$ with fixed ϵ_i (i.e., the training power $P_{i;T} \rightarrow \infty$).

We wish to determine P_i and ϵ_i , which maximize the achievable rate. Specifically, the channel estimate \hat{h}_i and variance θ_i determine the data power P_i and the pilot power in the *next* coherence block ϵ_{i+1} . We assume that the transmitter codes over many coherence blocks, and use the following

lower bound on ergodic capacity as the performance objective [3], [4]

$$R(P_i, \hat{\mu}_i, \theta_i) = \log \left(1 + \frac{P_i \hat{\mu}_i}{P_i \theta_i + \sigma_z^2} \right) \quad (8)$$

where $\hat{\mu}_i = |\hat{h}_i|^2$.

III. DYNAMIC PROGRAMMING FORMULATION

The pilot and data power control problem can be stated as

$$\begin{aligned} & \max_{\{P_i, \epsilon_i\}} \liminf_{n \rightarrow \infty} \frac{1}{n} E \left[\sum_{i=0}^{n-1} R(P_i, \hat{\mu}_i, \theta_i) \right] \\ & \text{subject to: } \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=0}^{n-1} \epsilon_i + \frac{1}{n} \sum_{i=0}^{n-1} P_i \right] \leq P_{av} \end{aligned} \quad (9)$$

where the expectation is over the sequence of channel gains. This is a discrete-time Markov control problem, and the solution can be formulated as a dynamic program. The system state at time (block) i is $S_i = (\hat{\mu}_i, \theta_i)$, and the action maps the state to the power pair (P_i, ϵ_{i+1}) . To see that S_i is the system state, note that $e_{i+1|i}$ and \hat{h}_i are independent random variables, hence it follows from (3) and (4) that the probability distribution of S_{i+1} is determined only by S_i and the action ϵ_{i+1} . The process $\{(\hat{\mu}_i, \theta_i)\}$ is therefore a Markov chain driven by the control $\{\epsilon_i\}$.

The average power constraint in (9) can be included in the objective through a Lagrange multiplier giving the relaxed problem

$$\max_{(P_i, \epsilon_i)} \lim_{n \rightarrow \infty} \frac{1}{n} E \left[\sum_{i=0}^{n-1} [R(P_i, \hat{\mu}_i, \theta_i) - \lambda (\epsilon_i + P_i)] \right] \quad (10)$$

where λ is chosen to enforce the constraint (9). If there exists a bounded function $V(\hat{\mu}, \theta)$ and a constant C , which satisfy the Bellman equation

$$V(\hat{\mu}, \theta) + C = \max_{(P, \epsilon)} [R(P, \hat{\mu}, \theta) - \lambda (\epsilon + P) + E_{\epsilon, (\hat{\mu}, \theta)}[V]] \quad (11)$$

then an optimal policy maximizes the right-hand side [10]. The function $V(\cdot, \cdot)$ is called an ‘‘auxiliary value function’’, and C is the maximum value of the objective in (10). The expectation $E_{\epsilon, (\hat{\mu}, \theta)}[\cdot]$ is over the conditional probability density of S_{i+1} given $S_i = (\hat{\mu}, \theta)$ and action $\epsilon_{i+1} = \epsilon$.

Using the channel state evolution equations derived in Section II, we have

$$E_{\epsilon, (\hat{\mu}, \theta)}[V] = \int_0^\infty V(u, \theta_{(i+1)}) f_r(u) du \quad (12)$$

where $f_r(u)$ is the conditional density of $\hat{\mu}_{i+1} = |\hat{h}_{i+1}|^2$ given $S_i = (\hat{\mu}_i, \theta_i) = (\hat{\mu}, \theta)$, and θ_{i+1} is given by (4) with θ_i replaced by θ . From (3) it follows that $f_r(u)$ is Ricean with noncentrality parameter $r^2 \hat{\mu}$ and variance $[(\theta_{i+1|i} \epsilon M) / \sigma_z^2] \theta_{i+1} + r^2 \hat{\mu}$, where θ_{i+1} and $\theta_{i+1|i}$ are given by (4) and (7), respectively, with θ_i replaced by θ .

IV. DIFFUSION LIMIT

The Bellman equation (11) is an integral fixed point equation, and appears to be difficult to solve analytically. To gain insight into properties of optimal policies, we consider a diffusion limit in which $M = \delta t$, $r = 1 - \rho(\delta t)$, and $\delta t \rightarrow 0$. That is, the coherence time goes to zero, and the correlation between adjacent coherence blocks goes to one at a specific rate determined by ρ .

In this limit, it can be shown that the discrete-time, complex, Gauss-Markov process $\{h_i\}$, described by (2), converges weakly to a continuous-time Ornstein-Uhlenbeck diffusion process $h(t)$ (e.g., see [9, Ch. 8]). Furthermore, the limiting channel process satisfies the stochastic differential equation (SDE)

$$dh(t) = -\rho h(t) dt + \sqrt{2\rho}\sigma_h dB(t) \quad (13)$$

where $B(t)$ is complex Brownian motion, and we assume that the initial state $h(0)$ is a CSCG random variable with zero mean and variance σ_h^2 . This is a stationary Gauss-Markov process, which is continuous in probability, and has autocorrelation function

$$\Phi(\tau) = \sigma_h^2 e^{-\rho\tau} \quad (14)$$

where τ is the lag normalized by the symbol duration. Hence ρ determines how fast the channel varies relative to the symbol rate.

In the diffusion limit considered, the Kalman filter continuously estimates the channel during each symbol, and the pilot and data powers are continuously updated. The channel estimate and estimation error updates given by (3) and (4), respectively, become the dynamical equations

$$\begin{aligned} d\hat{h}(t) &= \left[-\rho \hat{h}(t) + \theta(t) \frac{\epsilon(t)}{\sigma_z^2} (h(t) - \hat{h}(t)) \right] dt \\ &+ \left[\theta(t) \sqrt{\frac{\epsilon(t)}{\sigma_z^2}} \right] d\bar{B}(t) \end{aligned} \quad (15)$$

$$\frac{d\theta(t)}{dt} = -2\rho\theta(t) - \frac{\epsilon(t)\theta^2(t)}{\sigma_z^2} + 2\rho\sigma_h^2 \quad (16)$$

where $\bar{B}(t)$ is a complex Brownian process independent of $B(t)$, and $\epsilon(t)$ is the pilot power at time t . Furthermore, an SDE defining the evolution of $\hat{\mu}(t) = |\hat{h}(t)|^2$ can be obtained through an application of Ito's lemma [5].

If the average data power at time t is $P(t)$, then the achievable data rate corresponding to $h(t)$ is $R[P(t), \hat{\mu}(t), \theta(t)]$, where $R(\cdot)$ is given by (8). Our problem is to choose $\epsilon(t)$ and $P(t)$, given the state $(\hat{\mu}(t), \theta(t))$, to maximize the achievable data rate averaged over the channel process $h(t)$. Analogous to (9) we have the continuous-time control problem

$$\begin{aligned} &\max_{(P(t), \epsilon(t))} \liminf_{t \rightarrow \infty} \frac{1}{t} E \left[\int_0^t R(P(t), \hat{\mu}(t), \theta(t)) dt \right] \\ &\text{subject to: } \lim_{t \rightarrow \infty} \left[\frac{1}{t} \int_0^t \epsilon(t) dt + \frac{1}{t} \int_0^t P(t) dt \right] \leq P_{av} \end{aligned} \quad (17)$$

Analogous to (11), the Bellman equation can be written as

$$C = \max_{(P, \epsilon)} [R(P, \hat{\mu}, \theta) - \lambda(\epsilon + P) + A_\epsilon[V(\hat{\mu}, \theta)]] \quad (18)$$

where A_ϵ is generator of the state process $(\hat{\mu}(t), \theta(t))$ with pilot power $\epsilon(t)$ [5], and is given by

$$A_\epsilon[V] = \frac{E[dV]}{dt} = a + \epsilon b \quad (19)$$

where

$$a = \frac{\partial V}{\partial \hat{\mu}} [-2\rho\hat{\mu}] + \frac{\partial V}{\partial \theta} [-2\rho\theta + 2\rho\sigma_h^2] \quad (20)$$

$$b = \frac{\theta^2}{\sigma_z^2} \left[\frac{\partial V}{\partial \hat{\mu}} - \frac{\partial V}{\partial \theta} + \hat{\mu} \frac{\partial^2 V}{\partial \hat{\mu}^2} \right] \quad (21)$$

and the dependence on t is omitted for convenience. Here we ignore existence issues, and simply assume that there exists a bounded, continuous, and twice differentiable function $V(\cdot, \cdot)$ satisfying (18). Note that $V(\cdot, \cdot)$ is unique up to a constant [10].

Theorem 1: Given a limitation on the peak pilot power, i.e., $\epsilon \in [0, \epsilon_{max}]$, the optimal pilot power control policy is given by

$$\epsilon^* = \begin{cases} \epsilon_{max} & \text{if } b - \lambda > 0 \\ 0 & \text{if } b - \lambda \leq 0. \end{cases} \quad (22)$$

In words, the optimal pilot power control policy is bang-bang. This follows immediately from substituting the generator A_ϵ , given by (19)-(21), into (18), i.e.,

$$C = f(\hat{\mu}, \theta, \lambda) + \max_{\epsilon} [a + \epsilon(b - \lambda)] \quad (23)$$

where $f(\hat{\mu}, \theta, \lambda) = \max_P [R(P, \hat{\mu}, \theta) - \lambda P]$. Substituting (22) into (23) gives the final version of the Bellman equation

$$C = f(\hat{\mu}, \theta, \lambda) + a + \epsilon_{max}(b - \lambda)^+ \quad (24)$$

where $(x)^+ = \max\{0, x\}$. We remark that an alternative way to arrive at (23) and (24) is to pass the discrete-time Bellman equation (11) through the continuous-time limit.

It is easily shown that the optimal data power allocation is

$$P^* = \arg \max_P [R(P, \hat{\mu}, \theta) - \lambda P] \quad (25)$$

$$= \left(\frac{-\lambda\sigma_z^2(2\theta + \hat{\mu}) + \sqrt{\Delta}}{2\lambda\theta(\hat{\mu} + \theta)} \right)^+ \quad (26)$$

where $\Delta = \lambda^2 \hat{\mu}^2 \sigma_z^4 + 4\lambda \hat{\mu}^2 \theta \sigma_z^2 + 4\theta^2 \hat{\mu} \lambda \sigma_z^2$. Note that $P^* > 0$ for $\hat{\mu} > \lambda \sigma_z^2$. Finally, λ determines P_{av} in (17). We note that this power allocation is the same as that obtained in [15], which considers a fading channel with constant estimation error, as opposed to the time-varying estimation error in our model.

V. BEHAVIOR OF THE OPTIMAL POLICY

The boundary of the region, which defines the optimal pilot power control policy, hereafter called ‘‘free boundary’’, remains to be determined. Nevertheless, we can make the following observations.

Fig 1 illustrates the optimized pilot power control policy. The vertical and horizontal axes correspond to the channel

estimate $\hat{\mu}$ and estimation error variance θ , respectively. The shaded region, D_ϵ , is the region of the state space in which $\epsilon = \epsilon_{max}$, and $\epsilon = 0$ in the complementary region D_0 . These two regions are separated by the free boundary, AC . The penalty factor λ determines the position of this boundary, and the associated value of P_{av} .

The vertical line $A'A''$ in the figure corresponds to the estimation error variance θ^* , which results from taking $\epsilon = \epsilon_{max}$ for all t . Clearly, in steady state the estimation error variance cannot be lower than this value, hence the steady-state pdf of the state (μ, θ) is zero for $\theta < \theta^*$. Substituting $\epsilon = \epsilon_{max}$ in (16) and setting $\frac{d\theta}{dt} = 0$ gives $\theta^* = (\sqrt{1 + 2\sigma_h^2\gamma} - 1)/\gamma$ where $\gamma = \epsilon_{max}/(\rho\sigma_z^2)$.

Suppose that the initial state is in D_0 . With $\epsilon(t) = 0$ the equations (15) and (16) become $d\hat{h}(t) = -\rho\hat{h}(t)dt$ and $d\theta(t)/dt = -2\rho(\theta(t) - \sigma_h^2)$. This implies that the state trajectory is a straight line towards the point Z until it hits the free boundary, as illustrated in Fig. 1. If it hits the free boundary below the point B , then it is pushed back into D_0 . Otherwise, it continues into D_ϵ and settles along the line $A'A''$. For a discrete state space with small, positive δt , the state trajectory zig-zags around the boundary, as shown in Fig. 1. Hence if the free boundary AC intersects $A'A''$ at point B , then in steady state, the probability mass must be concentrated along the curve $A'BC$. In the continuous-time limit, this suggests that the probability associated with states not on this curve tends to zero.

We also observe that for $P_{av} > 0$, λ must be selected so that the point Z lies in D_ϵ . Otherwise, the state trajectory eventually drifts to Z , corresponding to $\epsilon = 0$ for all t , $P_{av} = 0$, and $R = 0$ (and stays there).

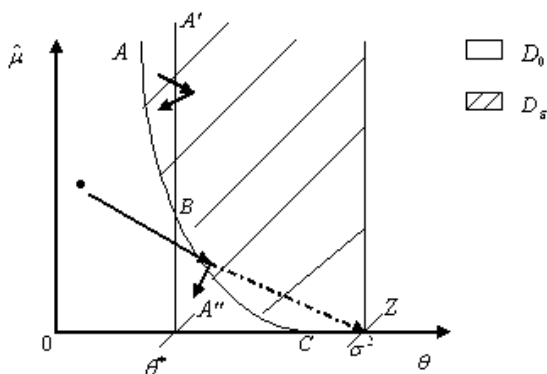


Fig. 1. System dynamics for a bang-bang pilot power control policy.

We remark that the PDE in region D_0 is a “transport equation” [6], which has an analytical solution containing an arbitrary function of a single variable. Determining this function and the constant C appears to be difficult, so that in the next section we describe a numerical approach to solving the free boundary problem.

VI. NUMERICAL SOLUTION TO FREE BOUNDARY PROBLEM

A. Optimization Formulation

Solving the free boundary PDE (24) includes specifying the domain associated with the control $\epsilon = \epsilon_{max}$. Because of this, none of the standard numerical methods for solving PDEs can be directly applied due to the absence of boundary conditions. Here we show how to obtain a solution numerically by reformulating the problem as a large scale quadratic program.

We first observe that (24) can be written as the variational inequality [7]

$$\begin{aligned} C - f - a &\geq 0 \\ C - f - a - \epsilon_{max}(b - \lambda) &\geq 0 \\ (C - f - a)(C - f - a - \epsilon_{max}(b - \lambda)) &= 0 \end{aligned} \quad (27)$$

A solution to (27) is a solution to (24) and vice versa. Now consider the following optimization problem,

$$\min w_0 \int u_1 u_2 d\theta d\hat{\mu} + \sum_{x \in X} w_x \int [(\partial_x u_1)^2 + (\partial_x u_2)^2] d\theta d\hat{\mu}$$

$$\begin{aligned} \text{Subject to: } C - f - a = u_1 &\geq 0 \\ u_1 - \epsilon_{max}(b - \lambda) = u_2 &\geq 0 \end{aligned} \quad (28)$$

where $\partial_x u_i = \frac{\partial u_i}{\partial x} dx$ for $x \in X = \{\hat{\mu}, \theta\}$ and $i = 1, 2$. If $w_0 > 0$, $w_\theta = 0$, and $w_{\hat{\mu}} = 0$, then the solution to (27) is a solution to (28). Also, a solution to (28) with zero objective value is a solution to (27). The second term in the objective function is included to regularize the numerical solution. The effect of this term can be controlled by changing the weights w_θ and $w_{\hat{\mu}}$. These weights affect both the accuracy of the results and also the rate at which the non-linear optimization algorithm converges. For the numerical results which follow, $w_0 = w_{\hat{\mu}} = 1$ and $w_\theta = 0$, which give accurate results (i.e., keep the first term in the objective small).

Fig 2 shows free boundaries obtained numerically for various values of the penalty factor (“water-level”) λ . In these examples, $\rho = 1$, which corresponds to fast fading, i.e., the correlation between channel values at the start and end of a symbol period is $1/e$. In each case the training region D_ϵ lies to the right of the boundary.

B. Capacity Comparison

To compute the achievable rate as a function of P_{av}/σ_z^2 , for given λ , we average the total (data plus pilot) power over the steady-state distribution of state variables $(\hat{\mu}, \theta)$.¹ The transition probabilities for the discretized state space can be computed from the conditional Ricean distribution on the channel gain, or the system dynamics equations (16) and (15). The steady state distribution over the state space can then be obtained from the transition probability matrix. Our numerical results confirm that for this discretized problem, the probability mass is highly concentrated along the free boundary.

¹We assume that the observed Markov process under the optimal policy is ergodic.

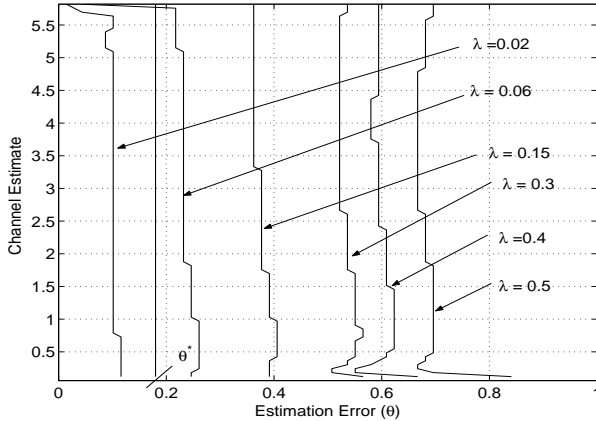


Fig. 2. Free boundary corresponding to various values of λ . System parameters are $\sigma_h^2 = 1$, $\sigma_z^2 = 0.2$ and the maximum allowable pilot power, $\epsilon_{max} = 10$ (17dB)

Fig. 3 shows plots of achievable rate versus $P_{av}\sigma_h^2/\sigma_z^2$ in dB with the following pilot power control policies: (1) optimal, corresponding to the free boundaries obtained in the previous section; (2) approximation of the optimal boundary with a vertical line; and (3) constant (optimized) pilot power with the optimized data power. The second policy takes $\epsilon = 0$ if $\theta < \theta_0$ and $\epsilon = \epsilon_{max}$ if $\theta \geq \theta_0$, where θ_0 is approximately aligned with the optimal boundary. The system parameters are the same as in the previous section. The figure shows that the optimal policy offers a significant increase in achievable rate, relative to constant pilot power, at low SNR's. We expect this gap to increase as the correlation parameter ρ decreases, since the more correlated the channel is in time, the less frequently the pilot power is set to ϵ_{max} . Also, increasing the allowable training power ϵ_{max} will also increase the gap. The rate obtained with the vertical line approximation to the free boundary is very close to optimal. Furthermore, the results are insensitive to small variations in θ_0 .

As the power budget increases, the three curves converge. This is consistent with Fig 2, which shows that the boundary corresponding to the highest P_{av} ($\lambda = 0.02$) lies to the left of the line $\theta = \theta^*$, corresponding to constant pilot power ϵ_{max} .

VII. CONCLUSIONS

We have studied the achievable rate for a flat Rayleigh fading channel, where both the data and pilot power are adapted based on estimated CSI. Taking a continuous limit in which the channel becomes a diffusion process gives a more realistic view of the channel than a correlated block fading model, and provides insight into optimal power control policies. Although determining the free boundary in which the pilot power switches between “on” and “off” is challenging, it can be computed numerically, and for the example shown, can be approximated as a vertical line with little loss in achievable rate.

Although the PDE, which specifies the free boundary appears to be difficult to solve, it may be possible to gain

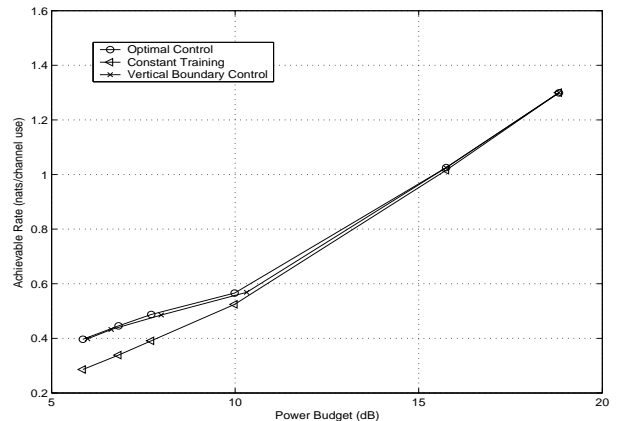


Fig. 3. Comparison of rates achieved with different pilot power control policies.

additional insight by considering various limits of the system parameters. Also, we plan to use this model to characterize optimal power control strategies over a wideband time-selective channel (e.g., containing several parallel flat fading channels, as in [4]).

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